

CHAPTER 17: SIMPLE HARMONIC MOTION

* Oscillations:

When a body moves to and fro about its mean position, then such motion is called vibratory or oscillatory motion.

OR

The complete round trip of the oscillating or vibrating body about its mean position is called vibration or oscillation.

* Terms Related to Oscillation:

⇒ Displacement:

The distance of vibrating body from mean position on either side at any instant of time.

SI unit = Meter (m)

Symbol = x

⇒ Amplitude:

The maximum displacement of a vibrating body from mean position.

SI unit = meter (m)

Symbol = x_0 or A

⇒ Time taken:

Time it takes to cover a displacement. It is the time measured for any number of cycles or events.

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SI unit = second (s)

Symbol = t

⇒ Time Period:

Time taken to complete one oscillation
or ^{one} complete round trip.

SI unit = seconds (s)

Symbol = T

⇒ Frequency:

The number of vibrations or oscillations (n)
per unit time / Number of waves per unit time / Number
of waves per second / Reciprocal of time period (T).

SI unit = Hertz (Hz)

Symbol = f

Formula = $f = \frac{n}{t}$

⇒ Angular Frequency:

Angular frequency tells how fast
something is oscillating in terms of radians per
second.

SI unit = radian per second (rads^{-1})

Symbol = ω

Formula = $\omega = \frac{2\pi}{T}$ or $2\pi f$

Jab poora
round ho jata
hai yani poori
1 oscillation hoti
hai tou uss case
mai angular velocity
angular freq
ban jaati hai

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* Angular Frequency

Definition

It describes how fast an object
oscillates/wave vibrates.

Angular Velocity

It describes how fast an
object is rotating.

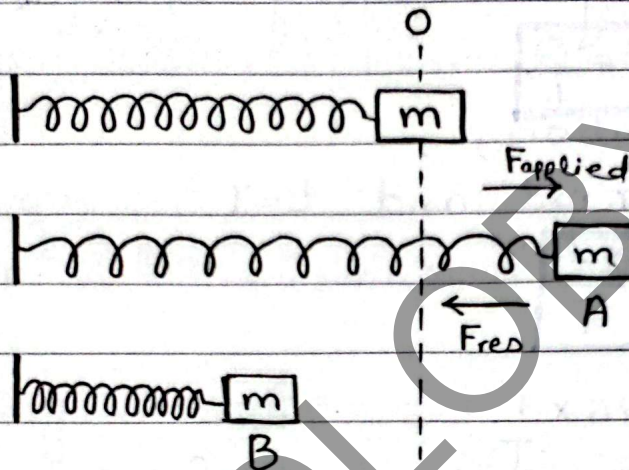
Example
It is typically used for: It would describe the speed of phenomena like spring's SHM. Spinning ferris wheel.

Unit
Its SI unit of rads^{-1} . Its SI unit is rads^{-1} .

Motion
It refers to oscillatory motion. It refers to rotational motion.

* Motion of mass attached to spring:-

• Figure:



Q Show that mass attached to spring executes SHM?

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• Derivation:

Since,

$$F_{\text{applied}} = ma \rightarrow \textcircled{i}$$

Since from Hooke's Law,

$$F_{\text{res}} = -kx \rightarrow \textcircled{ii}$$

$k = \text{Spring constant}$

Since, negative sign shows the restoring force is acting in opposite direction.

Now, from Newton's Third Law, compare eq (i) and (ii),

$$\Rightarrow ma = -kx$$

$$a = -\frac{k}{m}x$$

k = spring constant and m = mass, both are constants,

$$a = (\text{constant}) (-x)$$

$$a \propto -x$$

$$\therefore \frac{k}{m} = \text{constant} = \omega^2$$

Time Period and Frequency:

We know that angular velocity is,

$$\omega = \frac{\theta}{t}$$

Here, θ = one revolution,

$$\theta = 2\pi \text{ rad and } t = T$$

$$\omega = \frac{2\pi}{T}$$

$$\text{or } \omega = 2\pi \times \frac{1}{T}$$

$$\omega = 2\pi f$$

$$\therefore \frac{1}{T} = f$$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi} \times \omega$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \Rightarrow \text{Frequency}$$

$$T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow \text{Time Period}$$

$$\therefore f = \frac{1}{T}, T = \frac{1}{f}$$

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* Important MCQ's

→ Acceleration:

Mean Position: Min Extreme Position: Max

→ Velocity:

Mean Position: Max Extreme Position: Min

→ Potential Energy:

Mean Position: Min Extreme Position: Max

→ Kinetic Energy:

Mean Position: Max Extreme Position: Min

Q1. What is spring constant?

Ans k is the spring constant.

⇒ If value of k is more, the spring will be stiff and it would be difficult to displace.

⇒ If the value of k is less, the spring will be elastic/less stiff and would be easier to displace.

Q2. How do we know that k is the stiffness measure of the spring?

Ans k is the stiffness measure of the spring because whenever we displace spring to a small displacement, we multiply the value of k to the displacement which lets us find force. Suppose we displaced the spring to a large magnitude but its spring constant is small then less force would be required. However, a large magnitude

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of force would be required to displace the stiff spring even to displace it in smaller magnitude.

Q3. How SHM demonstrates Newton's Third Law?

Ans SHM is consistent with Newton's Third Law, which states,

"Action and Reaction forces are equal in magnitude and opposite in direction."

Action force is the ^{applied} force in simple pendulum by which bob is placed at extreme position after covering displacement.

Reaction force is the restoring force that brings the bob back to mean position.

Example 17.1

Given Data:-

$$x_1 = 8\text{cm} = \frac{8}{100} = 0.08\text{m}$$

$$m_1 = 10\text{kg}$$

$$x_2 = 5.5\text{cm} = \frac{5.5}{100} = 0.055\text{m}$$

To Find:-

$$(i) k = ? \quad (ii) m_2 = ?$$

Formula:-

$$F = kx, F = W = mg$$

Calculation:-

$$(i) kx_1 = mg$$

$$k = \frac{mg}{x}$$

$$= \frac{(10)(9.8)}{0.08}$$

$$k = 1225 \text{ Nm}^{-1}$$

$$(ii) kx_2 = m_2g$$

$$m_2 = \frac{kx_2}{g}$$

$$= \frac{(1225)(0.055)}{9.8}$$

$$m_2 = 6.875 \text{ kg}$$

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Result:-

- (i) The spring constant (k) is 1225 Nm^{-1} .
- (ii) The mass is 6.875 kg .

