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Subject:

Mathematics.

Class:

Eleventh (11th)

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Unit #01 (Complex Numbers)

Exercise 1.1:-

Q1: Evaluate the following:

(i) i^{31}

Sol:

$$\Rightarrow i^{31}$$

$$\Rightarrow i^{30} \cdot i$$

$$\Rightarrow (i^2)^{15} \cdot i$$

$$\Rightarrow (-1)^{15} \cdot i$$

$$\Rightarrow -1 \cdot i$$

$$\boxed{i^{31} \Rightarrow -i}$$

Ans

$$\boxed{i^2 = -1}$$

$$\boxed{(-1)^{\text{odd}} = -1}$$

$$\boxed{(-1)^{\text{even}} = +1}$$

(ii) $(-i)^6$

Sol:

$$\Rightarrow (-i)^6$$

$$\Rightarrow i^6$$

$$\Rightarrow (i^2)^3$$

$$\Rightarrow (-1)^3$$

$$\boxed{(-i)^6 = -1}$$

Ans

(iii) $(-1)^{-\frac{13}{2}}$

Sol:

$$\Rightarrow (-1)^{-\frac{13}{2}}$$

$$\Rightarrow (i^2)^{-\frac{13}{2}}$$

$$\Rightarrow \frac{1}{i^{13}}$$

$$\Rightarrow \frac{1}{i^{12} \cdot i}$$

$$\Rightarrow \frac{1}{(i^2)^6 \cdot i}$$

$$\Rightarrow \frac{1}{(-1)^6 \cdot i}$$

$$\Rightarrow \frac{1}{1 \cdot i}$$

$$\Rightarrow \frac{1}{i} \times \frac{-i}{-i}$$

$$\Rightarrow \frac{-i}{-i^2}$$

$$\Rightarrow \frac{-i}{(-1)}$$

$$\Rightarrow \frac{-i}{1}$$

$$\boxed{(-1)^{-\frac{13}{2}} \Rightarrow -i} \quad \text{Ans.}$$

$$(iv) \quad \frac{2}{(-1)^{3/2}}$$

Sol:

$$\Rightarrow \frac{2}{(-1)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{2}{(i^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{2}{i^3}$$

$$\Rightarrow \frac{2}{i^2 \cdot i}$$

$$\Rightarrow \frac{2}{-1 \cdot i}$$

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$$\Rightarrow \frac{2}{-i} \times \frac{i}{i}$$

$$\Rightarrow \frac{2i}{-i^2}$$

$$\Rightarrow \frac{2i}{-(-1)}$$

$$\Rightarrow \frac{2i}{1}$$

$$\boxed{\frac{2}{(-1)^{\frac{3}{2}}} \Rightarrow 2i} \quad \text{Ans.}$$

$$(v) i^{23} + i^{58} + i^{21}$$

Sol:

$$\Rightarrow i^{23} + i^{58} \cdot i^{21}$$

$$\Rightarrow i^{22} \cdot i + i^{58} + i^{20} \cdot i$$

$$\Rightarrow (i^2)^{11} \cdot i + (i^2)^{29} + (i^2)^{10} \cdot i$$

$$\Rightarrow (-1)^{11} \cdot i + (-1)^{29} + (-1)^{10} \cdot i$$

$$\Rightarrow -1 \cdot i + -1 + 1 \cdot i$$

$$\Rightarrow -i + i - 1$$

$$i^{23} + i^{58} + i^{21} \Rightarrow -1$$

Ans.

Q2: Write the following complex number in the form of $x + iy$

$$(i) (3+2i) + (2+4i)$$

Sol:

$$(3+2i) + (2+4i) = 3+2i+2+4i$$

$$\Rightarrow (3+2i) + (2+4i) = 5+6i$$

Ans.

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(ii) $(4+3i) - (2+5i)$

Sol:

$$\begin{aligned} (4+3i) - (2+5i) &= 4+3i-2-5i \\ \Rightarrow (4+3i) - (2+5i) &= 2-2i \end{aligned} \quad \text{Ans.}$$

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(iii) $(4+7i) + (4-7i)$

Sol:

$$\begin{aligned} (4+7i) + (4-7i) &= 4+7i+4-7i \\ \Rightarrow (4+7i) + (4-7i) &= 8+0i \end{aligned} \quad \text{Ans.}$$

(iv) $(2+5i) - (2-5i)$

Sol:

$$\begin{aligned} (2+5i) - (2-5i) &= 2+5i-2+5i \\ \Rightarrow (2+5i) - (2-5i) &= 0+10i \end{aligned} \quad \text{Ans.}$$

(v) $(3+2i)(4-3i)$

Sol:

$$\begin{aligned} (3+2i)(4-3i) &= 3(4-3i) + 2i(4-3i) \\ &= 12-9i+8i-6i^2 \\ &= 12-i-6(-1) \\ &= 12-i+6 \\ (3+2i)(4-3i) &= 18-i \end{aligned} \quad \text{Ans.}$$

$$(vi) (3, 2) \div (3, -1)$$

Sol:

$$\begin{aligned} (3, 2) \div (3, -1) &= 3+2i \div 3-i \\ &= \frac{3+2i}{3-i} \times \frac{3+i}{3+i} \\ &= \frac{9+3i+6i+2i^2}{(3)^2-(i)^2} \\ &= \frac{9+9i+2(-1)}{9+1} \\ &= \frac{9+9i-2}{10} \end{aligned}$$

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$$(3, 2) \div (3, -1) = \frac{7+9i}{10} \text{ OR } \frac{7}{10} + \frac{9}{10}i \quad \text{Ans}$$

$$(vii) (1+i)(1-i)(2+i)$$

Sol:

$$\begin{aligned} (1+i)(1-i)(2+i) &= [(i)^2 - (-i)^2] (2+i) \\ &= [1 - (-1)] (2+i) \\ &= (1+1)(2+i) \\ &= 2(2+i) \end{aligned}$$

$$(1+i)(1-i)(2+i) = 4+2i \quad \text{Ans.}$$

(viii) $\frac{1}{2+3i}$

Sol:

$$\frac{1}{2+3i} = \frac{1}{2+3i} \times \frac{2-3i}{2-3i}$$

$$= \frac{(2-3i)}{(2)^2 - (3i)^2}$$

$$= \frac{2-3i}{4-9i^2}$$

$$= \frac{2-3i}{4-9(-1)}$$

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$$\boxed{\frac{1}{2+3i} = \frac{2-3i}{13} \text{ OR } \frac{2}{13} - \frac{3i}{13}}$$

Ans.

Q3: Simplify the following:

(i) $\frac{(2+i)(3-2i)}{1+i}$

Sol:

$$\frac{(2+i)(3-2i)}{1+i} = \frac{2(3-2i)+i(3-2i)}{1+i}$$

$$= \frac{6-4i+3i-2i^2}{1+i}$$

$$= \frac{6-i-2(-1)}{1+i}$$

$$= \frac{6-i+2}{1+i}$$

$$= \frac{8-i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{8(1-i)-i(1-i)}{(1)^2-(i)^2}$$

$$= \frac{8-8i-i+i^2}{1-(-1)}$$

$$= \frac{8-9i+(-1)}{1+1}$$

$$= \frac{8-9i-1}{2}$$

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$$\frac{(2+i)(3-2i)}{1+i} = \frac{7-9i}{2} \text{ OR } \frac{7}{2} - \frac{9}{2}i$$

Ans.

(ii) $\frac{1+i}{(2+i)^2}$

Sol:

$$\frac{1+i}{(2+i)^2} = \frac{1+i}{(2)^2 + (i)^2 + 2(2)(i)}$$

$$= \frac{1+i}{4+i^2+4i}$$

$$= \frac{1+i}{4-1+4i}$$

$$= \frac{1+i}{3+4i} \times \frac{3-4i}{3-4i}$$

$$= \frac{1(3-4i) + i(3-4i)}{(3)^2 - (4i)^2}$$

$$= \frac{3-4i+3i-4i^2}{9-16i^2}$$

$$= \frac{3-i-4(-1)}{9-16(-1)}$$

$$= \frac{3-i+4}{9+16}$$

$$\frac{1+i}{(2+i)^2} = \frac{7-i}{25} \quad \text{OR} \quad \frac{7}{25} - \frac{1}{25}i$$

Ans.

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$$(iii) \frac{1}{3+i} - \frac{1}{3-i}$$

Sol:

- Can be solved by two methods
① Rationalization ② LCM

- We will solve using LCM as it seems easy to me.

$$\frac{1}{3+i} - \frac{1}{3-i} = \frac{(3-i) - (3+i)}{(3+i)(3-i)}$$

$$= \frac{3-i-3-i}{(3)^2 - (i)^2}$$

$$= \frac{-2i}{9+1}$$

$$= \frac{-2i}{10}$$

$$\boxed{\frac{1}{3+i} - \frac{1}{3-i} = -\frac{1}{5}i}$$

Ans.

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$$(iv) (1+i)^{-2} + (1-i)^{-2}$$

Sol:

$$(1+i)^{-2} + (1-i)^{-2} = \frac{1}{(1+i)^2} + \frac{1}{(1-i)^2}$$

$$= \frac{1}{(1)^2 + (i)^2 + 2(1)(i)} + \frac{1}{(1)^2 + (i)^2 - 2(1)(i)}$$

$$= \frac{1}{1 + i^2 + 2i} + \frac{1}{1 + i^2 - 2i}$$

$$= \frac{1}{1 - 1 + 2i} + \frac{1}{1 - 1 - 2i}$$

$$= \frac{1}{2i} + \frac{1}{-2i}$$

$$= \frac{1}{2i} - \frac{1}{2i}$$

$$= \frac{1-1}{2i}$$

$$(1+i)^{-2} + (1-i)^{-2} =$$

$$\frac{0}{2i}$$

$\Rightarrow$

$$0$$

Ans

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$$(v) (2+i)^2 + \frac{7-4i}{2+i}$$

Sol:

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$$(2+i)^2 + \frac{7-4i}{2+i} = \left[(2)^2 + (i)^2 + 2(2)(i) \right] + \left[\frac{7-4i}{2+i} \times \frac{2-i}{2-i} \right]$$

$$= [4 + i^2 + 4i] + \left[\frac{7(2-i) - 4i(2-i)}{(2)^2 - (i)^2} \right]$$

$$= (4 - 1 + 4i) + \left(\frac{14 - 7i - 8i + 4i^2}{4 - 1} \right)$$

$$= (3 + 4i) + \left(\frac{14 - 15i - 4}{4 - 1} \right)$$

$$= (3 + 4i) + \left(\frac{10 - 15i}{5} \right)$$

$$= (3 + 4i) + \left(\frac{10}{5} - \frac{15i}{5} \right)$$

$$= (3 + 4i) + (2 - 3i)$$

$$(2+i)^2 + \frac{7-4i}{2+i} = 5-i$$

Ans.

