4.8.2025

CHAPTER#17- SIMPLE HARMONIC MOTION

17.1 Oscillations

Definition:-

- when a body moves to and fro about its mean position, then such motion is called vibratory motion or oscillatory motion.
- The complete round trip of an oscillatory or vibrating body is called oscillation or vibration.

TERMS related to Oscillations:

(i) Displacement (x):-
Distance of a vibrating body from the mean position on either side at any instance of time.
SI Unit:- 'm'

(ii) Amplitude (x_0 or A):-
maximum displacement from mean position.

(iii) Time period (T):-
time taken to complete one oscillation or vibration is called time period
SI Unit:- Second (s)

(iv) frequency (f):-
The number of oscillations(n) per unit time (t).
SI Unit:- Hertz (Hz) $\longrightarrow$ $\boxed{f = 1/T}$

(v) Angular frequency (ω):-
Angular displacement per unit time
given by:- $\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$

(vi) Time taken (t)
The time taken for any number of oscillations, or a part of an oscillation.

17.2 SIMPLE HARMONIC MOTION

Definition:-

Such an oscillatory motion in which acceleration of a particle is directly proportional to its displacement from the mean position and is always directed towards the mean position.

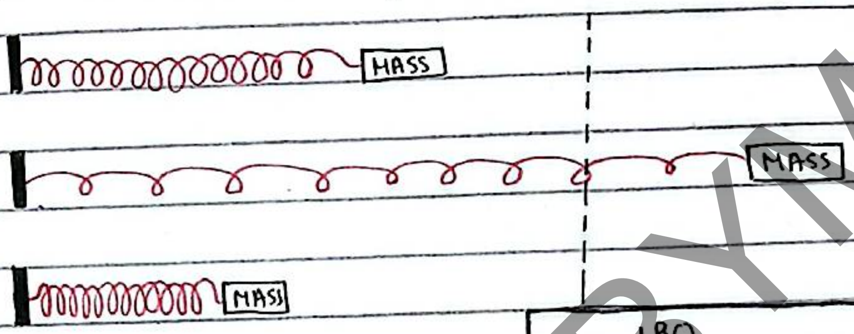
Mathematically:- $\boxed{a \propto -x}$

- This equation shows that:-

① acceleration (a) is directly proportional to displacement (x)

② (negative sign shows that) the acceleration is directed towards the mean position.

⟹ Motion of a mass attached to a spring:-



LRQ
- Show that mass attached to a spring executes S.H.M

Derivation:-

Since, $\boxed{F_{applied} = ma}$ ——(i)

from Hooke's law

$\boxed{F_{res} = -kx}$ —(ii)

As -ve sign shows that the restoring force is acting in the opposite direction.

⟹ from newton's third law of motion, we can

Comparing eq(i) and eq(ii)

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

where k = spring constant, m = mass and x = displacement here:-

$$\frac{k}{m} = \text{Constant} = w^2$$

$$a = -(\text{constant})\,x$$

$$a = (\text{constant})\,(-x)$$

$$\boxed{a = -x}$$

⟹ Time Period and frequency of an oscillatory Mass-Spring System:-

Since, Angular velocity is given by:-

$$w = \frac{\theta}{t}$$

for one complete revolution, $\theta = 2\pi$ rad and time taken becomes time period $t = T$ so:-

$$w = \frac{2\pi}{T}$$

$$\omega = 2\pi \times \frac{1}{T}$$

$$\omega = 2\pi f \qquad \because 1/T = f$$

we can derive the angular frequency from this equation as:-

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi} \times \omega$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

As $T = \frac{1}{f}$, so we can write it as:-

$$T = \frac{1}{f} = \frac{1}{\left[\frac{1}{2\pi}\sqrt{k/m}\right]}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

This can also be derived by using the formula $\omega = 2\pi/T$

SOLVED POSSIBLE QUESTIONS

1. What will the acceleration be at:- (MCQ)
   (a) Mean Position:- Minimum
   (b) Extreme Position:- Maximum
   Reason:- Acceleration is maximum where applied force is maximum

2. What will the velocity be at:- (MCQ)
   (a) Mean Position:- maximum
   (b) Extreme Position:- minimum
   Reason:- mass moves fastest where a & F_{res} is minimum

3. What will the potential energy be at:- (MCQ)
   (a) Mean Position:- Minimum
   (b) Extreme Position:- Maximum
   Reason:- P.E is maximum where displacement is maximum

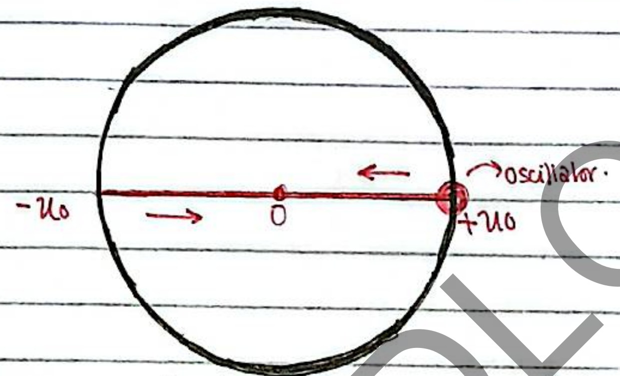
4. What will the kinetic energy be at:- (MCQ)
   (a) Mean Position:- Maximum
   (b) Extreme Position:- Minimum

Reason:- Kinetic energy is maximum where velocity is maximum.

5. Difference between Angular frequency & Angular Velocity.

	ANGULAR FREQUENCY	ANGULAR VELOCITY
1.	Rate of oscillations in rads per Sec	Rate of change of Ang. displacer.
2.	Scalar Quantity	Velocity Quantity.
3.	used in SHM wave Motion	in rotational motion of Rigid bodies.
4.	$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$	4. $\omega = 2\pi f$

17.3. UNIFORM CIRCULAR MOTION & SHM.



Possible Question (LAQ)
- Show that the projection of an oscillator moving with uniform Circular motion executes SHM

↳ Imagine an object moving in a perfect Circle at a Constant Speed.

↳ Consider a light Source (eg. a bulb) Shining on the object. The light Casts a Shadow of the object onto a flat Screen.

↳ As the Object travels around the circle, its shadow moves back and forth in a straight line of the Screen. This movement is the perfect example of SHM.

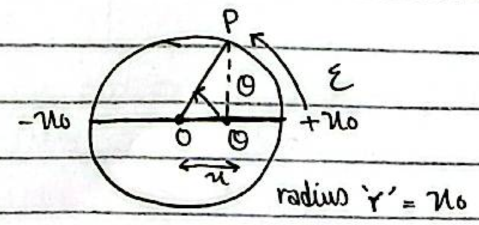
*So we Can Say that the projection of a uniform Circular motion is an example of SHM.