Assignment 17.1

Given Data:-

$$\text{mass} = m = \frac{500\text{g}}{1000} = 0.5\text{kg}$$

$$\text{displacement} = x = \frac{9.8\text{cm}}{100} = 0.098\text{m}$$

Calculation:-

$$k = \frac{mg}{x}$$

$$= \frac{(0.5)(9.8)}{(0.098)}$$

$$k = 50 \text{ Nm}^{-1}$$

To Find:-

Spring constant $= k = ?$

Formula:-

$$F = kx, F = W = mg$$

$$kx = mg$$

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Result:

The spring constant is 50 Nm^{-1} .

Q4. Show that the projection of an oscillator moving with uniform circular motion executes SHM?

Ans 1) Uniform Circular Motion and SHM:-

Expression for displacement:-

Since,

$$OQ = x \rightarrow (i)$$

$$OP = x_0 \rightarrow (ii)$$

Since, we have

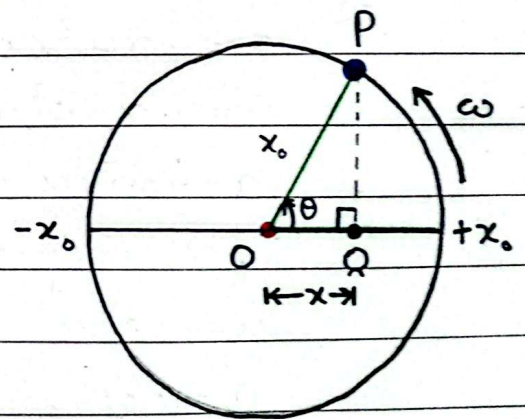
$$\cos \theta = \frac{\text{Base}}{\text{Hyp}}$$

From ΔOPQ :-

$$\cos \theta = \frac{OQ}{OP} = \frac{x}{x_0}$$

$$x = x_0 \cos \theta \rightarrow (iii) \quad \text{or} \quad x = x_0 \cos \omega t$$

$$\because \omega = \frac{\theta}{t}$$



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$$= \frac{(0.5)(9.8)}{(0.098)}$$

$$k = 50 \text{ Nm}^{-1}$$

Result:

The spring constant

Show that the projection of uniform circular motion

Uniform Circular Motion and
Expression for displacement

Since,

$$OQ = x \rightarrow (i)$$

$$OP = x_0 \rightarrow (ii)$$

pkmz

Imagine an object moving in a perfect circle at a constant speed. Now, picture a light source (a bulb) shining on this object. The light casts a shadow of the object onto a flat screen. As the object travels around the circle, its shadow moves back and forth in a straight line on the screen. This back and forth movement is a perfect example of SHM. The maximum distance the shadow moves from the centre of the screen is called amplitude (x_0 or A). This amplitude is always a positive value because it is a measure of distance.

Here, from eq (iii), we conclude that the value of 'x' depends on angle 'θ'.

2) Expression for acceleration:-

a_p = centripetal acceleration

a = acceleration of projection

Since, we have

$$\cos \theta = \frac{\text{Base}}{\text{Hyp}}$$

$$\cos \theta = \frac{OQ}{OP} = \frac{a}{a_p}$$

$$\boxed{a = a_p \cos \theta} \rightarrow (i)$$

Since, $a_p = -\frac{v^2}{r}$ $\therefore v = r\omega$

$$a_p = -\frac{r^2 \omega^2}{r}$$

$$\boxed{a_p = -r\omega^2} \rightarrow (ii)$$

Putting eq (ii) in (i), then

$$a = -r\omega^2 \cos \theta \quad \therefore r = x_0$$

$$a = -x_0 \omega^2 \cos \theta$$

$$a = -\omega^2 (x_0 \cos \theta)$$

$$\therefore x_0 \cos \theta = x$$

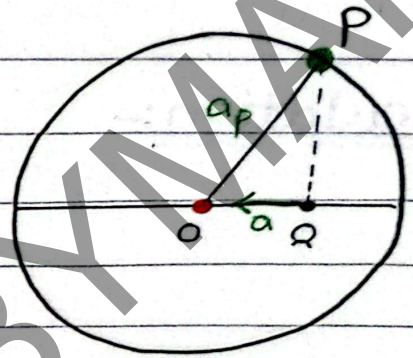
$$a = -\omega^2 x$$

$$a = \omega^2 (-x)$$

$$a = \text{constant} (-x)$$

$$a \propto -x$$

$\therefore \omega^2 = \text{constant}$ (Bcz angular velocity/frequency does not change)



3) Expression for Velocity:-

Since, $\cos \theta' = \frac{\text{Base}}{\text{Hyp}}$

$$\cos(90^\circ - \theta) = \frac{v}{v_p}$$

$$v = \cos(90^\circ - \theta) \cdot v_p$$

$$v = v_p \cos(90^\circ - \theta)$$

$$\boxed{v = v_p \sin \theta}$$

We know, that,

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

Taking square root on both sides,

$$\sqrt{\sin^2 \theta} = \sqrt{1 - \cos^2 \theta}$$

$$\sin \theta = \sqrt{1 - (\cos \theta)^2}$$

$$\sin \theta = \sqrt{1 - \left(\frac{x}{x_0}\right)^2}$$

$$\sin \theta = \sqrt{\frac{x_0^2 - x^2}{x_0^2}}$$

$$\boxed{\sin \theta = \frac{\sqrt{x_0^2 - x^2}}{x_0^2}} \rightarrow (ii)$$

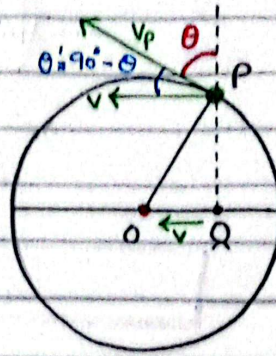
Since, $v = r\omega$

$\therefore r = x_0, v_p = v$

$$\boxed{v_p = x_0 \omega} \rightarrow (iii)$$

Putting eq (ii) and (iii) in (i), then

$$v = x_0 \omega \frac{\sqrt{x_0^2 - x^2}}{x_0^2}$$



v_p = velocity of particle

v = velocity of projection

$$\therefore \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos(90^\circ - \theta) = \cos 90^\circ \cdot \cos \theta + \sin 90^\circ \cdot \sin \theta$$

$$\cos(90^\circ - \theta) = 0 \cdot \cos \theta + 1 \cdot \sin \theta$$

$$\boxed{\cos(90^\circ - \theta) = \sin \theta}$$

Tricks:

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\therefore x_0 \cos \theta = x$$

$$\text{or } x = x_0 \cos \theta$$

$$\boxed{\cos \theta = \frac{x}{x_0}}$$

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$$V_0 = x_0 \omega \sqrt{\frac{x_0^2 - x^2}{x_0^2}}$$

$$V_0 = \frac{x_0 \omega \sqrt{x_0^2 - x^2}}{x_0}$$

$$V_0 = \omega \sqrt{x_0^2 - x^2} \rightarrow (iv)$$

At mean position, $x = 0$

$$\text{So, } v = \omega \sqrt{x_0^2 - (0)^2}$$

$$v = \omega \sqrt{x_0^2}$$

$$v = \omega x_0$$

At mean position, the velocity of projection is maximum.