Q4. Find the values of the real number x and y in each of the following:

(i) $(2+3i)x + (1+3i)y + 2 = 0$

Sol:

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$$(2+3i)x + (1+3i)y + 2 = 0$$

$$\Rightarrow 2x + 3xi + y + 3yi + 2 = 0$$

$$\Rightarrow (2x + y + 2) + (3x + 3y)i = 0$$

$\Rightarrow$ Comparing for real and imaginary part

$$2x + y + 2 = 0$$

$$2x + y = -2 \quad \text{--- ①}$$

$$3x + 3y = 0$$

$$3(x + y) = 0$$

$$x + y = 0 \quad \text{--- ②}$$

- Note:- Same Sign = Subtract, Different Sign = Addition

Subtracting eq ① and ②

$$\begin{array}{rcl} 2x + y & = & -2 \\ \oplus x & \oplus y & = 0 \\ \hline x & & = -2 \end{array}$$

So

$$\boxed{x = -2}$$

Put $x = -2$ in eq (2)

$$x + y = 0$$

$$\Rightarrow -2 + y = 0$$

$$\Rightarrow \boxed{y = 2}$$

Final Answers:

$$\boxed{x = -2}$$

$$\boxed{y = 2}$$

$$(ii) \frac{x}{1+i} + \frac{y}{1-2i} = 1$$

Sol:

$$\frac{x}{1+i} + \frac{y}{1-2i} = 1$$

• By taking L.C.M

$$\Rightarrow \frac{x(1-2i) + y(1+i)}{(1+i)(1-2i)} = 1$$

$$\Rightarrow \frac{x - 2xi + y + yi}{1(1-2i) + i(1-2i)} = 1$$

$$\Rightarrow \frac{(x+y) + (-2x+y)i}{1 - 2i + i - 2i^2} = 1$$

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$$\Rightarrow \frac{(x+y) + (-2x+y)i}{1-i+2} = 1$$

$$\Rightarrow \frac{(x+y) + (-2x+y)i}{3-i} = 1$$

$$\Rightarrow (x+y) + (-2x+y)i = 3-i$$

2) Comparing for Real and Imaginary

$$x+y=3 \text{ --- ①}$$

$$-2x+y=-1 \text{ --- ②}$$

Subtract eq ① and ②

$$\begin{array}{r} x + y = 3 \\ \ominus 2x + y = \ominus 1 \\ \hline 3x = 4 \end{array}$$

$$\Rightarrow \boxed{x = \frac{4}{3}}$$

Put $x = \frac{4}{3}$ in eq ①

$$\Rightarrow x+y=3$$

$$\Rightarrow \frac{4}{3} + y = 3$$

$$y = \frac{3}{1} - \frac{4}{3}$$

$$y = \frac{9-4}{3}$$

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$$\boxed{y = \frac{5}{3}}$$

Final Answers:

$$\boxed{x = \frac{4}{3}}$$

$$\boxed{y = \frac{5}{3}}$$

$$(iii) \frac{x}{2+i} = \frac{1-5i}{3-2i} + \frac{y}{2-i}$$

Sol:

$$\Rightarrow \frac{x}{2+i} = \frac{1-5i}{3-2i} + \frac{y}{2-i}$$

$$\Rightarrow \frac{x}{2+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i}$$

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$$\Rightarrow \frac{x(2-i) - y(2+i)}{(2+i)(2-i)} = \frac{1-5i}{3-2i} \times \frac{3+2i}{3+2i}$$

$$\Rightarrow \frac{2x - xi - 2y - yi}{(2)^2 - (i)^2} = \frac{1(3+2i) - 5i(3+2i)}{(3)^2 - (2i)^2}$$

$$\Rightarrow \frac{(2x - 2y) + (-x - y)i}{4 - (-1)} = \frac{3+2i - 15i - 10i^2}{9 - 4i^2}$$

$$\Rightarrow \frac{(2x - 2y) + (-x - y)i}{5} = \frac{3 - 13i + 10}{13}$$

$$\Rightarrow \frac{(2x-2y) + (-x-y)i}{5} = \frac{13}{13} - \frac{13i}{13}$$

$$\Rightarrow \frac{(2x-2y) + (-x-y)i}{5} = 1 - i$$

$$\Rightarrow (2x-2y) + (-x-y)i = 5-5i$$

$\Rightarrow$ Comparing for Real and Imaginary

$$2x - 2y = 5 \quad \text{--- (1)}$$

$$-x - y = -5$$

$$+(x+y) = +5$$

$$x + y = 5 \quad \text{--- (2)}$$

Multiply the eq (2) by "2"

$$\Rightarrow 2x + 2y = 10 \quad \text{--- (3)}$$

Add eq (1) and (2)

$$2x + 2y = 10$$

$$2x - 2y = 10$$

$$\begin{array}{r} 4x = 15 \\ x = \frac{15}{4} \end{array}$$

So,

$$\boxed{x = \frac{15}{4}}$$

Put $x = \frac{15}{4}$ in (2)

$$\Rightarrow x + y = 5$$

$$\Rightarrow \frac{15}{4} + y = 5$$

$$y = \frac{5}{1} - \frac{15}{4}$$

$$y = \frac{20-15}{4}$$

$$\boxed{y = \frac{5}{4}}$$

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Final Answers:

$$x = \frac{15}{4}$$

$$y = \frac{5}{4}$$

$$(iv) x(1+i)^2 + y(2-i)^2 = 3+10i$$

Sol:

$$\Rightarrow x[(1)^2 + (i)^2 + 2(1)(i)] + y[(2)^2 + (i)^2 - 2(2)(i)] = 3+10i$$

$$\Rightarrow x(1+1+2i) + y(4+1-4i) = 3+10i$$

$$\Rightarrow x(2+2i) + y(5-4i) = 3+10i$$

$$\Rightarrow x(2i) + y(5-4i) = 3+10i$$

$$\Rightarrow 2xi + 5y - 4yi = 3+10i$$

$$\Rightarrow (5y) + (2x-4y)i = 3+10i$$

$\Rightarrow$ Comparing for Real and Imaginary

$$5y = 3$$

$$y = \frac{3}{5}$$

$$y = 1$$

$$2x - 4y = 10 \quad \text{--- (1)}$$

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put $y = 1$ in eq (1)

$$2x - 4y = 10$$

$$\Rightarrow 2x - 4(+1) = 10$$

$$\Rightarrow 2x - 4 = 10$$

$$\Rightarrow 2x = 10 + 4$$

$$\Rightarrow x = \frac{14}{2}$$

$$\Rightarrow x = 7$$

Final Answers:

$$x = 7$$

$$y = 1$$

Q5: Find the complex number z if
 $4z - 3\bar{z} = \frac{1-18i}{2-i}$

Sol:

Let $z = x+yi$ ①, $\bar{z} = x-yi$

$$\Rightarrow 4z - 3\bar{z} = \frac{1-18i}{2-i}$$

$$\Rightarrow 4(x+yi) - 3(x-yi) = \frac{1-18i}{2-i} \times \frac{2+i}{2+i}$$

$$\Rightarrow 4x + 4yi - 3x + 3yi = \frac{1(2+i) - 18i(2+i)}{(2)^2 - (i)^2}$$

$$\Rightarrow (4x-3x) + (4y+3y)i = \frac{2+i-36i-18i^2}{4+1}$$

$$\Rightarrow x + 7yi = \frac{2-35i+18}{5}$$

$$\Rightarrow x + 7yi = \frac{20-35i}{5}$$

$$\Rightarrow x + 7yi = \frac{20}{5} - \frac{35}{5}i$$

$$\Rightarrow x + 7yi = 4 - 7i$$

Comparing for real and imaginary

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$$\boxed{x=4}$$

$$7y = -7$$

$$y = \frac{-7}{7} = -1$$

$$\boxed{y = -1}$$

put value of x and y in eq (1)

$$z = x + yi$$

$$\Rightarrow z = 4 + (-1)i$$

$$\Rightarrow z = 4 - i$$

So,

$$\boxed{z = 4 - i}$$

Final Answers:

$$\boxed{x=4}$$

$$\boxed{y=-1}$$

$$\boxed{z=4-i}$$

Q6: Find the conjugate of the following complex numbers:

(i) $4 - 3i$

Sol:

$$4 - 3i$$

$$\text{let } z = 4 - 3i$$

So,

$$\boxed{\bar{z} = 4 + 3i}$$

Ans

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(ii) $3i+8$

Sol:

Let $z = 3i+8$

So,

$\boxed{z = -3i+8}$ Ans

(iii) $2 + \sqrt{\frac{-1}{5}}$

Sol:

$2 + \sqrt{\frac{-1}{5}}$

Let $z = 2 + \sqrt{\frac{-1}{5}}$

$z = 2 + \frac{\sqrt{-1}}{\sqrt{5}}$

$z = 2 + \frac{i}{\sqrt{5}}$

So,

$\boxed{\bar{z} = 2 - \frac{i}{\sqrt{5}}}$ Ans

(iv) $\frac{5i}{2} - \frac{7}{8}$

Sol:

Let $z = \frac{5i}{2} - \frac{7}{8}$

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So,

$$\boxed{\bar{z} = -\frac{5i}{2} - \frac{7}{8}} \text{ Ans.}$$

Q7: Find the magnitude of the following complex numbers:

(i) $11 + 12i$

Sol:

$$|z| = \sqrt{[\operatorname{Re}(z)]^2 + [\operatorname{Im}(z)]^2}$$

$$z = 11 + 12i$$

$$|z| = \sqrt{(11)^2 + (12)^2}$$

$$|z| = \sqrt{121 + 144}$$

$$\boxed{|z| = \sqrt{265}} \text{ Ans.}$$

(ii) $(2+3i) - (2+6i)$

Sol:

$$\Rightarrow (2+3i) - (2+6i)$$

$$\Rightarrow 2+3i - 2 - 6i$$

$$\Rightarrow 0 - 3i$$

$$\text{Let } z = 0 - 3i$$

$$|z| = \sqrt{(0)^2 + (-3)^2}$$

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$$|z| = \sqrt{0+9}$$

$$\boxed{|z| = 3} \text{ Ans.}$$

(iii) $(2-i)(6+3i)$

Sol:

$$\begin{aligned} &\Rightarrow (2-i)(6+3i) \\ &\Rightarrow 2(6+3i) - i(6+3i) \\ &\Rightarrow 12 + 6i - 6i - 3i^2 \\ &\Rightarrow 12 - 3(-1) \\ &\Rightarrow 15 + 0i \end{aligned}$$

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Let $z = 15 + 0i$

$$|z| = \sqrt{(15)^2 + (0)^2}$$

$$|z| = \sqrt{(15)^2}$$

$$\boxed{|z| = 15} \text{ Ans}$$

(iv) $\frac{3-2i}{2+i}$

Sol:

$$\Rightarrow \frac{3-2i}{2+i}$$

$$\Rightarrow \frac{3-2i}{2+i} \times \frac{2-i}{2-i}$$

$$\Rightarrow \frac{3(2-i) - 2i(2-i)}{2^2 - i^2}$$

$$\Rightarrow \frac{6-3i-4i+2(-1)}{4+1}$$

$$\Rightarrow \frac{6-7i-2}{5}$$

$$\Rightarrow \frac{4-7i}{5} \Rightarrow \frac{4}{5} - \frac{7}{5}i$$

$$\text{Let } z = \frac{4}{5} - \frac{7}{5}i$$

$$|z| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(-\frac{7}{5}\right)^2}$$

$$|z| = \sqrt{\frac{16}{25} + \frac{49}{25}}$$

$$|z| = \sqrt{\frac{16+49}{25}}$$

$$|z| = \sqrt{\frac{65}{25}} \Rightarrow$$

$$|z| = \sqrt{\frac{13}{5}}$$

Ans.

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$$(✓) (\sqrt{3} - \sqrt{-8}) (\sqrt{3} + \sqrt{-8})$$

Sol:

$$\Rightarrow (\sqrt{3} - \sqrt{-8}) (\sqrt{3} + \sqrt{-8})$$

$$\Rightarrow (\sqrt{3})^2 - (\sqrt{-8})^2$$

$$\Rightarrow 3 - (-8)$$

$$\Rightarrow 3 + 8$$

$$z = 11 + 0i$$

$$|z| = \sqrt{(11)^2 + (0)^2}$$

$$|z| = \sqrt{(11)^2}$$

$$\boxed{|z| = 11} \text{ Ans.}$$

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* Exercise 1.2:-

Q1: Show that for any complex number.

$$(i) \operatorname{Re}(iz) = -\operatorname{Im}(z)$$

Sol:

$$\Rightarrow \operatorname{Re}(iz) = -\operatorname{Im}(z)$$

$$\text{Let } z = x + yi$$

$$\Rightarrow \operatorname{Re}[i(x + yi)] = -\operatorname{Im}(x + yi)$$

$$\Rightarrow \operatorname{Re}[xi + yi^2] = -y$$

$$\Rightarrow \operatorname{Re}[xi - y] = -y$$

$$\Rightarrow \boxed{-y = -y}$$

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Hence Proved
L.H.S = R.H.S

$$(ii) \operatorname{Im}(iz) = \operatorname{Re}(z)$$

Sol:

$$\Rightarrow \operatorname{Im}(iz) = \operatorname{Re}(z)$$

$$\text{Let } z = x + yi$$

$$\Rightarrow \operatorname{Im}[i(x+yi)] = \operatorname{Re}(x+yi)$$

$$\Rightarrow \operatorname{Im}[xi+yi^2] = x$$

$$\Rightarrow \operatorname{Im}[xi-y] = x$$

$$\Rightarrow \boxed{x = x}$$

Hence proved
L.H.S = R.H.S

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Q2: Solve the algebraic properties of complex numbers to prove that:

Properties:

1) Commutative Property:

$$z_1 z_2 = z_2 z_1$$

2) Associative Property:

$$(z_1 z_2) z_3 = z_1 (z_2 z_3)$$

OR

$$z_1 (z_2 z_3) = (z_1 z_2) z_3$$

Prove that,

$$(z_1 z_2)(z_3 z_4) = (z_1 z_3)(z_2 z_4) = z_3(z_1 z_2)z_4$$

Sol:

$$\Rightarrow (z_1 z_2)(z_3 z_4) = (z_1 z_3)(z_2 z_4) = z_3(z_1 z_2)z_4$$

$$\Rightarrow (z_1 z_2)(z_3 z_4)$$

$$= [(z_1 z_2) z_3] z_4$$

Associative Property

$$= [z_1(z_2 z_3)] z_4$$

Associative Property

$$= [z_1(z_3 z_2)] z_4$$

Commutative Property

$$= [(z_1 z_3) z_2] z_4$$

Associative Property

$$\Rightarrow [(z_1 z_3)(z_2 z_4)] = (z_1 z_3)(z_2 z_4)$$

Commutative Property

eq — 

So Now,

$$\Rightarrow (z_1 z_2)(z_3 z_4)$$

$$= z_1 [z_2 (z_3 z_4)]$$

Associative Property

$$= z_1 [(z_2 z_3) z_4]$$

Associative Property

$$= z_1 [(z_3 z_2) z_4]$$

Commutative Property

$$= z_1 [z_3 (z_2 z_4)]$$

Associative Property

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$$\begin{aligned}
 &= (z_1 z_3) (z_2 z_4) \\
 &= (z_3 z_1) (z_2 z_4) \\
 &= z_3 [z_1 (z_2 z_4)] \\
 &= z_3 [(z_1 z_2) z_4] \\
 &= z_3 (z_1 z_2) z_4
 \end{aligned}$$

Commutative Property

Commutative Property

Associative Property

Associative Property

From eq ① and ②

$$(z_1 z_2) (z_3 z_4) = (z_1 z_3) (z_2 z_4) = z_3 (z_1 z_2) z_4$$

Q3: Simplify the following:

Important Points:

$E \rightarrow$ Belongs to
 $C \rightarrow$ Complex Number
 $\text{iff} \rightarrow$ if and only if (symbol: $\Leftrightarrow$)

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(i) z is real iff $z = \bar{z}$

Sol:

• If z is real then $z = \bar{z}$
 Let $z = x + 0yi$
 $z = x \rightarrow \textcircled{i}$
 $\bar{z} = x \rightarrow \textcircled{ii}$

from (i) and (ii)

$$z = \bar{z} \quad (\text{Proved})$$

- If $z = \bar{z}$ then z is real

$$z = x + yi$$

$$\bar{z} = x - yi$$

As given

$$z = \bar{z}$$

$$\Rightarrow x + yi = x - yi$$

$$\Rightarrow x + \cancel{yi} - x + \cancel{yi} = 0$$

$$\Rightarrow 2yi = 0$$

$$\Rightarrow y = \frac{0}{2i}$$

$$\Rightarrow \boxed{y = 0}$$

Put $y = 0$ in z :

$$z = x + yi$$

$$z = x + 0i$$

$$\boxed{z = x}$$

Thus, z is real

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$$(ii) \frac{z - \bar{z}}{z + \bar{z}} = i \left(\frac{\text{Im } z}{\text{Re } z} \right)$$

Sol.

$$\Rightarrow \frac{z - \bar{z}}{z + \bar{z}} = i \left(\frac{\text{Im } z}{\text{Re } z} \right)$$

L.H.S:

$$\frac{z - \bar{z}}{z + \bar{z}}$$

$$\text{Let } z = x + yi$$

$$\bar{z} = x - yi$$

$$\Rightarrow \frac{(x + yi) - (x - yi)}{(x + yi) + (x - yi)}$$

$$\Rightarrow \frac{x + yi - x + yi}{x + yi + x - yi}$$

$$\Rightarrow \frac{2yi}{2x}$$

$$\Rightarrow \frac{yi}{x}$$

$$\Rightarrow i \left(\frac{y}{x} \right)$$

$$\Rightarrow i \left[\frac{\text{Im } (z)}{\text{Re } (z)} \right]$$

Hence Proved

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(iii) z is either real or pure Imaginary
iff $(\bar{z})^2 = z^2$

Sol:

1. (i) If z is real then $(\bar{z})^2 = (z)^2$

Let $z = x + 0yi$

$$\boxed{z = x}$$

$$\boxed{\bar{z} = x}$$

$$(\bar{z})^2 = (x)^2$$

$$(\bar{z})^2 = x^2 \quad \text{--- (1)}$$

$$(z)^2 = (x)^2$$

$$(z)^2 = x^2 \quad \text{--- (2)}$$

from (1) and (2)

$$(\bar{z})^2 = (z)^2$$

1. (ii) Now if z is imaginary then

$$(\bar{z})^2 = z^2$$

$$z = 0x + yi$$

$$z = yi$$

$$\bar{z} = -yi$$

$$(\bar{z})^2 = (-yi)^2$$

$$= y^2 i^2$$

$$= -y^2 \quad \text{--- (1)}$$

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$$\begin{aligned}
 (z)^2 &= (yi)^2 \\
 &= y^2 i^2 \\
 &= -y^2 \quad \text{--- ②}
 \end{aligned}$$

From ① and ②

$$(\bar{z})^2 = (z)^2$$

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2. (Conversely) If $(\bar{z})^2 = z^2$ then z is either real or pure Imaginary

$$z = x + yi$$

$$\bar{z} = x - yi$$

As given $(\bar{z})^2 = z^2$

$$(x - yi)^2 = (x + yi)^2$$

$$\Rightarrow (x)^2 + (yi)^2 - 2(x)(yi) = (x)^2 + (yi)^2 + 2(x)(yi)$$

$$\Rightarrow x^2 + y^2 i^2 - 2xyi = x^2 + y^2 i^2 + 2xyi$$

$$\Rightarrow x^2 - y^2 - 2xyi = x^2 - y^2 + 2xyi$$

$$-4xyi = 0$$

$$xy = \frac{0}{-4i}$$

$$\Rightarrow xy = 0$$

So, Either

$$\boxed{x=0} ; \boxed{y=0}$$

If $x=0$ then

$$z = x + yi$$

$$z = 0 + yi$$

$$\boxed{z = yi}$$

z is pure imaginary

If $y=0$ then

$$z = x + yi$$

$$z = x + 0i$$

$$\boxed{z = x}$$

z is real.

Hence Proved

If $(\bar{z})^2 = z^2$ then z is either real or pure imaginary.

Q4: If $z_1 = 2 - 3i$ and $|z_1 z_2| = 16$ find $|z_2|$.

Sol:

$$\text{As } |z_1 z_2| = |z_1| |z_2|$$

$$\Rightarrow |z_1| |z_2| = |z_1 z_2|$$

$$\Rightarrow \sqrt{(2)^2 + (-3)^2}$$

$$\Rightarrow \sqrt{4+9} \quad |z_2| = 16$$

$$\Rightarrow \sqrt{13} \quad |z_2| = 16$$

$$|z_2| = 16$$

$$\text{So, } \boxed{|z_2| = \frac{16}{\sqrt{13}}} \quad \text{Ans.}$$

Q5: If z_1 and z_2 are any two complex numbers then prove that:

$$|z_1 + z_2|^2 - |z_1 - z_2|^2 = 4 \operatorname{Re}(z_1) \operatorname{Re}(z_2)$$

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Sol:

$$\Rightarrow |z_1 + z_2|^2 - |z_1 - z_2|^2 = 4 \operatorname{Re}(z_1) \operatorname{Re}(z_2)$$

$$\text{Let } z_1 = a + bi \\ z_2 = c + di$$

L.H.S:

$$z_1 + z_2 = (a + bi) + (c + di) \\ = a + bi + c + di \\ z_1 + z_2 = (a + c) + (b + d)i$$

$$|z_1 + z_2| = \sqrt{(a+c)^2 + (b+d)^2}$$

Taking square on b/s

$$|z_1 + z_2|^2 = (\sqrt{(a+c)^2 + (b+d)^2})^2$$

$$|z_1 + z_2|^2 = a^2 + c^2 + 2ac + b^2 + d^2 + 2bd \quad \text{--- (1)}$$

Now,

$$z_1 - z_2 = (a + bi) - (c + di) \\ = a + bi - c - di \\ z_1 - z_2 = (a - c) + (b - d)i$$

$$|z_1 - z_2| = \sqrt{(a-c)^2 + (b-d)^2}$$

Taking square on b/s

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$$|z_1 - z_2|^2 = \left((a-c)^2 + (b-d)^2 \right)^2$$

$$|z_1 - z_2|^2 = a^2 + c^2 - 2ac + b^2 + d^2 - 2bd \quad \text{--- (2)}$$

Now from eq (1) and (2)

$$|z_1 + z_2|^2 - |z_1 - z_2|^2 =$$

$$(a^2 + c^2 + 2ac + b^2 + d^2 + 2bd) - (a^2 + c^2 - 2ac + b^2 + d^2 - 2bd)$$

$$|z_1 + z_2|^2 - |z_1 - z_2|^2 = \cancel{a^2} + \cancel{c^2} + 2ac + \cancel{b^2} + \cancel{d^2} + 2bd - \cancel{a^2} - \cancel{c^2} + 2ac$$

$$= 4ac + 4bd$$

$$= 4 \operatorname{Re}(z_1) \operatorname{Re}(z_2) +$$

$$4 \operatorname{Im}(z_1) \operatorname{Im}(z_2)$$

$$\because z_1 = a+bi$$

$$z_2 = c+di$$

Hence Proved
L.H.S = R.H.S

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**Q6: Find the value of λ ; if $\left| \frac{z_1}{z_2} + \lambda \right| = \sqrt{\lambda+2}$
where $z_1 = 3+i$ and $z_2 = 1+i$**

Sol:

Value of $\lambda = ?$

$$\left| \frac{z_1}{z_2} + \lambda \right| = \sqrt{\lambda+2}$$

where

$$z_1 = 3+i ; z_2 = 1+i$$

$$\Rightarrow \frac{z_1}{z_2} = \frac{3+i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{3-3i+i-i^2}{(1)^2 - (i)^2}$$

$$= \frac{3-2i+1}{2}$$

$$= \frac{4-2i}{2}$$

$$= \frac{4}{2} - \frac{2}{2}i$$

$$\boxed{\frac{z_1}{z_2} = 2-i}$$

$$\left| \frac{z_1}{z_2} + 1 \right| = \sqrt{1+2}$$

$$\Rightarrow |2-i+1| = \sqrt{1+2}$$

$$\Rightarrow |(2+1)-i| = \sqrt{1+2}$$

$$\Rightarrow \sqrt{(2+1)^2 + (-1)^2} = \sqrt{1+2}$$

Taking square on b/c

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$$\Rightarrow (\sqrt{(2)^2 + (1)^2 + 2(2)(1) + 1})^2 = (\sqrt{\lambda + 2})^2$$

$$\Rightarrow 4 + \lambda^2 + 4\lambda + 1 = \lambda + 2$$

$$\Rightarrow \lambda^2 + 4\lambda + 5 - \lambda - 2 = 0$$

$$\Rightarrow \lambda^2 + 3\lambda + 3 = 0$$

$$a = 1, b = 3, c = 3$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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$$\lambda = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(3)}}{2}$$

$$\lambda = \frac{-3 \pm \sqrt{9 - 12}}{2}$$

$$\lambda = \frac{-3 \pm \sqrt{-3}}{2}$$

$$\lambda = \frac{-3 \pm \sqrt{3} \sqrt{-1}}{2}$$

$$\begin{aligned} i^2 &= -1 \\ \sqrt{-1} &= i \\ i &= \sqrt{-1} \end{aligned}$$

$$\lambda = \frac{-3 \pm \sqrt{3} i}{2}$$

Q7: Verify that $\sqrt{2} |z| \geq |\operatorname{Re}(z)| + |\operatorname{Im}(z)|$
 Hint: (Start with $(|x| - |y|)^2 \geq 0$)

Sol:

Let $z = x + yi$

As

$$(|x| - |y|)^2 \geq 0$$

$$\Rightarrow |x|^2 + |y|^2 - 2|x||y| \geq 0$$

$$\Rightarrow 2|x|^2 - |x|^2 + 2|y|^2 - |y|^2 - 2|x||y| \geq 0$$

$$\Rightarrow 2|x|^2 + 2|y|^2 \geq |x|^2 + |y|^2 + 2|x||y|$$

$$\Rightarrow 2(|x|^2 + |y|^2) \geq (|x| + |y|)^2$$

$\Rightarrow$ Taking square root on b/s

$$\Rightarrow \sqrt{2(|x|^2 + |y|^2)} \geq \sqrt{(|x| + |y|)^2}$$

$$\Rightarrow \sqrt{2} \sqrt{|x|^2 + |y|^2} \geq |x| + |y|$$

$$\Rightarrow \sqrt{2} |z| \geq |x| + |y|$$

$$\Rightarrow \sqrt{2} |z| \geq |\operatorname{Re}(z)| + |\operatorname{Im}(z)|$$

Q8: Write the following equations and inequations in term of x and y by taking $z = x + yi$