① Expressions for displacement:-

Since,

$OQ = x$ ——(i)

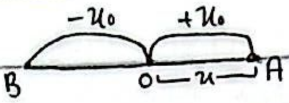
$OP = x_0$ ——(ii)



we know that :- $\cos\theta = \frac{\text{base}}{\text{hyp.}}$

from ΔOPQ :- $\cos\theta = \frac{OQ}{OP}$

$$\cos\theta = \frac{x}{x_0}$$



$$\boxed{x = x_0 \cos\theta} \text{ — (iii)}$$

OR From eq(iii) we conclude that the value of 'x' depends on Angle 'θ'

$\because W = \frac{\theta}{t} \Rightarrow \theta = \omega t$

$$\boxed{x = x_0 \cos\omega t}$$

② Expressions for Acceleration :-

Here, a_p = Centripetal acceleration

and, a = acceleration of projection

Since, $\cos\theta = \frac{\text{Base}}{\text{hyp.}}$

For ΔOPQ:-

$$\cos\theta = \frac{OQ}{OP} \quad , \quad \cos\theta = \frac{a}{a_p}$$

$$a_p \cos\theta = a$$

$$\boxed{a = a_p \cos\theta} \text{ — (i)}$$

Now, $a_p = \frac{-v^2}{r}$ and $v = r\omega$

$$a_p = \frac{-(r\omega)^2}{r} \quad \because r = x_0$$

$$\boxed{a_p = -x_0\omega^2} \text{ — (ii)}$$

putting eq(ii) in eq(i), we get:-

$$a = -x_0 \omega^2 \cos\theta$$

$$a = -\omega^2 (x_0 \cos\theta)$$

$$a = (-\omega^2)(x) \quad \because x = x_0\cos\theta$$

$$\boxed{a = -\omega^2 x}$$

To prove that the projection of UCM is SHM :-

we know that ω = Constant.

$$a = (\text{Constant})(-x)$$

$$\boxed{a \propto -x} \text{ — hence proved.}$$

SOLVED POSSIBLE QUESTIONS

Find the displacement (x) when the angle (θ) is :-

Used: $x = x_0 \cos\theta$

1. $\theta = 45^\circ$

$u = u_0 \cos(45^\circ)$

$u = u_0 \left(\frac{1}{\sqrt{2}}\right)$

$\boxed{u = \frac{u_0}{\sqrt{2}}}$

2. $\theta = 60^\circ$

$u = u_0 \cos(60^\circ)$

$u = u_0 \left(\frac{1}{2}\right)$

$\boxed{u = \frac{u_0}{2}}$

3. $\theta = 90^\circ$

$u = u_0 \cos(90^\circ)$

$u = u_0 (0)$

$\boxed{u = 0}$

4. $\theta = 180^\circ$

$u = u_0 \cos(180^\circ)$

$u = u_0 (-1)$

$\boxed{u = -u_0}$

5. $\theta = 135^\circ$

$u = u_0 \cos(135^\circ)$

$u = u_0 \left(\frac{-1}{\sqrt{2}}\right)$

$\boxed{u = -\frac{u_0}{\sqrt{2}}}$

6. $\theta = 225^\circ$

$u = u_0 \cos(225^\circ)$

$u = u_0 \left(\frac{-1}{\sqrt{2}}\right)$

$\boxed{u = -\frac{u_0}{\sqrt{2}}}$

7. $\theta = 270^\circ$

$u = u_0 \cos(270^\circ)$

$u = u_0 (0)$

$\boxed{u = 0}$

8. $\theta = 360^\circ$

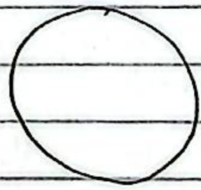
$u = u_0 \cos(360^\circ)$

$u = u_0 (1)$

$\boxed{u = u_0}$

③ Expressions for Velocity (of Project V)

Since, $\cos\theta' = \frac{\text{Base}}{\text{hyp.}}$



$\cos(90^\circ - \theta) = \frac{V}{V_p}$

$V_p \cdot \cos(90^\circ - \theta) = V$

$V_p \cdot \sin\theta = V$

$\boxed{V = V_p \sin\theta}$ — (i)

As $\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sqrt{(\sin^2\theta)} = \sqrt{1 - \cos^2\theta}$

$\sin\theta = \sqrt{1 - \cos^2\theta}$

now, $x = x_0 \cos\theta$ so $\cos\theta = \frac{x}{x_0}$

$$\sin\theta = \sqrt{1-\left(\frac{x}{x_0}\right)^2}$$

$$\sin\theta = \sqrt{1-\frac{x^2}{x_0^2}}$$

$$\boxed{\sin\theta = \sqrt{\frac{x_0^2 - x^2}{x_0^2}}} \quad \text{—(ii)}$$

Since, $v = r\omega$ and $r = x_0$, $V_p = V$

$$\boxed{V_p = x_0\omega} \quad \text{—(iii)}$$

putting eq(ii) and eq(iii) in eq(i):-

$$V = (x_0\omega)\left[\sqrt{\frac{x^2 - x^2}{x_0^2}}\right]$$

$$V = (x_0\omega)\sqrt{\frac{x^2 - x^2}{}}$$

$$V = x_0\omega \frac{\sqrt{x^2 - x^2}}{(\sqrt{x_0^2})}$$

$$V = x_0\omega \frac{(\sqrt{x_0^2 - x^2})}{x_0}$$

$$\boxed{V = \omega\sqrt{x_0^2 - x^2}}$$

↳ At Mean Position:-

$$x = 0 \longrightarrow V = \omega\sqrt{x_0^2 - (0)^2}$$

$$V = \omega\sqrt{x_0^2 - 0}$$

$$V = \omega\sqrt{x_0^2}$$

Here, Velocity of projection in maximum. $\boxed{V = \omega x_0}$

↳ At Extreme Position

$$x = x_0 \longrightarrow V = \omega\sqrt{x_0^2 - (x_0)^2}$$

$$V = \omega\sqrt{x_0^2 - x_0^2}$$

$$V = \omega\sqrt{0}$$

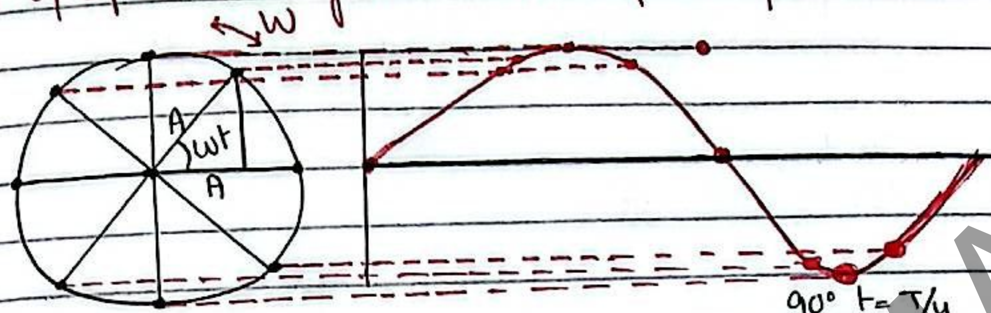
here Velocity of the projection is minimum. $V = \omega(0)$