At extreme position, $x = x_0$

$$\text{So, } v = \omega \sqrt{(x_0)^2 - (x_0)^2}$$

$$v = \omega \sqrt{x_0^2 - x_0^2}$$

$$v = \omega \sqrt{0}$$

$$v = \omega \times 0$$

$$v = 0$$

At extreme position, the velocity of projection will be zero.

Q5. What would be the value of displacement at angle:

(i) 90°

Sol We know that, $x = x_0 \cos \theta$
 $\theta = 90^\circ$

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$$x = x_0 \cos 90^\circ$$

$$x = x_0 (0)$$

$$\boxed{x = 0}$$

ii) 180°

Sol We know that, $x = x_0 \cos \theta$

$$\theta = 180^\circ$$

$$x = x_0 \cos 180^\circ$$

$$x = x_0 (-1)$$

$$\boxed{x = -x_0}$$

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iii) 60°

Sol We know that, $x = x_0 \cos \theta$

$$\theta = 60^\circ$$

$$x = x_0 \cos 60^\circ$$

$$x = x_0 \left(\frac{1}{2}\right)$$

$$\boxed{x = \frac{1}{2} x_0}$$

iv) 45°

Sol We know that, $x = x_0 \cos \theta$

$$\theta = 45^\circ$$

$$x = x_0 \cos 45^\circ$$

$$x = x_0 (0.7)$$

$$\boxed{x = 0.7 x_0}$$

(v) 135°

Sol We know that, $x = x_0 \cos \theta$

$$\theta = 135^\circ$$

$$x = x_0 \cos 135^\circ$$

$$x = x_0 (-0.7)$$

$$x = -0.7x_0$$

(vi) 225°

Sol We know that, $x = x_0 \cos \theta$

$$\theta = 225^\circ$$

$$x = x_0 \cos 225^\circ$$

$$x = x_0 (-0.7)$$

$$x = -0.7x_0$$

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(vii) 360°

Sol We know that, $x = x_0 \cos \theta$

$$\theta = 360^\circ$$

$$x = x_0 \cos 360^\circ$$

$$x = x_0 (1)$$

$$x = x_0$$

Theta batata hai kai kis angle pr oscillating body kis state pr hai aur woh wave bananay kai kis point pr hey. Kiya woh puri wave bana chuka hai ya half wave bana chuka hai, etc. Angle kai according oscillator kahan hai? Mean pr, ya trough pr ya crest pr hai?

* Phase(θ) :-

• Definition:

Phase tells the particle's position and motion state at a given time, measured in radians. It is a way to tell where the particle is in its vibration cycle and how it's moving at a certain time.

• Relation of Circular Motion with SHM:

Circular

motion is linked with SHM as phase can be seen as angle in uniform circular motion. SHM is projection of uniform circular motion on a straight line.

- Function:

Phase is angle (θ) that tells about the angle at which the body is oscillating and how it forms its waveform (the complete representation of wave). Either it will form full wave or half wave, like if the phase is 180° , then waveform will be half, if its 360° , then it will form a complete wave.

- Types of phase:

→ In phase (Crest overlaps crest or trough overlaps trough)

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→ Out phase (Crest overlaps trough)

- Derivation:

We know that,

$$x = x_0 \cos \theta$$

$$\therefore \theta = \omega t$$

$$\text{So, } x = x_0 \cos \omega t$$

This equation can also be written as,

$$x = x_0 \cos(\omega t + \phi)$$

Where ϕ gives information regarding the starting or initial phase or oscillation, and t is time taken.

$$\omega = 2\pi f \quad \therefore f = \frac{1}{T}$$

$$\omega = \frac{2\pi}{T}$$

$$\text{So, } x = x_0 \cos\left(\frac{2\pi}{T} x t + \phi\right)$$

Degree $\rightarrow$ Radians

$$0^\circ \rightarrow 0$$

$$90^\circ \rightarrow \frac{\pi}{2} \text{ rad}$$

$$180^\circ \rightarrow \pi \text{ rad}$$

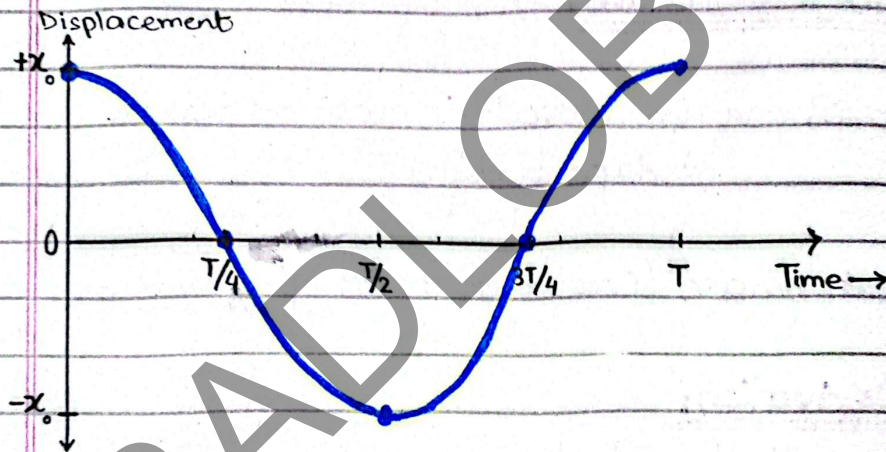
$$270^\circ \rightarrow \frac{3\pi}{2} \text{ rad}$$

$$360^\circ \rightarrow 2\pi \text{ rad}$$

(i) Case 01 :-

$$\phi = 0^\circ \text{ or } 0$$

$t=0$	$t=T/4$	$t=T/2$	$t=3T/4$	$t=T$
$x = x_0 \cos\left(\frac{2\pi}{T} \times t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \times t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \times t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \times t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \times t + \phi\right)$
$x = x_0 \cos\left(\frac{2\pi}{T} (0) + 0\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \left(\frac{T}{4}\right) + 0\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \left(\frac{T}{2}\right) + 0\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \left(\frac{3T}{4}\right) + 0\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} (T) + 0\right)$
$x = x_0 \cos(0+0)$	$x = x_0 \cos\left(\frac{2\pi}{4} + 0\right)$	$x = x_0 \cos(\pi+0)$	$x = x_0 \cos\left(\frac{3\pi}{2} + 0\right)$	$x = x_0 \cos(2\pi+0)$
$x = x_0 \cos(0)$	$x = x_0 \cos\left(\frac{\pi}{2}\right)$	$x = x_0 \cos(\pi)$	$x = x_0 \cos\left(\frac{3\pi}{2}\right)$	$x = x_0 \cos(2\pi)$
$x = x_0 (+1)$	$x = x_0 \cos(0)$	$x = x_0 (-1)$	$x = x_0 (0)$	$x = x_0 (+1)$
$x = +x_0$	$x = 0$	$x = -x_0$	$x = 0$	$x = +x_0$