(i) $|2z - i| = 4$

Sol:

$$|2z - i| = 4$$

$$\Rightarrow |2(x + yi) - i| = 4$$

$$\Rightarrow |2x + 2yi - i| = 4$$

$$\Rightarrow |2x + (2y - 1)i| = 4$$

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$$\begin{aligned}
 &\Rightarrow \sqrt{(2x)^2 + (2y-1)^2} = 4 \\
 &\Rightarrow \sqrt{4x^2 + (2y)^2 + 1^2 + 2(2y)(1)} = 4 \\
 &\Rightarrow \sqrt{4x^2 + 4y^2 + 1 + 4y} = 4 \\
 &\Rightarrow \text{Taking square on b/s} \\
 &\Rightarrow (\sqrt{4x^2 + 4y^2 + 1 + 4y})^2 = (4)^2 \\
 &\Rightarrow 4x^2 + 4y^2 + 4y + 1 = 16 \\
 &\Rightarrow 4x^2 + 4y^2 - 4y + 1 - 16 = 0 \\
 &\Rightarrow \boxed{4x^2 + 4y^2 - 4y - 15 = 0} \quad \text{Ans.}
 \end{aligned}$$

(ii) $|z-1| = |\bar{z}+i|$

Sol:

$$|z-1| = |\bar{z}+i|$$

$$z = x+yi$$

$$\bar{z} = x-yi$$

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$$\begin{aligned}
 &\Rightarrow |x+yi-1| = |x-yi+i| \\
 &\Rightarrow |(x-1)+yi| = |x+(y-1)i| \\
 &\Rightarrow \sqrt{(x-1)^2 + y^2} = \sqrt{x^2 + (y-1)^2} \\
 &\Rightarrow \sqrt{x^2 - 1 - 2x + y^2} = \sqrt{x^2 + 1 + y^2 - 2y} \\
 &\Rightarrow \text{Taking square on b/s} \\
 &\Rightarrow (\sqrt{x^2 - 1 - 2x + y^2})^2 = (\sqrt{x^2 + 1 + y^2 - 2y})^2 \\
 &\Rightarrow x^2 - 1 - 2x + y^2 = x^2 + 1 + y^2 - 2y \\
 &\Rightarrow \cancel{x^2} + 1 - 2x + \cancel{y^2} - \cancel{x^2} - 1 - \cancel{y^2} \\
 &\Rightarrow -2x = -2y \\
 &\Rightarrow x = y \\
 &\Rightarrow \boxed{x-y=0} \quad \text{Ans.}
 \end{aligned}$$

$$(iii) (z-4i) + (z+4i) = 10$$

Sol:

$$\Rightarrow |z-4i| + |z+4i| = 10$$

$$\Rightarrow |x+yi-4i| + |x+yi+4i| = 10$$

$$\Rightarrow |x+(y-4)i| + |x+(y+4)i| = 10$$

$$\Rightarrow \sqrt{x^2+(y-4)^2} + \sqrt{x^2+(y+4)^2} = 10$$

$$\Rightarrow \sqrt{x^2+y^2+4^2-2(y)(4)} + \sqrt{x^2+y^2+4^2+2(y)(4)} = 10$$

$$\Rightarrow \sqrt{x^2+y^2+16-8y} + \sqrt{x^2+y^2+16+8y} = 10$$

$$\Rightarrow \sqrt{x^2+y^2+16-8y} = (10 - \sqrt{x^2+y^2+16+8y})$$

$$\Rightarrow \sqrt{x^2+y^2+16-8y} = (10 - \sqrt{x^2+y^2+16+8y})$$

$$\Rightarrow x^2+y^2+16-8y = 100 + x^2+y^2+16+8y - 20\sqrt{x^2+y^2+16+8y}$$

$$\Rightarrow -8y - 8y - 100 = -20\sqrt{x^2+y^2+16+8y}$$

$$\Rightarrow -16y - 100 = -20\sqrt{x^2+y^2+16+8y}$$

$$\Rightarrow 4y + 25 = 5\sqrt{x^2+y^2+16+8y}$$

$$\Rightarrow (4y+25)^2 = (5\sqrt{x^2+y^2+16+8y})^2$$

$$\Rightarrow (4y)^2 + (25)^2 + 2(4y)(25) = 25(x^2+y^2+16+8y)$$

$$\Rightarrow 16y^2 + 625 + 200y = 25x^2 + 25y^2 + 400 + 200y$$

$$\Rightarrow 0 = 25x^2 + 9y^2 - 225$$

Ans.

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$$(iv) \frac{1}{2} \operatorname{Re}(i\bar{z}) = 4$$

Sol:

$$\frac{1}{2} \operatorname{Re}(i\bar{z}) = 4$$

$$z = x + yi$$

$$\bar{z} = x - yi$$

$$\Rightarrow \frac{1}{2} \operatorname{Re}[i(x - yi)] = 4$$

$$\Rightarrow \frac{1}{2} \operatorname{Re}[xi - yi^2] = 4$$

$$\Rightarrow \frac{1}{2} \operatorname{Re}[xi - y(-1)] = 4$$

$$\Rightarrow \frac{1}{2} \operatorname{Re}[xi + y] = 4$$

$$\Rightarrow \frac{1}{2} \operatorname{Re}$$

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$$\Rightarrow \frac{1}{2} y = 4$$

$$\Rightarrow y = 4 \times 2$$

$$\text{So } \boxed{y = 8} \text{ Ans.}$$

$$(v) \operatorname{Im}\left(\frac{z-1}{2i}\right) = -5$$

Sol:

$$\operatorname{Im}\left(\frac{z-1}{2i}\right) = -5$$

$$\Rightarrow \operatorname{Im}\left[\frac{x+yi}{2i}\right] = -5$$

$$\Rightarrow \operatorname{Im} \left[\frac{(x+1)+yi}{2i} \right]$$

$$\Rightarrow \operatorname{Im} \left[\frac{(x+1)+yi}{2i} \times \frac{-i}{-i} \right] = -5$$

$$2) \operatorname{Im} \left[\frac{-xi + i - yi^2}{-2i^2} \right] = -5$$

$$2) \operatorname{Im} \left[\frac{-xi + i + y}{-2(-1)} \right] = -5$$

$$\Rightarrow \operatorname{Im} \left[\frac{y + (-x+1)i}{2} \right] = -5$$

$$\Rightarrow \operatorname{Im} \left[\frac{y}{2} + \frac{(1-x)i}{2} \right] = -5$$

$$\Rightarrow \frac{1-x}{2} = -5$$

$$\Rightarrow 1-x = -5 \times 2$$

$$\Rightarrow -x = 10 - 1$$

$$\Rightarrow -x = 9$$

$$\Rightarrow \boxed{x = 11}$$

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$$(vi) -2 \leq \text{Im}(z+1) \leq 3$$

Sol:

$$\Rightarrow -2 \leq \text{Im}(z+i) \leq 3$$

$$\Rightarrow z = x+yi$$

$$\Rightarrow -2 \leq \text{Im}(x+yi+i) \leq 3$$

$$\Rightarrow -2 \leq \text{Im}(x+(y+1)i) \leq 3$$

$$\Rightarrow -2 \leq y+1 \leq 3$$

$$\Rightarrow$$

$$-2 \leq y+1$$

$$-2-1 \leq y$$

$$-3 \leq y$$

$$y+1 \leq 3$$

$$y \leq 3-1$$

$$y \leq 2$$

$$\boxed{-3 \leq y \leq 2} \text{ Ans.}$$

Q9: Find real and imaginary parts of the followings:

$$(i) (2+4i)^{-1}$$

Sol:

$$\Rightarrow (2+4i)^{-1}$$

$$\Rightarrow \frac{1}{(2+4i)}$$

$$\Rightarrow \frac{1}{2+4i} \times \frac{2-4i}{2-4i}$$

$$\Rightarrow \frac{2-4i}{(2)^2 - (4i)^2}$$

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$$\Rightarrow \frac{2-4i}{4-16i^2}$$

$$\Rightarrow \frac{2-4i}{4+16}$$

$$\Rightarrow \frac{2-4i}{20}$$

$$\Rightarrow \frac{2}{20} - \frac{4}{20}i$$

$$\Rightarrow \boxed{\frac{1}{10} - \frac{1}{5}i} \text{ Ans.}$$

Real part = $\frac{1}{10}$

Imaginary part = $-\frac{1}{5}$

(11) $(3 - \sqrt{-4})^{-2}$

Sol:

$$\Rightarrow (3 - \sqrt{-4})^{-2}$$

$$\Rightarrow (3 - \sqrt{4} \sqrt{-1})^{-2}$$

$$\Rightarrow (3 - 2i)^{-2}$$

$$\Rightarrow \frac{1}{(3-2i)^2}$$

$$\Rightarrow \frac{1}{(3^2 + (2i)^2 - 2(3)(2i))}$$

$$\Rightarrow \frac{1}{9 + 4 - 12i}$$

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$$\Rightarrow \frac{1}{5-12i} \times \frac{5+12i}{5+12i}$$

$$\Rightarrow \frac{5+12i}{(5)^2 - (12i)^2}$$

$$\Rightarrow \frac{5+12i}{25+144}$$

$$\Rightarrow \frac{5+12i}{169}$$

$$\Rightarrow \boxed{\frac{5}{169} + \frac{12}{169}i} \text{ Ans.}$$

$$(iii) \left(\frac{7+2i}{3-i} \right)^{-1}$$

Sol:

$$\Rightarrow \left(\frac{7+2i}{3-i} \right)^{-1}$$

$$\Rightarrow \left(\frac{3-i}{7+2i} \right)^{-1}$$

$$\Rightarrow \frac{3-i}{7+2i} \times \frac{7-2i}{7-2i}$$

$$\Rightarrow \frac{3(7-2i) - i(7-2i)}{(7)^2 - (2i)^2}$$

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$$\Rightarrow \frac{21 - 6i - 7i + 2i^2}{49 - 4i^2}$$

$$\Rightarrow \frac{21 - 13i - 2}{49 + 4}$$

$$\Rightarrow \frac{19 - 13i}{53}$$

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$$\Rightarrow \boxed{\frac{19}{53} - \frac{13}{53}i} - \text{Ans}$$

$$(iv) \left(\frac{4+2i}{2+5i} \right)^{-2}$$

Sol:

$$\Rightarrow \left(\frac{4+2i}{2+5i} \right)^{-2}$$

$$\Rightarrow \left(\frac{2+5i}{4+2i} \right)^2$$

$$\Rightarrow \frac{(2+5i)^2}{(4+2i)^2}$$

$$\Rightarrow \frac{(2)^2 + (5i)^2 + 2(2)(5i)}{(4)^2 + (2i)^2 + 2(4)(2i)}$$

$$\Rightarrow \frac{4 + 25i^2 + 20i}{16 + 4i^2 + 16i}$$

$$\Rightarrow \frac{-21+20i}{12+16i}$$

$$\Rightarrow \frac{-21+20i}{12+16i} \times \frac{12-16i}{12-16i}$$

$$\Rightarrow \frac{-21(12-16i)+20i(12-16i)}{(12)^2 - (16i)^2}$$

$$\Rightarrow \frac{-252 + 336i + 240i - 320i^2}{144 - 256i^2}$$

$$\Rightarrow \frac{-252 + 576i + 320}{144 + 256}$$

$$\Rightarrow \frac{68 + 576i}{400}$$

$$\Rightarrow \frac{68}{400} + \frac{576}{400}i$$

$$\Rightarrow \boxed{\frac{17}{100} + \frac{36}{25}i}$$

Ans

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$$(v) \left(\frac{5-4i}{5+4i} \right)^2$$

Sol:

$$\Rightarrow \left(\frac{5-4i}{5+4i} \right)^2$$

$$\Rightarrow \frac{(5-4i)^2}{(5+4i)^2}$$

$$\Rightarrow \frac{(5)^2 + (4i)^2 + 2(5)(4i)}{(5)^2 + (4i)^2 + 2(5)(4i)}$$

$$\Rightarrow \frac{25 + 16i^2 - 40i}{25 + 16i^2 + 40i}$$

$$\Rightarrow \frac{25 - 16 - 40i}{25 - 16 + 40i}$$

$$\Rightarrow \frac{9 - 40i}{9 + 40i} \times \frac{9 - 40i}{9 - 40i}$$

$$\Rightarrow \frac{9(9 - 40i) - 40i(9 - 40i)}{(9)^2 + (40)^2}$$

$$\Rightarrow \frac{81 - 720i + 1600i^2}{1681}$$

$$\Rightarrow \frac{81 - 720i - 1600}{1681}$$

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$$\Rightarrow \frac{-1519 - 720i}{1681}$$

$$\Rightarrow \boxed{\frac{-1519}{1681} - \frac{720}{1681}i} \text{ Ans}$$

(vi) $\frac{3-7i}{2+5i}$

Sol:

$$\Rightarrow \frac{3-7i}{2+5i}$$

$$\Rightarrow \frac{3-7i}{2+5i} \times \frac{2-5i}{2-5i}$$

$$\Rightarrow \frac{3(2-5i) - 7i(2-5i)}{(2)^2 + (5i)^2}$$

$$\Rightarrow \frac{6 - 15i - 14i + 35i^2}{4 - 25i^2}$$

$$\Rightarrow \frac{6 - 29i - 35}{29}$$

$$\Rightarrow \frac{-29 - 29i}{29}$$

$$\Rightarrow \frac{-29}{29} - \frac{29i}{29} \Rightarrow \boxed{-1-i} \text{ Ans}$$