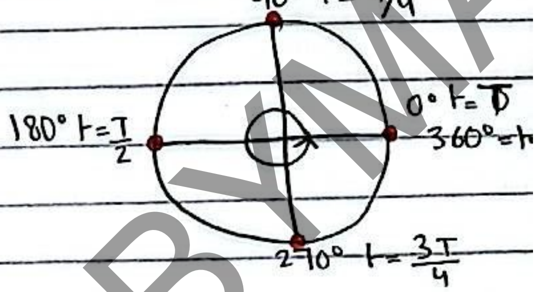
$$\boxed{V = 0}$$

17.4 PHASE

☒ Definition:-

The Angle $\theta = \omega t$ which Specifies the displacement as well as the direction of a point executing SHM is called phase of the motion.





$$x = x_0 \cos\theta$$

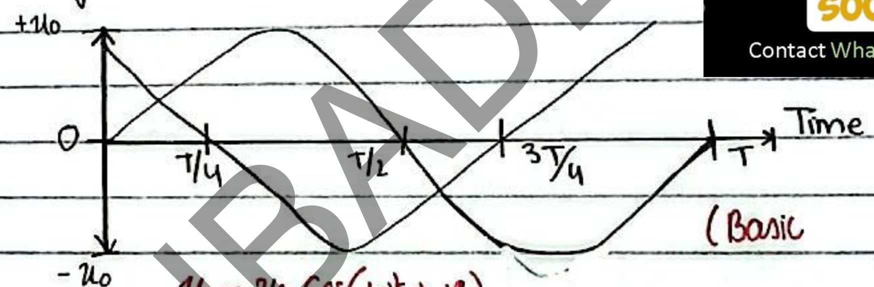
$$\theta = \omega t$$

$$x = x_0 \cos(\omega t)$$

In general:

$$\boxed{x = x_0 \cos(\omega t + \varphi)}$$

Here, φ is the initial phase. It is included to give information about the Starting or initial phase of oscillation.



$$x = x_0 \cos(\omega t + \varphi)$$

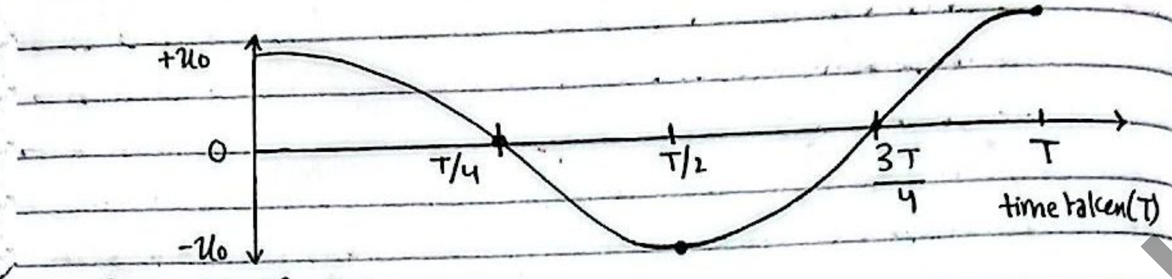
φ tells about the starting / initial Point.

$$\because \omega = 2\pi f = \frac{2\pi}{T}$$

$$\boxed{x = x_0 \cos\left(\frac{2\pi}{T} \times t + \varphi\right)}$$

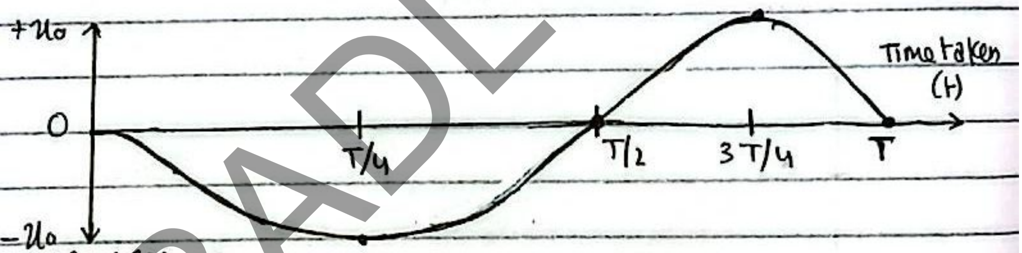
Case-01:- $\varphi = 0°$

T=0	t = T/4	t = T/2	t = 3T/4	t = T
$x = x_0 \cos\left[\frac{2\pi(0)}{T} + 0\right]$	$x = x_0 \cos\left[\frac{2\pi}{T} \times \frac{T}{4} + 0\right]$	$x = x_0 \cos\left[\frac{2\pi}{T} \times \frac{T}{2} + 0\right]$	$x = x_0 \cos\left[\frac{2\pi}{T} \times \frac{3T}{4} + 0\right]$	$x = x_0 \cos\left[\frac{2\pi}{T} \times T + 0\right]$
$x = x_0 \cos(0)$	$x = x_0 \cos(\pi/2)$	$x = x_0 \cos(\pi)$	$x = x_0 \cos(3\pi/2)$	$x = x_0 \cos(2\pi)$
$x = x_0(+1)$	$x = x_0(0)$	$x = x_0(-1)$	$x = x_0(0)$	$x = x_0(+1)$
$\boxed{x = +x_0}$	$\boxed{x = 0}$	$\boxed{x = x_0}$	$\boxed{x = 0}$	$\boxed{x = +x_0}$



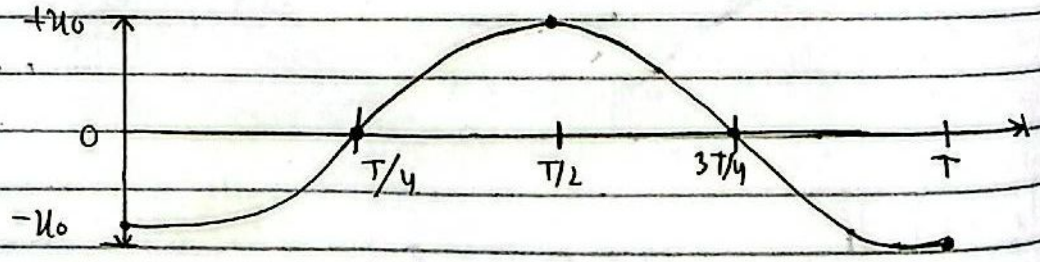
Case -02 $\varphi = 90° = \frac{\pi}{2}$