(ii) Case 02 :-

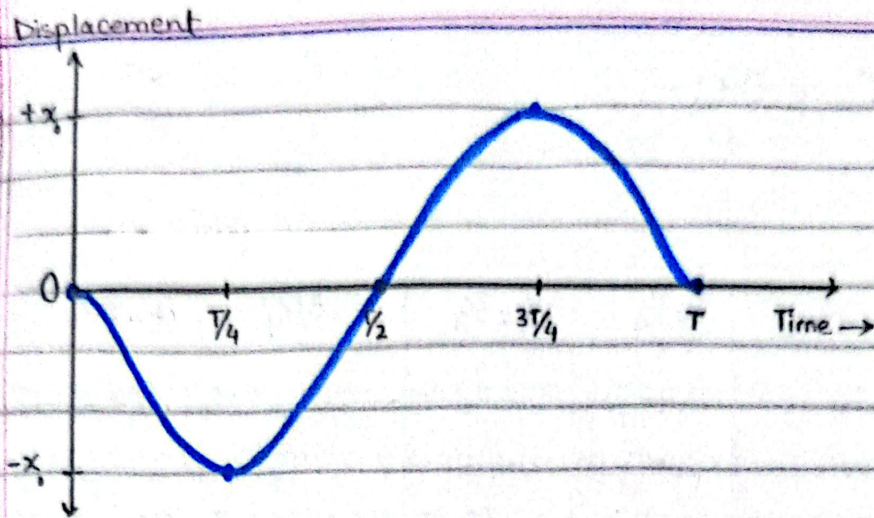
$$\phi = 90^\circ \text{ or } \frac{\pi}{2}$$

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$t=0$	$t=T/4$	$t=T/2$	$t=3T/4$	$t=T$
$x = x_0 \cos\left(\frac{2\pi}{T} \times t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \times t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \times t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \times t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \times t + \phi\right)$
$x = x_0 \cos\left(\frac{2\pi}{T} (0) + \frac{\pi}{2}\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \left(\frac{T}{4}\right) + \frac{\pi}{2}\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \left(\frac{T}{2}\right) + \frac{\pi}{2}\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} \left(\frac{3T}{4}\right) + \frac{\pi}{2}\right)$	$x = x_0 \cos\left(\frac{2\pi}{T} (T) + \frac{\pi}{2}\right)$
$x = x_0 \cos\left(0 + \frac{\pi}{2}\right)$	$x = x_0 \cos\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$	$x = x_0 \cos\left(\pi + \frac{\pi}{2}\right)$	$x = x_0 \cos\left(\frac{3\pi}{2} + \frac{\pi}{2}\right)$	$x = x_0 \cos\left(2\pi + \frac{\pi}{2}\right)$
$x = x_0 \cos\left(\frac{\pi}{2}\right)$	$x = x_0 \cos(\pi)$	$x = x_0 \cos\left(\pi + \frac{\pi}{2}\right)$	$x = x_0 \cos\left(\frac{3\pi}{2} + \frac{\pi}{2}\right)$	$x = x_0 \cos\left(2\pi + \frac{\pi}{2}\right)$
$x = x_0 (0)$	$x = x_0 (-1)$	$x = x_0 \cos\left(2\pi + \frac{\pi}{2}\right)$	$x = x_0 \cos\left(\frac{4\pi}{2}\right)$	$x = x_0 \cos\left(\frac{5\pi}{2}\right)$
$x = 0$	$x = -x_0$	$x = x_0 \cos\left(\frac{3\pi}{2}\right)$	$x = x_0 \cos(2\pi)$	$x = x_0 \cos\left(\frac{5\pi}{2}\right)$
		$x = x_0 (0)$	$x = x_0 (1)$	$x = x_0 (0)$
		$x = 0$	$x = x_0$	$x = 0$