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Q10 For $z_1 = -3+2i$ and $z_2 = 1-3i$
verify the followings:

(i) $|z_1| = |-z_1| = |\bar{z}_1| = |\overline{-z_1}|$
Soln

$$\Rightarrow |z_1| = |-z_1| = |\bar{z}_1| = |\overline{-z_1}|$$

$$\Rightarrow |z_1| = |-3+2i|$$

$$= \sqrt{(-3)^2 + (2)^2}$$

$$= \sqrt{9+4}$$

$$|z_1| = \sqrt{13} \quad \text{--- (1)}$$

$$|-z_1| = | -(-3+2i) |$$

$$= |3-2i|$$

$$= \sqrt{(3)^2 + (-2)^2}$$

$$= \sqrt{9+4}$$

$$|-z_1| = \sqrt{13} \quad \text{--- (2)}$$

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$$|\bar{z}_1| =$$

$$\boxed{\bar{z}_1 = -3 - 2i}$$

$$|\bar{z}_1| = |-3 - 2i|$$

$$|\bar{z}_1| = \sqrt{9+4}$$

$$|\bar{z}_1| = \sqrt{13} \quad \text{--- (3)}$$

$$|z_1| = | -(-3 - 2i) |$$

$$= |3 + 2i|$$

$$= \sqrt{(3)^2 + (2)^2}$$

$$= \sqrt{9+4}$$

$$= \sqrt{13} \quad \text{--- (4)}$$

from (1), (2), (3), (4)

$$|z_1| = |z_1| = |\bar{z}_1| = |\bar{z}_1|$$

Hence Proved

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$$(ii) \left(\frac{\overline{z_1}}{z_2} \right) = \frac{\overline{z_1}}{\overline{z_2}}$$

Sol:

$$\left(\frac{z_1}{z_2} \right) = \frac{z_1}{z_2}$$

L.H.S =

$$\frac{z_1}{z_2} = \frac{-3+2i}{1-3i} \times \frac{1+3i}{1+3i}$$

$$= \frac{-3(1+3i) + 2i(1+3i)}{(1)^2 - (3i)^2}$$

$$= \frac{-3-7i-6}{1+9}$$

$$= \frac{-9-7i}{10}$$

$$\frac{z_1}{z_2} = \frac{-9}{10} - \frac{7}{10}i$$

$$\left(\frac{z_1}{z_2} \right) = \frac{-9}{10} + \frac{7}{10}i \quad \text{--- (1)}$$

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13.H.S

$$\frac{\overline{z_1}}{\overline{z_2}}$$

$$\overline{z_1} = -3 - 2i$$

$$\overline{z_2} = 1 + 3i$$

$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{-3-2i}{1+3i} \times \frac{1-3i}{1-3i}$$

$$= \frac{-3+9i-2i+6i^2}{1-9i^2}$$

$$= \frac{-3+7i-6}{1+9}$$

$$= \frac{-9+7i}{10}$$

$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{-9}{10} + \frac{7i}{10} \quad \text{--- (2)}$$

From (1) and (2)

$$\left(\frac{\overline{z_1}}{\overline{z_2}} \right) = \frac{\overline{z_1}}{\overline{z_2}}$$

Hence proved.

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$$(iii) \overline{z_1 z_2} = \overline{z_1} \overline{z_2}$$

Sol:

$$\overline{z_1 z_2} = \overline{z_1} \overline{z_2}$$

L.H.S

$$z_1 z_2 = (-3+2i)(1-3i)$$

$$= -3+9i+2i-6i^2$$

$$= -3+11i+6$$

$$= 3+11i$$

$$\overline{z_1 z_2} = 3-11i \quad \text{--- ①}$$

R.H.S:

$$\overline{z_1} = -3-2i$$

$$\overline{z_2} = 1+3i$$

$$\overline{z_1} \overline{z_2} = (-3-2i)(1+3i)$$

$$= -3-9i-2i-6i^2$$

$$= -3-11i+6$$

$$= 3-11i \quad \text{--- ②}$$

from ① and ②

$$\overline{z_1 z_2} = \overline{z_1} \overline{z_2}$$

Hence Proved

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(iv) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$
 Sol:

$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

L.H.S

$$z_1 + z_2 = -3 + 2i + 1 - 3i$$

$$= -2 - i$$

$$\overline{z_1 + z_2} = -2 + i \quad \text{--- (1)}$$

R.H.S:-

$$\overline{z_1} = -3 - 2i$$

$$\overline{z_2} = 1 + 3i$$

$$\overline{z_1} + \overline{z_2} = -3 - 2i + 1 + 3i$$

$$= -2 + i \quad \text{--- (2)}$$

from (1) and (2)

$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

Hence Proved

(v) $|z_1 z_2| = |z_1| |z_2|$

Sol:

$$|z_1 z_2| = |z_1| |z_2|$$

L.H.S:

$$z_1 z_2 = (-3 + 2i)(1 - 3i)$$

$$= -3 + 9i + 2i - 6i^2$$

$$= -3 + 11i + 6$$

$$z_1 z_2 = 3 + 11i$$

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$$|z_1 z_2| = |3 + 11i|$$

$$= \sqrt{(3)^2 + (11)^2}$$

$$= \sqrt{9 + 121}$$

$$|z_1 z_2| = \sqrt{130} \quad \text{--- ①}$$

R.H.S =

$$|z_1| |z_2| = |-3 + 2i| |1 - 3i|$$

$$|z_1| = \sqrt{(-3)^2 + (2)^2} \quad \sqrt{(1)^2 + (-3)^2}$$

$$= \sqrt{9+4} \quad \sqrt{1+9}$$

$$= \sqrt{13} \quad \sqrt{10}$$

$$= \sqrt{13 \times 10}$$

$$= \sqrt{130} \quad \text{--- ②}$$

from ① and ②

$$|z_1 z_2| = |z_1| |z_2|$$

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(vi) $|z_1 + z_2| \leq |z_1| + |z_2|$
 Sol:

L.H.S $|z_1 + z_2| \leq |z_1| + |z_2|$

$$z_1 + z_2 = -3 + 2i + 1 - 3i$$

$$= -2 - i$$

$$|z_1 + z_2| = |-2 - i|$$

$$= \sqrt{(-2)^2 + (-1)^2}$$

$$= \sqrt{4+1}$$

$$= \sqrt{5}$$

$$|z_1| = |-3 + 2i|$$

$$= \sqrt{(-3)^2 + (2)^2}$$

$$= \sqrt{9+4}$$

$$= \sqrt{13}$$

$$|z_2| = |1 - 3i|$$

$$= \sqrt{(1)^2 + (-3)^2}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10}$$

$$|z_1| + |z_2| = \sqrt{13} + \sqrt{10}$$

Now,

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

$$\sqrt{5} \leq \sqrt{13} + \sqrt{10}$$

$$2.23 \leq 3.60 + 3.16$$

$$2.23 \leq 6.76$$

Hence proved

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Exercise 1.3:-

Q1: Factorize the following Polynomials into Linear functions:

(i) $z^2 + 169$

Sol:

$$\begin{aligned} &\Rightarrow z^2 + 169 \\ &\Rightarrow z^2 - (-169) \\ &\Rightarrow z^2 - (i^2 13^2) \\ &\Rightarrow (z)^2 - (13i)^2 \\ &\Rightarrow (z - 13i)(z + 13i) \end{aligned}$$

Ans

(ii) $2z^2 + 18$

Sol:

$$\begin{aligned} &\Rightarrow 2z^2 + 18 \\ &\Rightarrow 2(z^2 + 9) \\ &\Rightarrow 2[(z)^2 - (-9)] \\ &\Rightarrow 2[(z)^2 - (i^2 3^2)] \\ &\Rightarrow 2[(z)^2 - (3i)^2] \\ &\Rightarrow 2(z - 3i)(z + 3i) \end{aligned}$$

Ans

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(iii) $3z^2 + 363$

Sol:

$$\Rightarrow 3z^2 + 363$$

$$\Rightarrow 3[z^2 + 121]$$

$$\Rightarrow 3[z^2 - (-121)]$$

$$\Rightarrow 3[z^2 - (i^2 11^2)]$$

$$\Rightarrow 3[(z)^2 - (11i)^2]$$

$$\Rightarrow \boxed{3(z - 11i)(z + 11i)} \quad \text{Ans}$$

(iv) $z^2 + \frac{3}{25}$

Sol:

$$\Rightarrow z^2 + \frac{3}{25}$$

$$\Rightarrow z^2 - \left(-\frac{3}{25}\right)$$

$$\Rightarrow z^2 - \left(i^2 \frac{(\sqrt{3})^2}{5^2}\right)$$

$$\Rightarrow (z)^2 - \left(\frac{\sqrt{3}}{5}i\right)^2$$

$$\Rightarrow \left(z - \frac{\sqrt{3}}{5}i\right) \left(z + \frac{\sqrt{3}}{5}i\right)$$

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(v) $2z^3 + 3z^2 - 10z - 15$
 Sol:

$$\begin{aligned} &\Rightarrow 2z^3 + 3z^2 - 10z - 15 \\ &\Rightarrow z^2(2z+3) - 5(2z+3) \\ &\Rightarrow (z^2-5)(2z+3) \\ &\Rightarrow (2z+3)[z^2-(\sqrt{5})^2] \end{aligned}$$

$$\Rightarrow (2z+3)(z-\sqrt{5})(z+\sqrt{5}) \quad \text{Ans}$$

(vi) $z^3 - 7z + 6$
 Sol:

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & \downarrow & & & \\ & 1 & 1 & -6 & 0 \end{array}$$

$$\begin{aligned} &\Rightarrow (z-1)(z^2+z-6) \\ &\Rightarrow (z-1)(z^2+3z-2z-6) \\ &\Rightarrow (z-1)[z(z+3)-2(z+3)] \\ &\Rightarrow (z-1)(z-2)(z+3) \end{aligned}$$

Ans.

(vii) $z^3 + 2z^2 - 23z - 60$
 Sol:

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -23 & -60 \\ & \downarrow & & & \\ & 1 & -1 & 3 & +60 \\ & & & & 0 \end{array}$$

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$$\Rightarrow (z+3)(z^2 - z - 20)$$

$$\Rightarrow (z+3)(z^2 - 5z + 4z - 20)$$

$$\Rightarrow (z+3)[z(z-5) + 4(z-5)]$$

$$\Rightarrow \boxed{(z+3)(z+4)(z-5)} \text{ Ans.}$$

(viii) $2z^3 + 9z^2 - 11z - 30$

Sol:

	2	9	-11	-30
2	↓	4	26	36
	2	13	15	0

$$\Rightarrow (z-2)(2z^2 + 13z + 15)$$

$$\Rightarrow (z-2)(2z^2 + 10z + 3z + 15)$$

$$\Rightarrow (z-2)[2z(z+5) + 3(z+5)]$$

$$\Rightarrow \boxed{(z-2)(z+5)(2z+3)} \text{ Ans}$$

(ix) $z^2 - 7z - 8$

Sol:

$$\Rightarrow z^2 - 7z - 8$$

$$\Rightarrow z^2 - 8z + z - 8$$

$$\Rightarrow z(z-8) + 1(z-8)$$

$$\Rightarrow \boxed{(z-8)(z+1)} \text{ Ans.}$$

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$$(x) \quad 4z^2 - 7z - 11$$

Sol:

$$\Rightarrow 4z^2 - 7z - 11$$

$$\Rightarrow 4z^2 - 11z + 4z - 11$$

$$\Rightarrow z(4z-11) + 1(4z-11)$$

$$\Rightarrow (z+1)(4z-11)$$

Ans.