$t=0$	$t=T/4$	$t=T/2$	$t=3T/4$	$t=T$
$x=x_0\cos\left[\frac{2\pi}{T}\times 0+\frac{\pi}{2}\right]$	$x=x_0\cos\left[\frac{2\pi}{T}\times\frac{T}{4}+\frac{\pi}{2}\right]$	$x_0\cos\left[\frac{2\pi}{T}\times\frac{T}{2}+\frac{\pi}{2}\right]$	$x_0\cos\left[\frac{2\pi}{T}\times\frac{3T}{4}+\frac{\pi}{2}\right]$	$x_0\cos\left[\frac{2\pi}{T}\times T+\frac{\pi}{2}\right]$
$x=x_0\cos\left[0+\frac{\pi}{2}\right]$	$x=x_0\cos\left[\frac{\pi}{2}+\frac{\pi}{2}\right]$	$x=x_0\cos\left[\pi+\frac{\pi}{2}\right]$	$x=x_0\cos\left[\frac{3\pi}{2}+\frac{\pi}{2}\right]$	$x=x_0\cos\left[2\pi+\frac{\pi}{2}\right]$
$x=x_0\cos(\pi/2)$	$x=x_0\cos(\pi)$	$x=x_0\cos(3\pi/2)$	$x=x_0\cos(2\pi)$	$x=x_0\cos(3\pi/2)$
$x=x_0(0)$	$x=x_0(-1)$	$x=x_0(0)$	$x=x_0(+1)$	$x=x_0(0)$
$\boxed{x=0}$	$\boxed{x=-x_0}$	$\boxed{x=0}$	$\boxed{x=+x_0}$	$\boxed{x=0}$



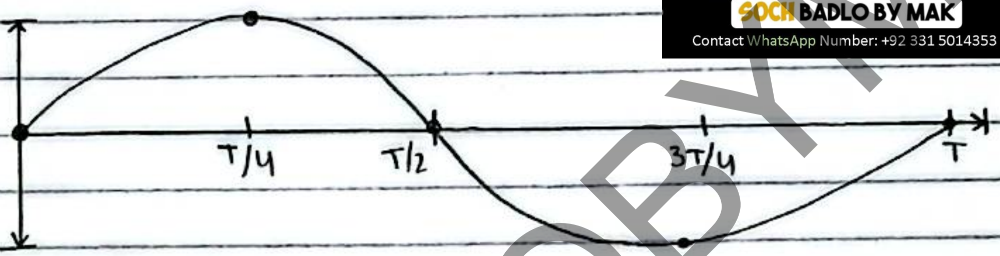
Case-03, $\varphi = 180° = \pi$

$t=0$	$t=T/4$	$t=T/2$	$T=3T/4$	$t=T$
$x=x_0\cos\left[\frac{2\pi}{T}\times 0+\pi\right]$	$x_0\cos\left[\frac{2\pi}{T}\times\frac{T}{4}+\pi\right]$	$x_0\cos\left[\frac{2\pi}{T}\times\frac{T}{2}+\pi\right]$	$x_0\cos\left[\frac{2\pi}{T}\times\frac{3T}{4}+\pi\right]$	$x_0\left[\frac{2\pi}{T}\times T+\pi\right]$
$x=x_0\cos(\pi)$	$x=x_0\cos(3\pi/2)$	$x=x_0\cos(2\pi)$	$x=x_0\cos(5\pi/2)$	$x=x_0(3\pi)$
$x=x_0(-1)$	$x=x_0(0)$	$x=x_0(+1)$	$x=x_0(0)$	$x=x_0(-1)$
$\boxed{x=-x_0}$	$\boxed{x=0}$	$\boxed{x=+x_0}$	$\boxed{x=0}$	$\boxed{x=-x_0}$



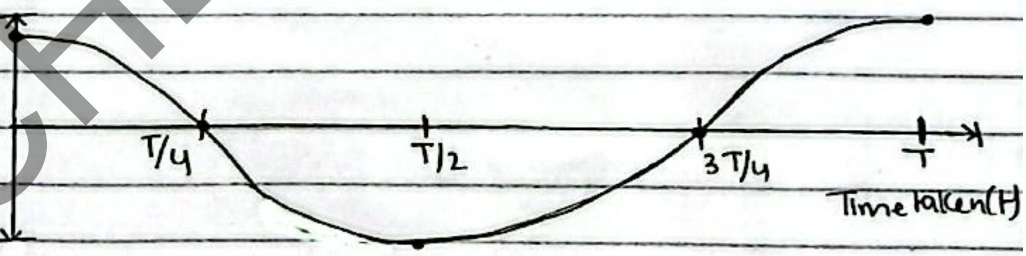
Case-04: $\varphi = 270° = 3\pi/2$.

$t=0$	$t=T/4$	$t=T/2$	$t=3T/4$	$t=T$
$x_0\cos\left[\frac{2\pi\times 0}{T}+\frac{3\pi}{2}\right]$	$x_0\cos\left[\frac{2\pi}{T}\times\frac{T}{4}+\frac{3\pi}{2}\right]$	$x_0\cos\left[\frac{2\pi}{T}\times\frac{T}{2}+\frac{3\pi}{2}\right]$	$x_0\cos\left[\frac{2\pi}{T}\times\frac{3T}{4}+\frac{3\pi}{2}\right]$	$\cos\left[\frac{2\pi}{T}\times T+\frac{3\pi}{2}\right]$
$x=x_0\cos(3\pi/2)$	$x=x_0\cos(2\pi)$	$x=x_0\cos(5\pi/2)$	$x=x_0\cos(3\pi)$	$x=x_0\cos(7\pi/2)$
$x=x_0(0)$	$x=x_0(+1)$	$x=x_0(0)$	$x=x_0(-1)$	$x=x_0(0)$
$\boxed{x=0}$	$\boxed{x=+x_0}$	$\boxed{x=0}$	$\boxed{x=-x_0}$	$\boxed{x=0}$