(ii) Case 03:-

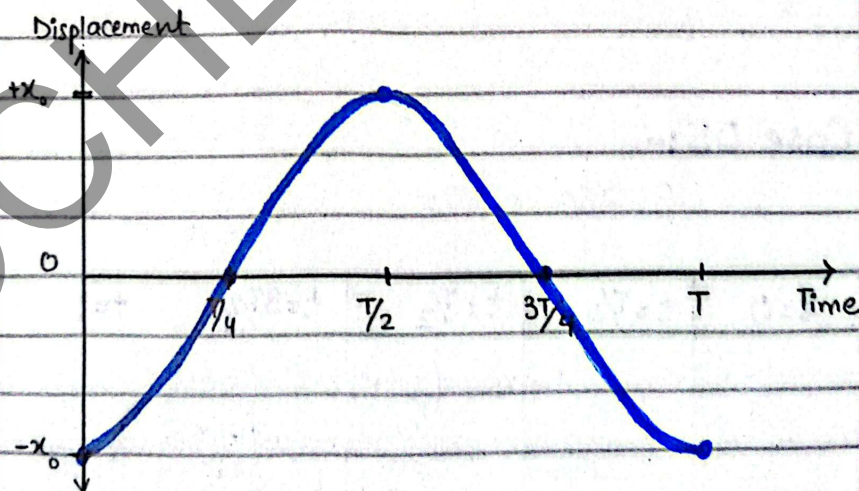
$$\phi = 180^\circ \text{ or } \pi$$

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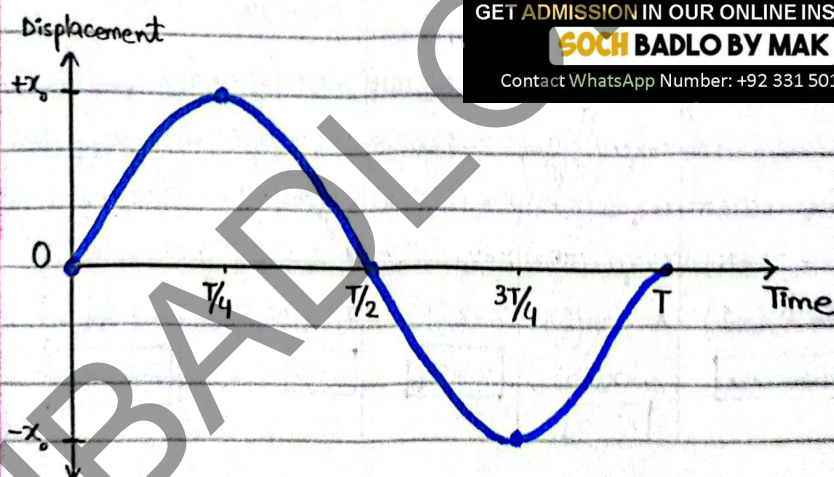
$t=0$	$t=T/4$	$t=T/2$	$t=3T/4$	$t=T$
$x = x_0 \cos\left(\frac{2\pi}{T}(t) + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}(t) + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}(t) + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}(t) + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}(t) + \phi\right)$
$x = x_0 \cos\left(\frac{2\pi}{T}(0) + \pi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}\left(\frac{T}{4}\right) + \pi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}\left(\frac{T}{2}\right) + \pi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}\left(\frac{3T}{4}\right) + \pi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}(T) + \pi\right)$
$x = x_0 \cos(0 + \pi)$	$x = x_0 \cos\left(\frac{\pi}{2} + \pi\right)$	$x = x_0 \cos(\pi + \pi)$	$x = x_0 \cos\left(\frac{3\pi}{2} + \pi\right)$	$x = x_0 \cos(2\pi + \pi)$
$x = x_0 \cos(\pi)$	$x = x_0 \cos\left(\frac{\pi + 2\pi}{2}\right)$	$x = x_0 \cos(2\pi)$	$x = x_0 \cos\frac{3\pi + 2\pi}{2}$	$x = x_0 \cos(3\pi)$
$x = x_0(-1)$	$x = x_0 \cos\left(\frac{3\pi}{2}\right)$	$x = x_0(1)$	$x = x_0 \cos\left(\frac{5\pi}{2}\right)$	$x = x_0(-1)$
$x = -x_0$	$x = x_0(0)$	$x = x_0$	$x = 0$	$x = -x_0$
	$x = 0$			



(iv) Case 04:-

$$\theta = 270^\circ \text{ or } \frac{3\pi}{2}$$

$t=0$	$t=T/4$	$t=T/2$	$t=3T/4$	$t=T$
$x = x_0 \cos\left(\frac{2\pi}{T}t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}t + \phi\right)$
$x = x_0 \cos\left(\frac{2\pi}{T}(0) + \frac{3\pi}{2}\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}\left(\frac{T}{4}\right) + \frac{3\pi}{2}\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}\left(\frac{T}{2}\right) + \frac{3\pi}{2}\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}\left(\frac{3T}{4}\right) + \frac{3\pi}{2}\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}(T) + \frac{3\pi}{2}\right)$
$x = x_0 \cos\left(0 + \frac{3\pi}{2}\right)$	$x = x_0 \cos\left(\frac{\pi}{2} + \frac{3\pi}{2}\right)$	$x = x_0 \cos\left(\pi + \frac{3\pi}{2}\right)$	$x = x_0 \cos\left(\frac{3\pi}{2} + \frac{3\pi}{2}\right)$	$x = x_0 \cos\left(2\pi + \frac{3\pi}{2}\right)$
$x = x_0 \cos\left(\frac{3\pi}{2}\right)$	$x = x_0 \cos\left(\frac{\pi+3\pi}{2}\right)$	$x = x_0 \cos\left(\frac{2\pi+3\pi}{2}\right)$	$x = x_0 \cos\left(\frac{3\pi+3\pi}{2}\right)$	$x = x_0 \cos\left(\frac{4\pi+3\pi}{2}\right)$
$x = x_0 \cos(2\pi)$	$x = x_0 \cos(2\pi)$	$x = x_0 \cos(2\pi)$	$x = x_0 \cos(2\pi)$	$x = x_0 \cos(2\pi)$
$x = x_0$	$x = x_0$	$x = x_0$	$x = x_0$	$x = x_0$
$x = x_0$	$x = x_0$	$x = x_0$	$x = x_0$	$x = x_0$



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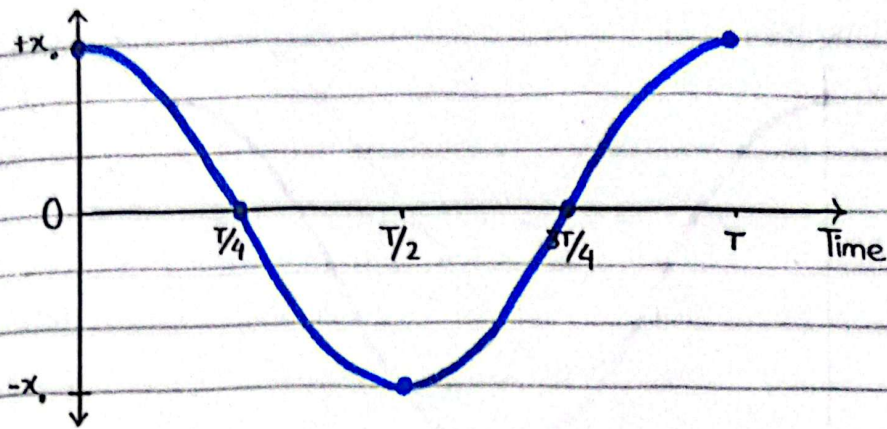
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(v) Case 05:-

$$\theta = 360^\circ \text{ or } 2\pi$$

$t=0$	$t=T/4$	$t=T/2$	$t=3T/4$	$t=T$
$x = x_0 \cos\left(\frac{2\pi}{T}t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}t + \phi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}t + \phi\right)$
$x = x_0 \cos\left(\frac{2\pi}{T}(0) + 2\pi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}\left(\frac{T}{4}\right) + 2\pi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}\left(\frac{T}{2}\right) + 2\pi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}\left(\frac{3T}{4}\right) + 2\pi\right)$	$x = x_0 \cos\left(\frac{2\pi}{T}(T) + 2\pi\right)$
$x = x_0 \cos(0 + 2\pi)$	$x = x_0 \cos\left(\frac{\pi}{2} + 2\pi\right)$	$x = x_0 \cos(\pi + 2\pi)$	$x = x_0 \cos\left(\frac{3\pi}{2} + 2\pi\right)$	$x = x_0 \cos(2\pi + 2\pi)$
$x = x_0 \cos(2\pi)$	$x = x_0 \cos\left(\frac{\pi+4\pi}{2}\right)$	$x = x_0 \cos(3\pi)$	$x = x_0 \cos\left(\frac{3\pi+4\pi}{2}\right)$	$x = x_0 \cos(4\pi)$
$x = x_0$	$x = x_0$	$x = x_0$	$x = x_0$	$x = x_0$
$x = x_0$	$x = x_0$	$x = x_0$	$x = x_0$	$x = x_0$