Q2: Solve the following equations by Completing square method.

$$(i) \quad z^2 - 6z + 2 = 0$$

Sol:

$$\Rightarrow z^2 - 6z + 2 = 0$$

$$\Rightarrow z^2 - 6z = -2$$

$$\Rightarrow z^2 - 6z + (-3)^2 = -2 + (-3)^2$$

$$\Rightarrow z^2 - 6z + 9 = -2 + 9$$

$$\Rightarrow (z-3)^2 = -2 + 9$$

$$\Rightarrow (z-3)^2 = 7$$

Taking square root on b/s

$$\Rightarrow \sqrt{(z-3)^2} = \sqrt{7}$$

$$\Rightarrow z-3 = \pm\sqrt{7}$$

$$\Rightarrow z = 3 \pm \sqrt{7}$$

Ans.

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$$(ii) -\frac{1}{2}z^2 - 5z + 2 = 0$$

Sol:

$\Rightarrow$ Multiplying by "-2" on both sides

$$\Rightarrow z^2 + 10z - 4 = 0$$

$$\Rightarrow z^2 + 10z = 4$$

$$\Rightarrow z^2 + 10z + (5)^2 = 4 + (5)^2$$

$$\Rightarrow (z+5)^2 = 4 + 25$$

$$\Rightarrow (z+5)^2 = 29$$

Taking square root on b/s

$$\Rightarrow \sqrt{(z+5)^2} = \sqrt{29}$$

$$\Rightarrow z+5 = \pm \sqrt{29}$$

$$\Rightarrow z = -5 \pm \sqrt{29} \text{ Ans}$$

$$(iii) 4z^2 + 5z = 14$$

Sol:

$$\Rightarrow 4z^2 + 5z = 14$$

$\Rightarrow$ Divide by "4" on b/s

$$\Rightarrow z^2 + \frac{5}{4}z = \frac{7}{2}$$

$$\Rightarrow z^2 + \frac{5}{4}z + \left(\frac{5}{8}\right)^2 = \frac{7}{2} + \left(\frac{5}{8}\right)^2$$

$$\Rightarrow \left(z + \frac{5}{8}\right)^2 = \frac{7}{2} + \frac{25}{64}$$

$$\Rightarrow \left(z + \frac{5}{8}\right)^2 = \frac{249}{64}$$

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Taking square root on both sides

$$\Rightarrow \sqrt{\left(\frac{z+5}{8}\right)^2} = \sqrt{\frac{249}{64}}$$

$$\Rightarrow \frac{z+5}{8} = \pm \frac{\sqrt{249}}{8}$$

$$\Rightarrow z = \frac{-5 \pm \sqrt{249}}{8}$$

$$\Rightarrow \boxed{z = \frac{-5 \pm \sqrt{249}}{8}}$$

Ans.

(iv) $z^2 = 5z - 3$

Sol:

$$\Rightarrow z^2 - 5z = -3$$

$$\Rightarrow z^2 - 5z + \left(\frac{-5}{2}\right)^2 = -3 + \left(\frac{-5}{2}\right)^2$$

$$\Rightarrow \left(\frac{z-5}{2}\right)^2 = \frac{-3}{1} + \frac{25}{4}$$

$$\Rightarrow \left(\frac{z-5}{2}\right)^2 = \frac{13}{4}$$

Taking square root on b/s

$$\Rightarrow \sqrt{\left(\frac{z-5}{2}\right)^2} = \sqrt{\frac{13}{4}}$$

$$\Rightarrow \frac{z-5}{2} = \pm \sqrt{\frac{13}{4}}$$

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$$z = \frac{5}{2} \pm \frac{\sqrt{13}}{2}$$

$$z = \frac{5 \pm \sqrt{13}}{2}$$

Ans

Q3: Solve the following quadratic equations

$$(i) \frac{1}{3}z^2 + 2z - 16 = 0$$

Sol:

Multiplying by "3" on b/s

$$\Rightarrow z^2 + 6z - 48 = 0$$

$$\Rightarrow a = 1, b = 6, c = -48$$

$$\Rightarrow \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-48)}}{2(1)}$$

$$\Rightarrow \frac{-6 \pm \sqrt{36 + 192}}{2}$$

$$\Rightarrow \frac{-6 \pm \sqrt{228}}{2}$$

$$\Rightarrow \frac{-6 \pm 2\sqrt{57}}{2}$$

$$\Rightarrow \cancel{2} \frac{(-3 \pm \sqrt{57})}{\cancel{2}}$$

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$$z = -3 \pm \sqrt{57}$$

Ans

(ii) $z^2 - \frac{1}{2}z + 17 = 0$
Sol:

Multiplying by "2" in b/l

$$\Rightarrow 2z^2 - z + 34 = 0$$

$$\Rightarrow a = 2, b = -1, c = 34$$

$$\Rightarrow \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(34)}}{2(2)}$$

$$\Rightarrow \frac{1 \pm \sqrt{1 - 272}}{4}$$

$$\Rightarrow \frac{1 \pm \sqrt{-271}}{4}$$

$$\Rightarrow \frac{1 \pm \sqrt{271} \sqrt{-1}}{4}$$

$$\Rightarrow \boxed{\frac{1 \pm \sqrt{271}i}{4}} \text{ Ans}$$

(iii) $z^2 - 6z + 25 = 0$
Sol:

$$a = 1, b = -6, c = 25$$

$$z = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(25)}}{2(1)}$$

$$z = \frac{6 \pm \sqrt{36 - 100}}{2}$$

$$z = \frac{6 \pm \sqrt{-64}}{2}$$

$$z = \frac{6 \pm \sqrt{64} \sqrt{-1}}{2}$$

$$z = \frac{6 \pm \sqrt{64}i}{2}$$

$$z = \frac{6 \pm 8i}{2}$$

$$z = \frac{36 \pm 84}{2, 21}$$

$$z = 3 \pm 4i$$

Ans.

$$(iv) z^2 - 9z + 11 = 0$$

Sol:

$$a = 1, b = -9, c = 11$$

$$z = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(11)}}{2(1)}$$

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$$\Rightarrow \frac{9 \pm \sqrt{37}}{2}$$

Ans

(i) $(1-i)z + (1+i)w = 3; 2z - (2+5i)w = 2+3i$

$$\Rightarrow (1-i)z + (1+i)w = 3; \quad 2z - (2+5i)w = 2+3i$$

$$\begin{aligned}(1-i)z + (1+i)w &= 3 \quad \text{--- (1)} \\ 2z - (2+5i)w &= 2+3i \quad \text{--- (2)}\end{aligned}$$

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Multiplying eq ① by "2" and eq ② by "1 - i" and then subtracting eq ② from ①

eq (1) $\Rightarrow$

$$2(1-i)^2 + 2(1+i)w = 6$$

eq (2) $\Rightarrow$

$$2(1-i)z - (1-i)(2+5i)w = (2+3i)(1-i)$$

Subtracting ② from ①

$$\frac{\begin{aligned} &2(1-i)z + 2(1+i)w = 6 \\ &\oplus 2(1-i)z \ominus (1-i)(2+5i)w \oplus (2+3i)(1-i) \end{aligned}}{2(1+i)} = 6$$

$$2(1+i)w + (1-i)(2+5i)w = 6 - (1-i)(2+3i)$$

$$\Rightarrow w [2(1+i) + (1-i)(2+3i)] = 6 - (1-i)(2+3i)$$

$$\Rightarrow w [2+2i+2+5i-2i-5i^2] = 6 - (2+3i-2i-3i^2)$$

$$\Rightarrow w (4+5i+5) = 6 - (2+i+3)$$

$$\Rightarrow w (9+5i) = 6 - 5 - i$$

$$\Rightarrow w = \frac{1-i}{9+5i} \times \frac{9-5i}{9-5i}$$

$$\Rightarrow w = \frac{9-5i-9i+5i^2}{(9)^2 - (5i)^2}$$

$$\Rightarrow w = \frac{9-14i-5}{81+25}$$

$$\Rightarrow w = \frac{4-14i}{106}$$

$$\Rightarrow w = \frac{2(2-7i)}{106}$$

$$\Rightarrow w = \frac{2-7i}{53}$$

$$\boxed{w = \frac{2}{53} - \frac{7}{53}i}$$

Put $\boxed{w = \frac{2-7i}{53}}$ in eq (2)

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$$2z - (2+5i)w = 2+3i$$

$$\Rightarrow 2z - (2+5i)\left(\frac{2-7i}{53}\right) = 2+3i$$

$$\Rightarrow 2z - \frac{(14-14i+10i-35i^2)}{53} = 2+3i$$

$$\Rightarrow 2z - \frac{(4-4i+35)}{53} = 2+3i$$

$$\Rightarrow 2z - \frac{(39-4i)}{53} = 2+3i$$

$$\Rightarrow 2z = \frac{2+3i}{1} + \frac{39-4i}{53}$$

$$\Rightarrow 2z = \frac{53(2+3i) + 39-4i}{53}$$

$$\Rightarrow 2z = \frac{106+159i+39-4i}{53}$$

$$\Rightarrow 2z = \frac{145+155i}{53}$$

$$\Rightarrow z = \frac{145+155i}{106}$$

$$\Rightarrow z = \frac{145}{106} + \frac{155}{106}i$$

$$(z, w) = \left(\frac{145}{106} + \frac{155}{106}i, \frac{2-7i}{53} \right)$$

(ii) $2iz =$
Sol:
 $\Rightarrow 2iz =$

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(ii) $2iz + (3-2i)w = 1+i$; $(1-2i)z + (3+2i)w = 5+6i$
Sol:

$$\Rightarrow 2iz + (3-2i)w = 1+i; (1-2i)z + (3+2i)w = 5+6i$$

$$2iz + (3-2i)w = 1+i \quad \text{--- ①}$$

$$(1-i)z + (3+2i)w = 5+6i \quad \text{--- ②}$$

$\Rightarrow$ Multiply eq ① by $(1-2i)$ and eq ② by $2i$ and then subtracting ② from ①

$$\text{①} \Rightarrow 2i(1-2i)z + (1-2i)(3-2i)w = (1-2i)(1+i)$$

$$\text{②} \Rightarrow 2i(1-i)z + 2i(3+2i)w = 2i(5+6i)$$

$$(1-2i)(3-2i)w - 2i(3+2i)w = (1-2i)(1+i) - 2i(5+6i)$$

$$\Rightarrow w [(1-2i)(3-2i) - 2i(3+2i)] = (1-2i)(1+i) - 2i(5+6i)$$

$$\Rightarrow w [3-2i-6i+4i^2 - 6i-4i^2] = 1+i-2i-2i^2-10i-12i^2$$

$$\Rightarrow w [3-14i] = 1-11i-14i^2$$

$$\Rightarrow w [3-14i] = 1-11i+14$$

$$\Rightarrow w [3-14i] = 15-11i$$

$$\Rightarrow w = \frac{15-11i}{3-14i} \times \frac{3+14i}{3+14i}$$

$$\Rightarrow w = \frac{45 + 210i - 33i - 154i^2}{(3)^2 - (14i)^2}$$

$$\Rightarrow w = \frac{45 + 177i + 154}{9 + 196}$$

$$\Rightarrow w = \frac{199}{205} + \frac{177}{205}i$$

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Put $w = \frac{199 + 177i}{205}$ in eq (1)

$$(1) \Rightarrow 2iz + (3 - 2i)w = 1 + i$$

$$2iz + (3 - 2i) \left(\frac{199 + 177i}{205} \right) = 1 + i$$

$$\Rightarrow \frac{2iz + 597 + 531i - 398i - 354i^2}{205} = 1 + i$$

$$\Rightarrow \frac{2iz + 597 + 133i + 354}{205} = 1 + i$$

$$\Rightarrow \frac{2iz + 951 + 133i}{205} = 1 + i$$

$$\Rightarrow 2iz = 1 + i - \left(\frac{951 + 133i}{205} \right)$$

$$\Rightarrow 2iz = 1 + i - \frac{951}{205} - \frac{133i}{205}$$

$$\Rightarrow 2iz = \left(1 - \frac{951}{205} \right) + \left(1 - \frac{133i}{205} \right)$$

$$\Rightarrow 2iz = \frac{-746}{205} + \frac{72i}{205}$$

$$z = \left(\frac{-746}{205} + \frac{72i}{205} \right) \times \frac{1}{2i}$$

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Sol:

$\Rightarrow$

$\Rightarrow$

$\Rightarrow \frac{1}{2}$

$\Rightarrow$

$$z = \left(\frac{-746}{205} + \frac{72}{205}i \right) \times \frac{1}{2i} \times \frac{2i}{2i}$$

$$z = \left(\frac{-746}{205} + \frac{72}{205}i \right) \frac{2i}{24i^2}$$

$$z = \left(\frac{-746}{205} + \frac{72}{205}i \right) \frac{i}{-2}$$

$$z = \frac{373}{205}i - \frac{36}{205}i^2$$

$$z = \frac{373}{205}i + \frac{36}{205}$$

$$z = \frac{36}{205} + \frac{373}{205}i$$

$$(z, w) = \left(\frac{36}{205} + \frac{373}{205}i, \frac{199}{205} + \frac{177}{205}i \right)$$