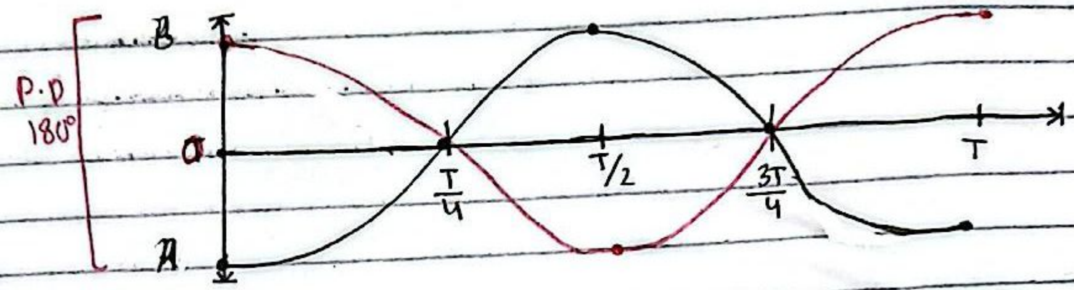


Case-05: $\varphi = 360° = 2\pi$

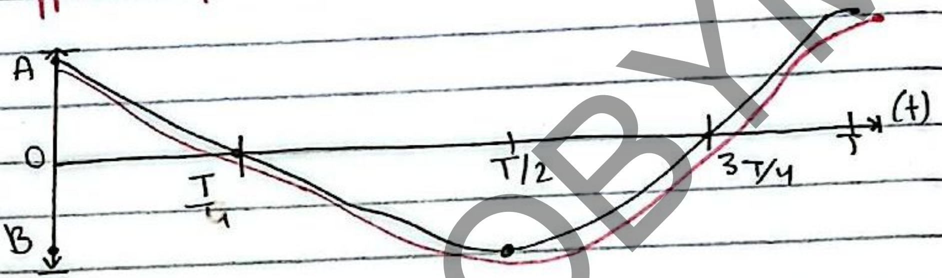
$t=0$	$t=T/4$	$t=T/2$	$t=3T/4$	$t=T$
$\cos\left[\frac{2\pi}{T}\times 0+2\pi\right]$	$\cos\left[\frac{2\pi}{T}\times\frac{T}{4}+2\pi\right]$	$\cos\left[\frac{2\pi}{T}\times\frac{T}{2}+2\pi\right]$	$\cos\left[\frac{2\pi}{T}\times\frac{3T}{4}+2\pi\right]$	$\cos\left[\frac{2\pi}{T}\times T+2\pi\right]$
$x=x_0\cos(2\pi)$	$x=x_0\cos(5\pi/2)$	$x=x_0\cos(3\pi)$	$x=x_0\cos(7\pi/2)$	$x=x_0\cos(4\pi)$
$x=x_0(+1)$	$x=x_0(0)$	$x=x_0(-1)$	$x=x_0(0)$	$x=x_0(+1)$
$\boxed{x=+x_0}$	$\boxed{x=0}$	$\boxed{x=-x_0}$	$\boxed{x=0}$	$\boxed{x=+x_0}$



- **Out of Phase:-**
  When the difference b/w two waves/oscillating systems is 180°, they are said to be oscillating out of phase.
- **In Phase:-**
  When the difference b/w two waves/oscillating systems is 0° or 360°, they are said to be oscillating in phase.



↳ These two waves, A & B, are out of phase as the phase difference b/w them is 180°. This is because the angles of both the waves produce opposite amplitude $(+x_0, -x_0)$



→ The two waves with $\varphi = 0°$ & $360°$ have the same starting point in the graph, so they are in phase. Their angles produce the same Amplitude.

MATHEMATICAL PROOF OF IN-PHASE OSCILLATIONS

$\varphi = 0°$

$x = x_0 \cos(\omega t + \varphi)$

$x = x_0 \cos\left[\frac{2\pi}{T} \times t + \varphi\right]$

let $t = 0$, then

$x = x_0 \cos\left[\frac{2\pi}{T} \times 0 + 0\right]$

$x = x_0 \cos(0+0)$

$x = x_0 \cos(0)$

$x = x_0 (+1)$

$\boxed{x = +x_0}$

$\varphi = 360°$

$x = x_0 \cos(\omega t + \varphi)$

$x = x_0 \cos\left[\frac{2\pi}{T} \times t + \varphi\right]$

let $t = 0$, then :-

$x = x_0 \cos\left[\frac{2\pi}{T} \times 0 + 360°\right]$

$x = x_0 \cos[0 + 360]$

$x = x_0 \cos(360)$

$x = x_0 (+1)$

$\boxed{x = +x_0}$

↳ Clearly, $\varphi = 0°$ & $\varphi = 360°$ with produce same Amplitude

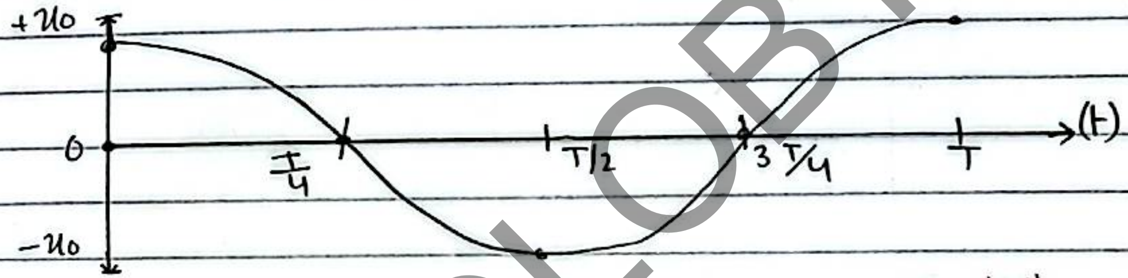
17.5: Graphical Repre[n]tation of Displacement, Velocity & Acceleration