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Graphical Representation of Displacement, Velocity and Acceleration:-

Displacement:-

$$x = x_0 \cos \omega t \Rightarrow x = x_0 \cos(2\pi f \times t)$$

$$\Rightarrow x = x_0 \cos\left(\frac{2\pi}{T} \times t\right)$$

(a) $t = 0$

$$x = x_0 \cos\left(\frac{2\pi}{T} \times t\right)$$

$$x = x_0 \cos\left(\frac{2\pi}{T} \times 0\right)$$

$$x = x_0 \cos(0)$$

$$x = x_0 (1)$$

$$\boxed{x = x_0}$$

(b) $t = T/4$

$$x = x_0 \cos\left(\frac{2\pi}{T} \times t\right)$$

$$x = x_0 \cos\left(\frac{2\pi}{T} \times \frac{T}{4}\right)$$

$$x = x_0 \cos\left(\frac{\pi}{2}\right)$$

$$x = x_0 (0)$$

$$\boxed{x = 0}$$

(c) $t = T/2$

$$x = x_0 \cos\left(\frac{2\pi}{T} \times t\right)$$

$$x = x_0 \cos\left(\frac{2\pi}{T} \times \frac{T}{2}\right)$$

$$x = x_0 \cos(\pi)$$

$$\boxed{x = -x_0}$$

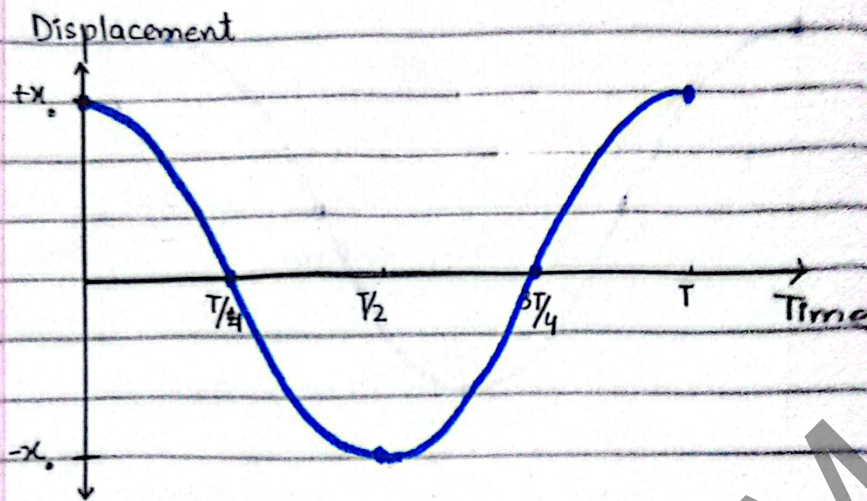
(d) $t = 3T/4$

$$x = x_0 \cos\left(\frac{2\pi}{T} \times t\right)$$

$$x = x_0 \cos\left(\frac{2\pi}{T} \times \frac{3T}{4}\right)$$

$$x = x_0 \cos\left(\frac{3\pi}{2}\right)$$

$$\boxed{x = 0}$$



2) Velocity:-

Since,

$$v = x_0 \omega \sin \theta$$

$$\therefore \theta = \omega t$$

$$v = (x_0 \omega) \sin \omega t$$

$$v = x_0 \omega \sin(2\pi f \times t)$$

$$v = x_0 \omega \sin\left(\frac{2\pi}{T} \times t\right)$$

(a) $t=0$

$$v = x_0 \omega \sin\left(\frac{2\pi}{T} \times 0\right)$$

$$v = x_0 \omega \sin(0)$$

$$v = x_0 \omega (0)$$

$$\boxed{v=0}$$

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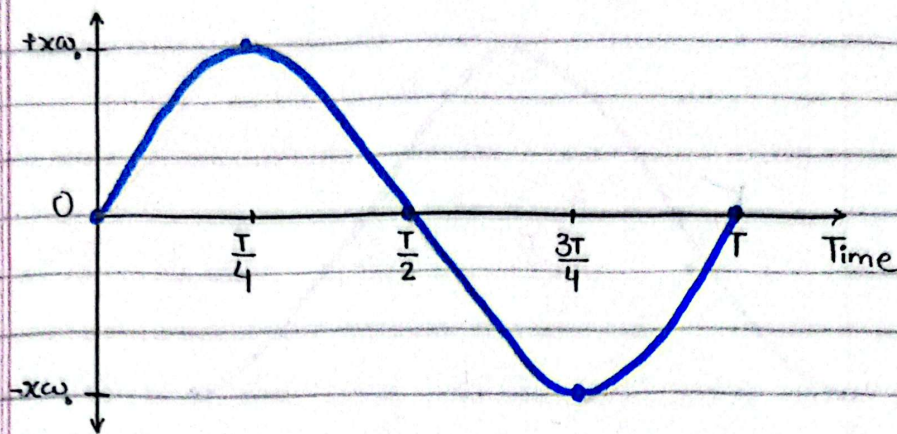
(b) $t = \frac{T}{4}$

$$v = x_0 \omega \sin\left(\frac{2\pi}{T} \times \frac{T}{4}\right)$$

$$v = x_0 \omega \sin\left(\frac{\pi}{2}\right)$$

$$v = x_0 \omega (1)$$

$$\boxed{v = +x_0 \omega}$$



3) Acceleration:-

Since,

$$a = -x_0 \omega^2 \cos \omega t$$

$$a = -(x_0 \omega^2) \cos\left(\frac{2\pi}{T} \times t\right)$$

(a) $t=0$

$$a = -(x_0 \omega^2) \cos\left(\frac{2\pi}{T} \times 0\right)$$

$$a = -(x_0 \omega^2) \cos(0)$$

$$a = -(x_0 \omega^2) (+1)$$

$$\boxed{a = -x_0 \omega^2}$$

(b) $t = \frac{T}{4}$

$$a = -(x_0 \omega^2) \cos\left(\frac{2\pi}{T} \times \frac{T}{4}\right)$$

$$a = -(x_0 \omega^2) \cos\left(\frac{\pi}{2}\right)$$

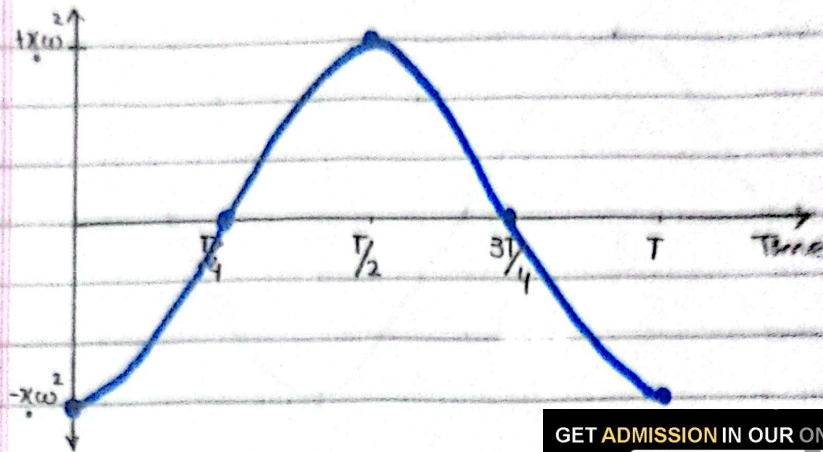
$$a = -(x_0 \omega^2) (0)$$

$$\boxed{a = 0}$$

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Q6. Why phase is represented as θ ?