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$$(iii) \frac{3}{i}z - (6+2i)w=5; \frac{i}{2}z + \left(\frac{3}{4} - \frac{1}{2}i\right)w = \frac{1}{2} + 2i$$

Sol:

$$\Rightarrow \frac{3}{i}z - (6+2i)w=5; \frac{i}{2}z + \left(\frac{3}{4} - \frac{1}{2}i\right)w = \frac{1}{2} + 2i$$

$$\Rightarrow \frac{3}{i}z - (6+2i)w=5 \quad \text{--- (1)}$$

$$\Rightarrow \frac{i}{2}z + \left(\frac{3}{4} - \frac{1}{2}i\right)w = \frac{1}{2} + 2i \quad \text{--- (2)}$$

$$\textcircled{1} \Rightarrow \left(\frac{3}{i} \times \frac{i}{i} \right) z - (6+2i)w=5$$

$$\Rightarrow \frac{3i}{2}z - (6+2i)w = 5 \quad (3)$$

$$\Rightarrow 3iz - (6+2i)w = 5$$

Multiplying eq (2) by "6"

$$3iz + \left(\frac{9}{2} - 3i\right)w = 3 + 12i \quad (4)$$

Adding eq (3) and (4)

$$\begin{array}{r} -3iz - (6+2i)w = 5 \\ 3iz + \left(\frac{9}{2} - 3i\right)w = 3 + 12i \\ \hline -(6+2i)w + \left(\frac{9}{2} - 3i\right)w = 5 + 3 + 12i \end{array}$$

$$\Rightarrow w \left[-6 - 2i + \frac{9}{2} - 3i \right] = 8 + 12i$$

$$\Rightarrow w \left(\frac{-3}{2} - \frac{5i}{1} \right) = 8 + 12i$$

$$\Rightarrow w \left(\frac{-3 - 10i}{2} \right) = 8 + 12i$$

$$\Rightarrow w(-3 - 10i) = 16 + 24i$$

$$\Rightarrow w = \frac{16 + 24i}{-3 - 10i} \times \frac{-3 + 10i}{-3 + 10i}$$

$$\Rightarrow w = \frac{-48 + 160i - 72i + 240i^2}{(-3)^2 - (10i)^2}$$

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$$\Rightarrow w = \frac{-48 + 88i - 240}{9 + 100}$$

$$\Rightarrow w = \frac{-288 + 88i}{109}$$

$$\Rightarrow w = \frac{-288}{109} + \frac{88}{109}i$$

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Put $w = \frac{-288 + 88i}{109}$ in eq ①

$$\textcircled{1} \Rightarrow \frac{3}{i} z - (6 + 2i)w = 5$$

$$\Rightarrow \frac{3}{i} z - (6 + 2i) \left(\frac{-288 + 88i}{109} \right) = 5$$

$$\Rightarrow \frac{3}{i} z - \left(\frac{-1728 + 528i - 576i + 176i^2}{109} \right) = 5$$

$$\Rightarrow \frac{3}{i} z - \left(\frac{-1728 - 48i - 176}{109} \right) = 5$$

$$\Rightarrow \frac{3}{i} z - \left(\frac{-1904 - 48i}{109} \right) = 5$$

$$\Rightarrow \frac{3}{i} z = 5 + \frac{-1904}{109} + \frac{48}{109}i$$

$$\Rightarrow \frac{3}{i} z = \left(5 - \frac{1904}{109} \right) - \frac{48}{109}i$$

$$\Rightarrow \frac{3}{i} z = \frac{-1359}{109} - \frac{48i}{109}$$

$$\Rightarrow \frac{3}{i} z = 3 \left[\frac{-453}{109} - \frac{16i}{109} \right]$$

$$\Rightarrow z = \left[\frac{-453i}{109} - \frac{16}{109} i^2 \right]$$

$$\Rightarrow z = \frac{-453i}{109} + \frac{16}{109}$$

$$\Rightarrow \boxed{z = \frac{16}{109} - \frac{453i}{109}}$$

$$(z, w) = \left(\frac{16}{109} - \frac{453i}{109}, \frac{-288 + 88i}{109} \right)$$

Ans.

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$$(iv) \frac{1}{1-i} z + (1+i) w = 3 \quad \text{①}$$

Sol,

$$\therefore \frac{1}{1-i} \times \frac{1+i}{1+i} = \frac{1+i}{1-i^2} = \frac{1+i}{1+1} = \frac{1+i}{2}$$

$$\therefore \frac{2 \times i}{i} = \frac{2i}{i^2} = \frac{2i}{-1} = -2i$$

②

$$\frac{1+i}{2}z + (1+i)w = 3 \quad \text{--- (3)} \quad -2iz - (2-3i)w = 2+6i \quad \text{--- (4)}$$

Multiplying eq (3) by $2i$ and eq (4) by $\frac{1+i}{2}$ and then adding eq (3) and (4)

~~(3) + (4)~~

$$(3) \Rightarrow (2i) \frac{1+i}{2}z + 2i(1+i)w = 2i \times 3$$

$$(4) \Rightarrow - (2i) \frac{1+i}{2}z - \frac{1+i}{2}(2-3i)w = \frac{1+i}{2}(2+6i)$$

$$\begin{aligned} & 2i(1+i)w - \frac{1+i}{2}(2-3i)w = 6i + \frac{1+i}{2}(2+6i) \\ \Rightarrow w \left[2i(1+i) - \frac{1+i}{2}(2-3i) \right] &= 6i + \frac{(1+i)2(1+3i)}{2} \end{aligned}$$

$$\Rightarrow w \left[\frac{2i+2i^2 - (2-3i+2i-3i^2)}{2} \right] = 6i + 1+3i+i+3i^2$$

$$\Rightarrow w \left[\frac{2i-2 - (2-i+3)}{2} \right] = 1+10i-3$$

$$\Rightarrow w \left[\frac{2i-2 - (5-i)}{2} \right] = -2+10i$$

$$\Rightarrow w \left[\frac{4i-4-5+i}{2} \right] = -2+10i$$

$$\Rightarrow w \left[\frac{-9+5i}{2} \right] = -2+10i$$

$$\Rightarrow w(-9+5i) = -4+20i$$

$$\Rightarrow w = \frac{-4+20i}{-9+5i} \times \frac{-9-5i}{-9-5i}$$

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$$\Rightarrow \frac{36 + 20i - 180i - 100i^2}{(-9)^2 - (5i)^2}$$

$$\Rightarrow \frac{36 - 160i + 100}{81 + 25}$$

$$\Rightarrow \frac{136 - 160i}{106}$$

$$\Rightarrow \frac{136}{106} - \frac{160i}{106}$$

$$w \Rightarrow \frac{68}{53} - \frac{80i}{53}$$

Put value of w in eq ①

$$\text{eq ①} \Rightarrow \frac{1}{1-i} z + (1+i) \left(\frac{68-80i}{53} \right) = 3$$

$$\Rightarrow \frac{1}{1-i} z + \frac{68 - 80i + 68i - 80i^2}{53} = 3$$

$$\Rightarrow \frac{1}{1-i} z + \frac{68 - 12i + 80}{53} = 3$$

$$\Rightarrow \frac{1}{1-i} z = 3 - \frac{148}{53} + \frac{12i}{53}$$

$$\Rightarrow z = \left(\frac{11}{53} + \frac{12i}{53} \right) (1-i)$$

$$\Rightarrow z = \frac{11}{53} - \frac{11i}{53} + \frac{12i}{53} - \frac{12i^2}{53}$$

$$\Rightarrow z = \frac{11}{53} + \frac{1i}{53} + \frac{12}{53}$$

$$z = \frac{23}{53} + \frac{1i}{53}$$

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Exercise 1.4:

Basic Concept:

Complex number

$$x + yi$$

$$= r [\cos \theta + i \sin \theta]$$

Euler's formula
 $= r e^{i\theta}$

- Rectangular form
- Algebraic form
- Cartesian form

- Polar form
- Trigonometric form

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right) \quad \text{— argument of } z$$

$\theta = ?$

II $\theta = \pi - \alpha$	I $\theta = \alpha$
III $\theta = -\pi + \alpha$	IV $\theta = -\alpha$

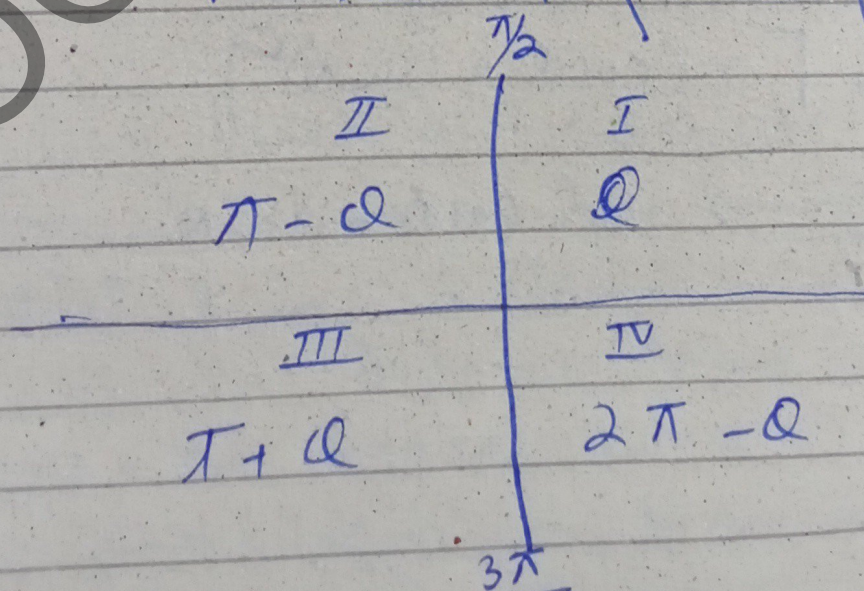
Principle Argument of z
 $-\pi < \theta \leq \pi$

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Normal Trigonometry



1. Rectangular form / ~~Trigonometric~~ Cartesian form: $(x+yi)$ Algebraic form
2. Polar form / Trigonometric form: $r(\cos \theta + i \sin \theta)$
3. Euler's Formula: $e^{i\theta}$
4. Argument of z : $\arg(z) = \alpha = \tan^{-1} \left(\frac{y}{x} \right)$
5. Principle Argument of z : $\text{Arg}(z) = \theta = ?$
 $\theta = ?$

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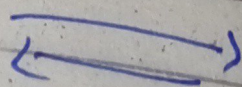
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Complex Numbers

→ If the words used are (Rectangular form OR Algebraic form OR Cartesian form) then it means $(x+yi)$

⇒ If words used are (Polar form OR Trigonometric form) then it means $[r(\cos \theta + i \sin \theta)]$

$x+yi$

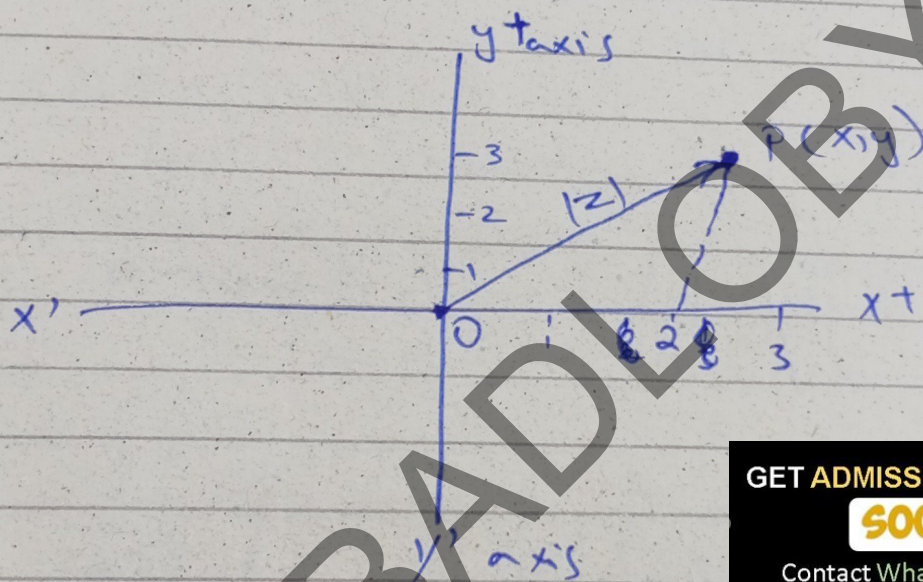


$r(\cos \theta + i \sin \theta)$

1. Rectangular Form / Algebraic Form / Cartesian form.

→ Rectangular Coordinate System

$(x+iy)$ OR (x,y) → same
i.e. $(2+3i)$ OR $(2,3)$ → same



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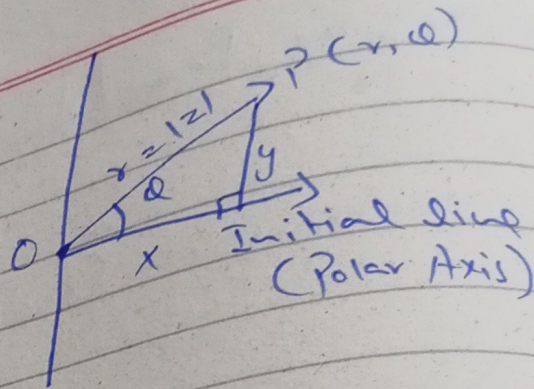
2. Polar Form / Trigonometric Form
 $\sigma (\cos \theta + i \sin \theta)$

→ O is a fixed point called pole.