① Displacement

$x = x_0 \cos \omega t$

$x = x_0 \cos\left[\frac{2\pi}{T} \times t\right]$

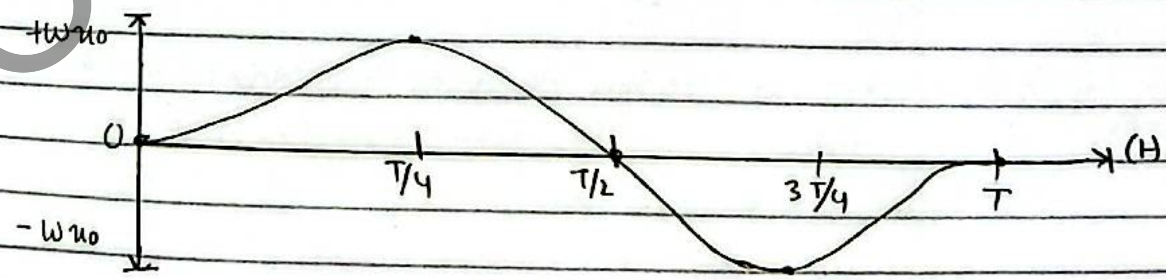
$t=0$	$t=T/4$	$t=T/2$	$t=3T/4$	$t=T$
$\cos\left[\frac{2\pi}{T}\times 0\right]$	$\cos\left[\frac{\cancel{2}\pi}{\cancel{T}}\times\frac{\cancel{T}}{\cancel{4}_2}\right]$	$\cos\left[\frac{\cancel{2}\pi}{\cancel{T}}\times\frac{\cancel{T}}{\cancel{2}}\right]$	$\cos\left[\frac{\cancel{2}\pi}{\cancel{T}}\times\frac{3\cancel{T}}{4}\right]$	$\cos\left[\frac{2\pi}{\cancel{T}}\times\cancel{T}\right]$
$x = x_0\cos(0)$ ①	$x = x_0\cos(\pi/2)$, $x_0 = 0$	$x_0\cos(\pi)$, $x_0 = -1$	$x_0\cos(3\pi/2)$	$x_0\cos 2\pi$
$\boxed{x = +x_0}$	$\boxed{x = 0}$	$\boxed{x = -x_0}$	$\boxed{x = 0}$	$\boxed{x = +x_0}$



② Velocity Since, $V = (x_0\omega)\sin\omega t \Rightarrow V = (x_0\omega)\sin(\omega t)$ → Constant; Shows variation with time.

$V = x_0\omega \sin\left[\frac{2\pi}{T} \times t\right]$

$t=0$	$t=T/4$	$t=T/2$	$t=3T/4$	$t=T$
$x_0\omega\sin\left[\frac{2\pi}{T}\times 0\right]$	$x_0\omega\sin\left[\frac{\cancel{2}\pi}{\cancel{T}}\times\frac{\cancel{T}}{\cancel{4}_2}\right]$	$x_0\omega\sin\left[\frac{\cancel{2}\pi}{\cancel{T}}\times\frac{\cancel{T}}{\cancel{2}}\right]$	$x_0\omega\sin\left[\frac{\cancel{2}\pi}{T}\times\frac{3T}{\cancel{4}}\right]$	$x_0\omega\sin\left[\frac{2\pi}{\cancel{T}}\times\cancel{T}\right]$
$x_0\omega\sin(0)$	$V = x_0\omega\sin(\pi/2)$	$V = x_0\omega\sin(\pi)$	$V = x_0\omega\sin(3\pi/2)$	$x_0\omega\sin(2\pi)$
$V = x_0\omega(0)$	$V = x_0\omega(+1)$	$V = x_0\omega(0)$	$V = x_0\omega(-1)$	$V = x_0\omega(0)$
$\boxed{V=0}$	$\boxed{V = +\omega x_0}$	$\boxed{V = 0}$	$\boxed{V = -\omega x_0}$	$\boxed{V=0}$

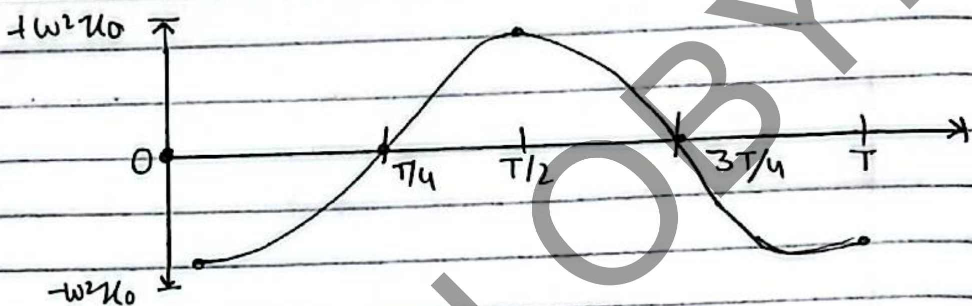


③ Acceleration:-

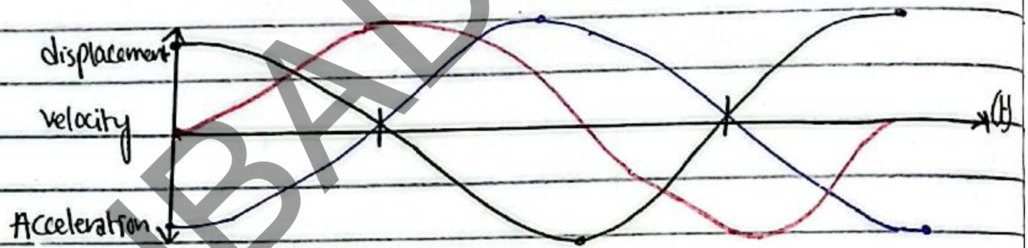
Since, $a = -(x_0 \omega^2) \cos(\omega t)$

$a = -x_0\omega^2 \cos\left[\frac{2\pi}{T} \times t\right]$

$t=0$	$t = T/4$	$t = T/2$	$t = 3T/4$	$t = T$
$-x_0\omega^2 \cos\left[\frac{2\pi \times 0}{T}\right]$	$-x_0\omega^2 \cos\left[\frac{2\pi}{T} \times \frac{T}{4}\right]$	$-x_0\omega^2 \cos\left[\frac{2\pi}{T} \times \frac{T}{2}\right]$	$-x_0\omega^2 \cos\left[\frac{2\pi}{T} \times \frac{3T}{4}\right]$	$-x_0\omega^2 \cos\left[\frac{2\pi}{T}\right]$
$a = -x_0\omega^2 \cos(0)$	$a = -x_0\omega^2 \cos(\pi/2)$	$-x_0\omega^2 \cos(\pi)$	$a = -x_0\omega^2 \cos(3\pi/2)$	$a = -x_0\omega^2 \cos$
$\boxed{a = -\omega^2 x_0}$	$\boxed{a = 0}$	$\boxed{a = +\omega^2 x_0}$	$\boxed{a = 0}$	$\boxed{a = -\omega^2 x_0}$