Ans Phase is represented as θ because SHM is mathematically linked to uniform circular motion, where position is described by an angular displacement (θ). Since phase in SHM corresponds to this angular measure, we use the same symbol θ to show it.

Q7. Write all possible definitions of phase?

Ans In SHM, the phase describes the object's position and direction of motion at a given time.

OR

Phase tells about particle's current position, about where it is and also represents it graphically.

OR

Phase is defined as angle which helps us find displacement of an object.

OR

Phase is the angle that tells the position and direction of a particle in its SHM cycle. It shows the current state of the

oscillation at a given time.
OR

Phase is a way to tell where the particle is in its vibration cycle and how its moving at a certain time.

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Q8. What is meant by lead and lag in phase?

ms Lead:-

In phase, lead means one oscillation is ahead of another in its cycle.

Example:

If wave A reaches a certain position earlier than wave B, we say wave A leads B in phase.

Lag:-

In phase, lag means one oscillation is behind another in its cycle.

Example:

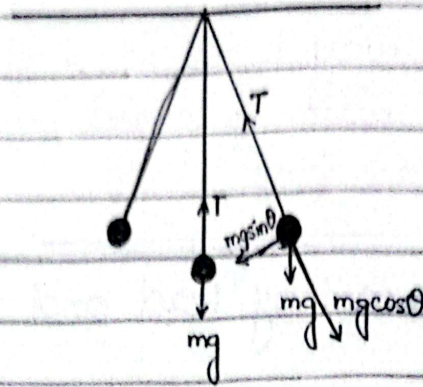
If wave A reaches a certain position later than wave B, we say wave A lags B in phase.

Q9. Show how that simple pendulum exhibits SHM?

Ans Simple Pendulum:-

Simple pendulum consists of a mass (called a bob) hanging from a string or rod fixed at top.

Figure:-



Derivation:-

$$mg \cos \theta = T \rightarrow (i)$$

$$-mg \sin \theta = F_{\text{res}} \rightarrow (ii)$$

Since, $\sin \theta = \frac{\text{Perp}}{\text{Hyp}}$

$$\sin \theta = \frac{x}{l} \rightarrow (iii)$$

Putting (iii) in (ii), then

$$F_{\text{res}} = -mg \left(\frac{x}{l} \right) \rightarrow (iv)$$

Since, from applied force,

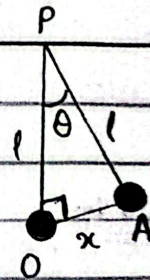
$$F_{\text{applied}} = ma \rightarrow (v)$$

Now, comparing eq (iv) and (v), then

$$ma = -mg \left(\frac{x}{l} \right)$$

$$a = -g \left(\frac{x}{l} \right)$$

$$a = +\frac{g}{l} (-x)$$



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$$\therefore \frac{g}{l} = \text{constant}$$

$$a = \text{constant} (-x)$$

$$a \propto -x$$

So, simple pendulum exhibits SHM.

Q10. Why we resolved weight into its components?

Ans Tension must be balanced at extreme point by weight. But at extreme point, weight is acting downward and not balancing tension. So to balance tension, we resolve weight into its perpendicular components. So x -component of weight will balance tension and its y -component acts as restoring force.

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* Simple Pendulum:-

$$a = -\frac{g}{l}(x) \rightarrow (i)$$

$$a = -\omega^2 x \rightarrow (ii)$$

By comparing eq (i) and (ii), then

$$+\omega^2 x = +\frac{g}{l}(x)$$

$$\omega^2 = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}} \rightarrow (iii)$$

Time Period and Frequency of Oscillating Mass-Spring System:

Since, $\omega = 2\pi f \rightarrow (iv)$

Putting eq (iv) in (iii), then

$$\omega = \sqrt{\frac{g}{l}}$$

$$\omega = 2\pi f$$

$$2\pi f = \sqrt{\frac{g}{l}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$

$$\therefore f = \frac{1}{T} \text{ or } T = \frac{1}{f}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$\Rightarrow$ This is time period of simple pendulum.

$\Rightarrow$ It is independent of mass.

$\Rightarrow$ It is independent of amplitude.

Q11. What is the time period of simple pendulum when length doubles?

Ans $l' = 2l$

$$T' = 2\pi \sqrt{\frac{l'}{g}}$$

$$T' = 2\pi \sqrt{\frac{2l}{g}}$$

$$T' = 2\pi \sqrt{2} \sqrt{\frac{l}{g}}$$

$$T' = \sqrt{2} \left(2\pi \sqrt{\frac{l}{g}} \right)$$

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$$T' = \sqrt{2} T$$

Q12. What is the time period of simple pendulum when $g' = \frac{g}{6}$?

Ans $g' = \frac{g}{6}$

$$T' = 2\pi \sqrt{\frac{l}{g'}}$$

$$T' = 2\pi \sqrt{\frac{l}{\frac{g}{6}}} \Rightarrow T' = 2\pi \sqrt{\frac{6l}{g}}$$

$$T' = \sqrt{6} \left(2\pi \sqrt{\frac{l}{g}} \right)$$

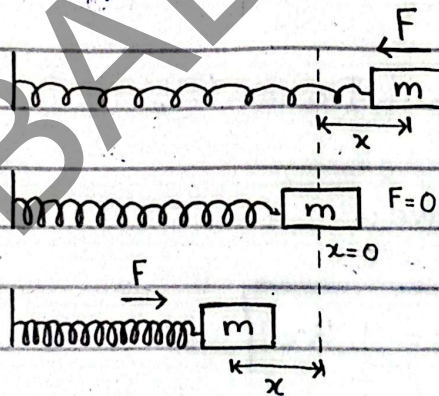
$$T' = \sqrt{6} T$$

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* Energy Conservation in SHM:-
 • Figure:-



• Derivation:

Since, $W = F_{av} x \rightarrow (i)$

$$\therefore F_{av} = \frac{F_i + F_f}{2}$$

$$\therefore F_i = 0, F_f = F$$

$$F_{av} = \frac{0+F}{2}$$

$$F_{av} = \frac{F}{2} \quad , (ii)$$

Putting eq (ii) in (i),

$$W = F_{av} x$$

$$W = \left(\frac{F}{2} \right) \cdot x$$

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Since, from Hooke's law,

$$F = -kx$$

$$W = \left(\frac{kx}{2} \right) x$$

$$W = \frac{kx^2}{2}$$

$$\Rightarrow W = \frac{1}{2} kx^2$$

Applied force = - Restoring force

Applied force = -(-kx)

Applied force = kx

From work energy principle,

$$\text{Work Done} = P.E$$

So,

we have:-

$$P.E = \frac{1}{2} kx^2 \quad \rightarrow (iii)$$

a) At mean position, $x = 0$

$$P.E = \frac{1}{2} k(0)^2$$

$$P.E = 0$$

This means P.E will be minimum at mean position.

b) At extreme position, $x = x_0$

$$P.E = \frac{1}{2} k(x_0)^2$$

$$P.E = \frac{1}{2} kx_0^2$$

P.E is maximum at extreme position.

* Kinetic Energy:-

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$$\text{Since, } K.E = \frac{1}{2} mv^2 \rightarrow (iv)$$

$$\therefore v = \omega \sqrt{x_0^2 - x^2} \rightarrow (v)$$

Putting eq (v) in eq (iv),

$$K.E = \frac{1}{2} m(\omega \sqrt{x_0^2 - x^2})^2$$

$$K.E = \frac{1}{2} m\omega^2 (\sqrt{x_0^2 - x^2})^2$$

$$K.E = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$$

$$\therefore \omega = \sqrt{\frac{k}{m}}$$

$$\omega^2 = \frac{k}{m}$$

$$m\omega^2 = k$$

$$K.E = \frac{1}{2} k(x_0^2 - x^2) \rightarrow (vi)$$

a) At mean position, $x = 0$

$$K.E = \frac{1}{2} k(x_0^2 - (0)^2)$$

$$K.E = \frac{1}{2} kx_0^2$$

Kinetic energy is maximum at mean position.

b) At extreme position, $x = x_0$:

$$K.E = \frac{1}{2} K (x_0^2 - x_0^2)$$

$$K.E = \frac{1}{2} K (0)$$

$$K.E = 0$$

Kinetic energy is minimum at extreme position.

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* Total Energy:-

$$T.E = P.E + K.E$$

$$T.E = \frac{1}{2} K x^2 + \frac{1}{2} K (x_0^2 - x^2)$$

$$T.E = \frac{1}{2} K x^2 + \frac{1}{2} K x_0^2 - \frac{1}{2} K x^2$$

$$T.E = \frac{1}{2} K x_0^2 \quad \text{or} \quad T.E = \frac{1}{2} m \omega^2 x_0^2$$

Q13. At what displacement, the $K.E = P.E$?

Ans. $K.E = P.E$ when the displacement is 70% is from the total displacement or 7cm from mean position.

Q14. In simple harmonic oscillator, $K.E$ becomes equal to $P.E$ when the displacement is _____ of the amplitude.

(a) 50%

(b) 60%

☒ (c) 70%

(d) 80%

* Free Oscillation:-

A body is said to be executing free oscillation if it is oscillating at natural frequency under the action of restoring force without the need of external force after initial disturbance.

OR

Free oscillations occur when an object, after an initial disturbance, oscillates at its natural frequency solely under the influence of a restoring force, with no continuous external force.

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* Forced Oscillation:-

If external forces cause the body to oscillate at the frequency of applied force rather than its natural frequency, such oscillations are forced oscillations.

OR

When an external force drives an object to oscillate at the frequency of applied force, not its natural frequency.

* Damped Oscillation:-

Damped Oscillations are oscillations where the amplitude gradually decrease over time due to a resistive or damping force, like friction or air resistance, which converts the system's energy into heat.

⇒ Heavy Damping (Overdamping) :-

- Rapid Energy Loss
- No oscillations
- System returns to equilibrium slowly.

⇒ Light Damping (Underdamping) :-

- Gradual Energy Loss
- Oscillations continue with decreasing amplitude.
- System oscillates around equilibrium.

⇒ Critical Damping :-

- Fastest return to equilibrium without oscillations.
- No overshoot or oscillations
- System returns to equilibrium in shortest time possible.

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Q15. Explain critical damping with reference to shock absorber?

Ans Critical damping quickly brings the oscillation back to the equilibrium position hereby avoiding annoying situation of light and heavy damping. If critical damping is absent, it will take comparatively larger time to its oscillations to come at equilibrium position.

* There is natural frequency of every object, even tiniest object.

Reason:

Objects are made of atoms which

vibrate and produce natural frequency. Even in solids, the atoms vibrate, so it is understood that every matter in the universe has natural frequency.

Now suppose there is an object containing driven frequency. Natural frequency is 5Hz + Driven frequency is 5Hz, then resonance will occur, i.e. the amplitude and energy will increase.

⇒ When a lady sings at a very high pitch and her voice frequency matches the natural frequency of a glass, resonance will occur. The vibrations keep increasing in amplitude and can even cause the glass to break.

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Q16. How can you describe resonance in a pendulum?

Ans Resonance is a phenomenon where a pendulum's amplitude dramatically increases when a small, repeated force is applied at its natural swinging frequency.

* Sharpness of resonance means how clear and strong the vibrations are at a particular frequency.

→ If damping is high, vibrations die out and resonance becomes weak.

→ If damping is low, vibrations last longer and resonance is stronger.

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* Sharpness of resonance means how clear and strong the vibrations are at a particular frequency.

→ If damping is high, vibrations die out and resonance becomes weak.

→ If damping is low, vibrations last longer and resonance is stronger.

Q17. At what displacement, the K.E and P.E are equal?

Sol $K.E = P.E$

$$\Rightarrow K.E = \frac{1}{2} k(x_0^2 - x^2)$$

$$\Rightarrow P.E = \frac{1}{2} kx^2$$

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$$\frac{1}{2} k(x_0^2 - x^2) = \frac{1}{2} kx^2$$

$$x_0^2 - x^2 = x^2$$

$$x_0^2 = x^2 + x^2$$

$$x_0^2 = 2x^2$$

$$x^2 = \frac{x_0^2}{2}$$

$$\sqrt{x^2} = \sqrt{\frac{x_0^2}{2}}$$

$$x = \frac{x_0}{\sqrt{2}}$$

$$x = x_0 \times \frac{1}{\sqrt{2}}$$

$$\frac{x}{x_0} = \frac{1}{\sqrt{2}}$$

For any value of x_0 , $x = 0.70 \times 100$
 $= 70\%$