SOLVED POSSIBLE MCQS



Q. What is the phase difference b/w displacement & Acceleration?
180°, ie they are out of phase

Q. What is the phase difference between displacement & velocity?
90°

Q. Is the velocity wave leading or is the acceleration wave leading?
velocity wave is leading, Acceleration is lagging.

Q. Is displacement leading or acceleration?
wave of displacement is leading.

Q. Are displacement & Acceleration in phase or out of phase?
They have a phase difference of 180° — out of phase.

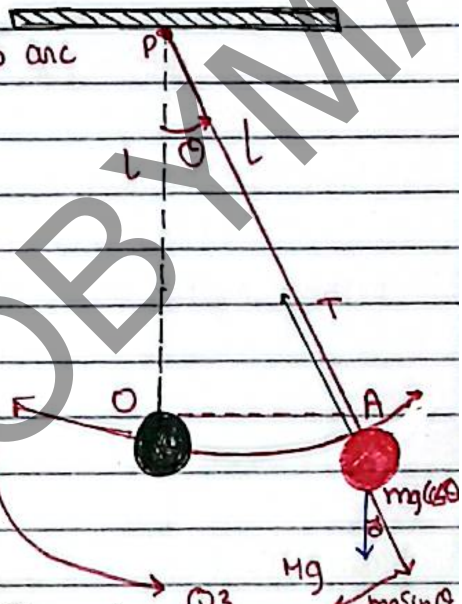
Q. Are displacement & velocity in phase or out of phase?
Displacement & Velocity are neither in phase, nor out of phase, as they don't have a phase difference of 180°, or 0/360°

17.6: SIMPLE PENDULUM

We know that

$\boxed{Mg\cos\theta = T}$ —— (i) → Mg Cos θ is along the string

$\boxed{-mg\sin\theta = F_{res}}$ —— (ii) → Mgsin θ is tangent to arc



Since, $\frac{\sin\theta}{\sin\theta} = \frac{Perp}{hyp}$

$\boxed{\sin\theta = \frac{x}{L}}$ —— (iii)

putting eq (iii) in eq (ii), we get

$\boxed{F_{res} = -mg\left[\frac{x}{L}\right]}$ —— (iv)

from Applied force:-

$\boxed{F_{applied} = ma}$ —— (v)

now Comparing eq (iv) and (v) we get:-

$\cancel{m}a = -\cancel{m}g\left[\frac{x}{L}\right]$

$a = -g\frac{x}{L}$

$\frac{g}{L}$ = Constant

$a = (-x)\left(\frac{g}{L}\right)$

$a = (\text{Constant})(-x)$

$\boxed{a \propto -x}$

Q3
* This is taken for very very small Angle θ where θ < 10° or 0.2 rad.

Q2.
* Negative Sign is present as the restoring force is opposing the motion of the pendulum.

Qs:-
Q1. What is Countering the tension 'T'? mg Cos θ
Q2. What is the role of -ve Sign in Fres?
Q3 Is Sin θ = θ for all values of 'θ'?

Since, $\boxed{a = -\frac{g}{L}(x)}$ —— (i)

& $\frac{g}{L} = \omega^2$ ⟹ $\boxed{a = -\omega^2 x}$ —— (ii) by Comparing eq(i) & eq(ii), we get:-

$$-\omega^2 x = -\frac{g}{L}$$

$$\omega^2 = \frac{g}{L}$$

Adding Square root on both Sides

$$\sqrt{\omega^2} = \sqrt{\frac{g}{L}}$$

$$\boxed{\omega = \sqrt{\frac{g}{L}}} \quad \text{—(iii)}$$

$\omega = \theta/t$

$\because \theta = 1$, rev $= 2\pi$ &

$t = 1$ for 1 rev $= T$

$\omega = \frac{2\pi}{T}$

$\because \frac{1}{T} = f$

$\omega = 2\pi f$

Since we know that:-

$$\boxed{\omega = 2\pi f} \quad \text{—(iv)}$$

putting eq(iv) in eq(iii), we get:-

$$2\pi f = \sqrt{\frac{g}{L}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}}$$

Now, $T = \frac{1}{f}$

So, $T = 2\pi\sqrt{\frac{L}{g}}$

* Time period of a Simple Pendulum depends on the length of the Pendulum & the acceleration due to gravity.

* Time period is independent of the mass of the object.

POSSIBLE PBA Qs (MCQs or SRQs)

Q. If $L' = 2L$, what will value of T become?

$T = 2\pi\sqrt{L/g}$ $\quad L' = 2L$

$T' = 2\pi\sqrt{2L/g}$

$T' = 2\pi\sqrt{2L/g}$

$T' = 2\pi \frac{\sqrt{2} \cdot \sqrt{L}}{g}$

$T' = \sqrt{2}\left[2\pi\sqrt{\frac{L}{g}}\right]$

$$\boxed{T' = \sqrt{2}\, T}$$

Q. If $g' = g/6$, then what will T' be?

$T = 2\pi\sqrt{L/g}$ $\quad g' = g/6$

$T' = 2\pi\sqrt{L/g'}$

$T' = 2\pi\sqrt{L/g/6}$

$T' = 2\pi\sqrt{6L/g}$

$T' = \sqrt{6}\left[2\pi\left[\frac{L}{g}\right]\right]$

$$\boxed{T' = \sqrt{6}\, T}$$

Q.17 $l' = 2l$, what will f' become?

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \qquad f' = \frac{1}{2\pi}\sqrt{\frac{g}{l'}}$$

$\because l' = 2l$

$$f' = \frac{1}{2\pi}\sqrt{\frac{g}{\sqrt{2}\sqrt{l}}}$$

$$f' = \frac{1}{\sqrt{2}}\left[\frac{1}{2\pi}\sqrt{\frac{g}{l}}\right]$$

$$\boxed{f' = \frac{1}{\sqrt{2}} f}$$

Q.17 $g' = g/6$, what will f' become?

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \qquad f' = \frac{1}{2\pi}\sqrt{\frac{g'}{l}}$$

$\because g' = g/6$

$$f' = \frac{1}{2\pi}\sqrt{\frac{g/6}{l}}$$

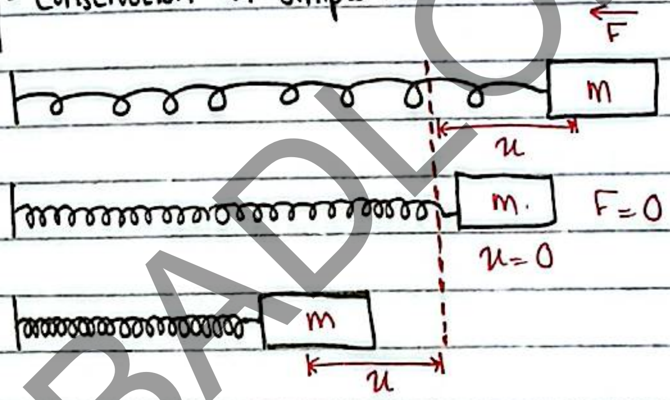
$$f' = \frac{1}{2\pi}\sqrt{\frac{g}{6l}}$$

$$f' = \frac{1}{\sqrt{6}}\left[\frac{1}{2\pi}\sqrt{\frac{g}{l}}\right]$$

$$\boxed{f' = \frac{1}{\sqrt{6}} f}$$

Energy-Conservation in Simple Harmonic Motion

$\Rightarrow$ figure



• **Derivation:**

Since, $\boxed{W = F_{av}\, x}$ —(i)

$\because F_{av} = \dfrac{F_i + F_f}{2}$

as $F_i = 0$, $F_f = F$

so $F_{av} = \dfrac{0+F}{2}$

$$\boxed{F_{av} = \frac{F}{2}} \quad \text{—(ii)}$$

putting eq (ii) in (i),

$$W = F_{av}\, x$$

$$W = \left(\frac{F}{2}\right)\cdot x$$

$$W = \frac{kx^2}{2}$$

$$\Rightarrow W = \frac{1}{2}kx^2$$

[Applied force = −restoring force
Applied force = $-(-kx)$
Applied force = kx]

from the work energy principle,
Work done = P.E

$$\boxed{P.E = \frac{1}{2}kx^2} \longrightarrow (iii)$$

So, we have :-

$$P.E = \frac{1}{2}k(0)^2$$

$$\boxed{P.E = 0}$$

at mean position $x = 0$

This means P.E will be minimum at mean position.

→ for extreme position :- $x = x_0$

$$P.E = \frac{1}{2}k(x_0)^2$$

$$\boxed{P.E = \frac{1}{2}kx_0^2}$$

P.E will be max. at extreme position

Kinetic Energy

→ The energy possessed by a body due to its motion.

Since $K.E = \frac{1}{2}mv^2 \longrightarrow (iv)$

$$\because V = \omega\sqrt{x_0^2 - x^2} \longrightarrow (v)$$

putting eq (v) in eq (iv)

$$K.E = \frac{1}{2}m\left(\omega\sqrt{x_0^2 - x^2}\right)^2$$

$$K.E = \frac{1}{2}m\omega^2\left(\sqrt{x_0^2 - x^2}\right)^2$$

$$K.E = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

$$\because \omega = \sqrt{\frac{k}{m}}$$

$$\omega^2 = \frac{k}{m}$$

$$m\omega^2 = k$$

$$\boxed{\text{Kinetic energy} = \frac{1}{2}K(x_0^2 - x^2)} \quad \text{—(vi)}$$

(a) at mean position, $x = 0$

$$K.E = \frac{1}{2}K(x_0^2 - (0)^2)$$

$$\boxed{K.E = \frac{1}{2}Kx_0^2}$$ → Kinetic energy is maximum at mean position.

(b) At extreme position, $x = x_0$:-

$$\text{Kinetic energy} = \frac{1}{2}K(x_0^2 - x_0^2)$$

$$K.E = \frac{1}{2}K(0)$$

$$\boxed{K.E = 0}$$ → Kinetic energy is minimum at extreme position.

⟹ Total energy:-

$$T.E = P.E + K.E$$

$$T.E = \frac{1}{2}Kx^2 + \frac{1}{2}K(x_0^2 - x^2)$$

$$T.E = \frac{1}{2}Kx^2 + \frac{1}{2}Kx_0^2 - \frac{1}{2}Kx^2$$

$$\boxed{T.E = \frac{1}{2}Kx_0^2} \quad \text{or} \quad \boxed{T.E = \frac{1}{2}m\omega^2 x_0^2}$$

(Question:) At what displacement, the K.E = P.E ?

K.E & P.E when the displacement is 70% is from the total displacement or 7cm from the mean position.

→ MCQ:- In simple harmonic oscillator, K.E becomes equal to P.E when the displacement is 70% of the Amplitude.

*Free Oscillation:-

→ A body is said to be executing free oscillation if it is oscillating at natural frequency under the action of restoring force without the need of external force after initial disturbance.

OR

Free oscillations occur when an object, after an initial disturbance, oscillation at its natural frequency solely under the influence of a restoring force, with no continuous external forces.

* Forced Oscillation:-

If external forces cause the body to oscillate at the frequency of applied force rather than its natural frequency, such oscillations are forced oscillation.

OR

When an external forces drives an object to oscillate at the frequency of applied force rather than its natural frequency, such oscillations are forced oscillations.

* Damped Oscillations:-

are oscillations where the amplitude gradually decrease over time due to a resistive or damping force, like friction or air resistance, which converts the systems energy into heat.

⟹ Heavy damping (overdamping):-
- No oscillations
- Rapid energy loss
- System returns to equilibrium slowly

⟹ Light damping (underdamping):-
- Gradual energy loss
- Oscillations continue with decreasing amplitude.
- System oscillations around equilibrium.

⟹ Critical damping:-
- Fastest return to equilibrium without oscillations.
- No overshoot or oscillations
- System returns to equilibrium in shortest time possible.

* RESONANCE IN A PENDULUM

- Resonance is a phenomenon where a pendulum's amplitude dramatically increases when a small, repeated force is applied at its natural swinging frequency.

→ Sharpness of resonance means how clean & strong the vibrations are at a particular frequency.

→* If damping is high, vibrations die out & resonance becomes weak.

→* If damping is low, vibrations last longer and resonance is stronger.

Ø vibration and production of Natural frequency.

→ Even in Solids, the atoms Vibrate, so it is understood that every matter in the universe has its own Natural frequency.

Scenarios / Considerations:-

• Suppose there is an object containing driven frequency. Natural frequency is 5Hz + Driven frequency is 5Hz then resonance will occur i.e the amplitude & energy will increase.

• When a lady sings at a very high pitch and her voice frequency matches the natural frequency of a glass, resonance will occur.

↳ The Vibrations keep increasing in Amplitude and can even cause the glass to break.

[Question] At what displacement, K.E & P.E are equal?

$$K.E = P.E$$

$$\boxed{K.E = \frac{1}{2}k(x_0^2 - x^2)}$$

$$\boxed{P.E = \frac{1}{2}kx^2}$$

$$\cancel{\frac{1}{2}}\cancel{k}(x_0^2 - x^2) = \cancel{\frac{1}{2}}\cancel{k}x^2$$

$$x_0^2 - x^2 = x^2$$

$$x_0^2 = x^2 + x^2$$

$$x_0^2 = 2x^2 \rightarrow x^2 = \frac{x_0^2}{2}$$

$$\sqrt{x^2} = \sqrt{\frac{x_0^2}{2}}$$

$$x = \frac{x_0}{\sqrt{2}}$$

$$x = \frac{x_0}{\sqrt{2}} \rightarrow x = x_0 \times \frac{1}{\sqrt{2}}$$

$$\boxed{\frac{x}{x_0} = \frac{1}{\sqrt{2}}}$$

For any value of x_0, $x = 0.70 \times 100 = 70\%$