→ Horizontal line emerging from pole is called initial line (Polar axis)

→ σ is the distance from O to P

→ θ is the angle which is measured in anticlockwise from initial line.



By Pythagoras Theorem
 $(h)^2 = (b)^2 + (p)^2$

$$r^2 = x^2 + y^2$$

Taking " $\sqrt{\quad}$ " on b/s

$$\sqrt{r^2} = \sqrt{x^2 + y^2}$$

$$r = \sqrt{x^2 + y^2}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hyp}}$$

$$\cos \theta = \frac{x}{r}$$

$$r \cos \theta = x \quad \text{--- (1)}$$

$$\sin \theta = \frac{\text{Perp}}{\text{Hyp}}$$

$$\sin \theta = \frac{y}{r}$$

$$r \sin \theta = y \quad \text{--- (2)}$$

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from (1) and (2)

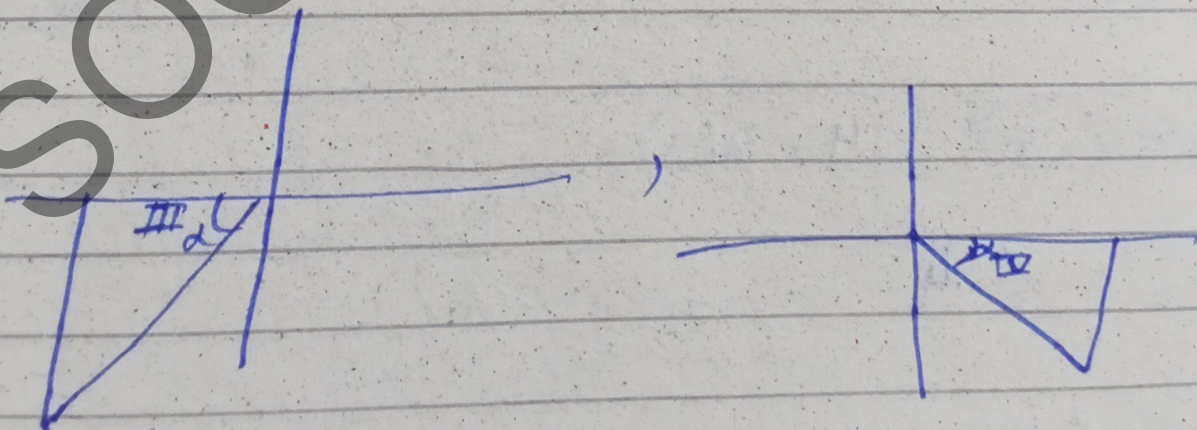
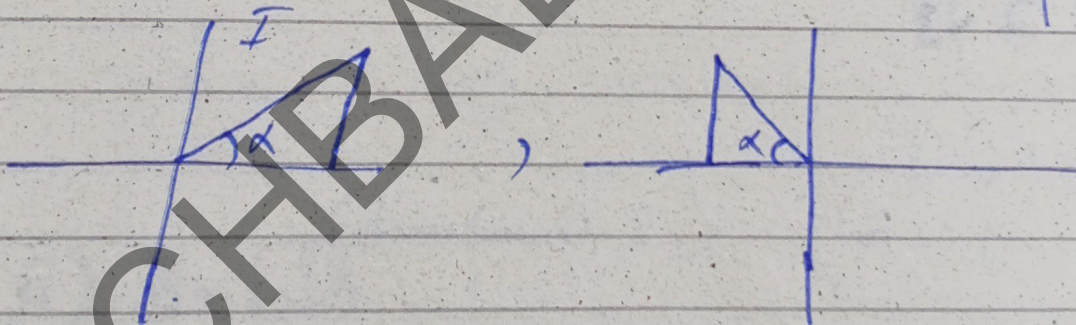
$$\begin{aligned}x + yi &= r(\cos \alpha + i \sin \alpha) \\&= r(\cos \alpha + i \sin \alpha) \\&\text{OR} \\&r(\cos \alpha + i \sin \alpha)\end{aligned}$$

3. Euler's Formula

$$x + yi = r(\cos \alpha + i \sin \alpha) = r e^{i\alpha}$$

$\cos \alpha + i \sin \alpha = e^{i\alpha}$ is known as Euler's formula

4. argument of z / Angle of $z : \alpha$
 $\arg(z) = \alpha = \tan^{-1} \left| \frac{y}{x} \right|$



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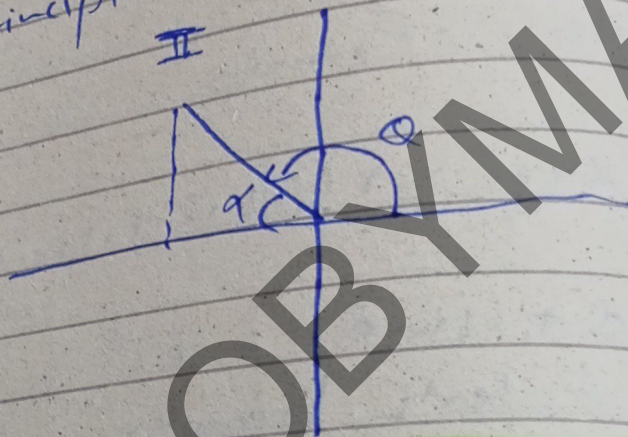
5. Principle Argument of z / Principle value of $\text{Arg}(z)$

$$\text{Arg}(z) = \theta = ?$$

θ that lies in the interval
is called Principle

$$-\pi < \theta \leq \pi$$

Argument of z



Q1: Write following complex number in polar form.

(i) $2 + i2\sqrt{3}$

Sol:

$$z = 2 + i2\sqrt{3}$$

$$x = 2, y = 2\sqrt{3} \quad (\text{I Quadrant})$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(2)^2 + (2\sqrt{3})^2} = \sqrt{4 + 4(3)}$$

$$r = \sqrt{16} = 4$$

$$\theta = \tan^{-1} \left| \frac{y}{x} \right|$$

(1)

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(ii)
Sol:

$$\arg z = \tan^{-1} \left| \frac{2\sqrt{3}}{2} \right|$$

$$\arg z = \tan^{-1} (\sqrt{3})$$

$$\arg z = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$$

For I Quadrant the Principle of argument

$$\theta = \alpha$$

$$\theta = \frac{\pi}{3}$$

from (1) and (2)

$$z = r (\cos \theta + i \sin \theta)$$

$$z = 4 \left[\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right]$$

$$z = 4 \left[\cos \left(\frac{\pi}{3} + 2k\pi \right) + i \sin \left(\frac{\pi}{3} + 2k\pi \right) \right] \quad \text{Ans}$$

(ii) $3 - i\sqrt{3}$

Sol:

Let $z = 3 - i\sqrt{3}$

$x = +3, y = -\sqrt{3}$ (IV Quadrant)

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(3)^2 + (-\sqrt{3})^2}$$

$$r = \sqrt{9 + 3} \Rightarrow r = \sqrt{12}$$

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$$r = \sqrt{4 \times 3}$$

$$\boxed{r = 2\sqrt{3}} \quad \text{--- (1)}$$

$$\alpha = \tan^{-1} \left| \frac{y}{x} \right|$$

$$\alpha = \tan^{-1} \left| \frac{-\sqrt{3}}{3} \right|$$

$$\alpha = \tan^{-1} \left(\frac{\sqrt{3}}{3} \right)$$

$$\alpha = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$\alpha = 30^\circ = \frac{30 \times \pi}{180} = \frac{\pi}{6}$$

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For IV Quadrant Principal argument

$$\theta = -\alpha$$

$$\boxed{\theta = -\frac{\pi}{6}} \quad \text{--- (2)}$$

from (1) and (2)

$$z = r (\cos \theta + i \sin \theta)$$

$$z = 2\sqrt{3} \left[\cos \left(-\frac{\pi}{6} \right) + i \sin \left(-\frac{\pi}{6} \right) \right]$$

$$z = 2\sqrt{3} \left[\cos \frac{\pi}{6} - i \sin \frac{\pi}{6} \right]$$

(iii)
Sol:

$$z = 2\sqrt{3} \left[\cos\left(\frac{\pi}{6} + 2k\pi\right) - i \sin\left(\frac{\pi}{6} + 2k\pi\right) \right] \quad \text{Ans}$$

(iii) $-2 - i2$

Sol:

Let $z = -2 - i2$

$x = -2, y = -2$ (III Quadrant)

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-2)^2 + (-2)^2}$$

$$r = \sqrt{4 + 4} = \sqrt{8}$$

$$r = \sqrt{4 \times 2}$$

$$r = 2\sqrt{2} \quad \text{--- (1)}$$

$$\alpha = \tan^{-1} \left| \frac{y}{x} \right|$$

$$\alpha = \tan^{-1} \left| \frac{-2}{-2} \right|$$

$$\alpha = \tan^{-1} (1)$$

$$\alpha = \frac{45^\circ}{\frac{180}{4}} \times \frac{\pi}{4} = \frac{\pi}{4}$$

Q For III Quadrant, the principle argument

$$\alpha = -\pi + \alpha$$

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$$\Rightarrow -\pi + \frac{\pi}{4}$$

$$\theta = \frac{-3\pi}{4}$$

from ① and ②

$$z = r(\cos \theta + i \sin \theta)$$

$$z = 2\sqrt{2} \left[\cos \left(\frac{-3\pi}{4} \right) + i \sin \left(\frac{-3\pi}{4} \right) \right]$$

$$z = 2\sqrt{2} \left[\cos \frac{3\pi}{4} - i \sin \frac{3\pi}{4} \right]$$

$$z = 2\sqrt{2} \left[\cos \left(\frac{3\pi}{4} + 2k\pi \right) - i \sin \left(\frac{3\pi}{4} + 2k\pi \right) \right] \text{ Ans.}$$

$$(iv) \frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$$

Sol:

$$\text{Let } z = \frac{i-1}{\cos 60 + i \sin 60}$$

$$= \frac{i-1}{\frac{1}{2} + i \frac{\sqrt{3}}{2}} = \frac{i-1}{\frac{1+i\sqrt{3}}{2}}$$

$$\Rightarrow \frac{2(i-1)}{1+i\sqrt{3}} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = 2 \left[\frac{i(1-\sqrt{3}i) - 1(1-\sqrt{3}i)}{(1)^2 - (\sqrt{3}i)^2} \right]$$

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$$\Rightarrow 2 \left[\frac{i - \sqrt{3}i - 1 + \sqrt{3}i}{1 + 3} \right] = 2 \left[\frac{i + \sqrt{3} - 1 + \sqrt{3}i}{4} \right]$$

$$\Rightarrow 2 \left(\frac{(\sqrt{3} - 1) + (\sqrt{3} + 1)i}{4} \right) = \frac{\sqrt{3} - 1}{2} + \frac{\sqrt{3} + 1}{2}i$$

$$\Rightarrow x = \frac{\sqrt{3} - 1}{2}, \quad y = \frac{\sqrt{3} + 1}{2} \quad (\text{I Quadrant})$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{\left(\frac{\sqrt{3} - 1}{2}\right)^2 + \left(\frac{\sqrt{3} + 1}{2}\right)^2}$$

$$r = \sqrt{\frac{3 + 1 - 2\sqrt{3}}{4} + \frac{3 + 1 + 2\sqrt{3}}{4}}$$

$$r = \sqrt{\frac{4 + 3 + 1}{4}}$$

$$r = \sqrt{\frac{8}{4}} \Rightarrow \boxed{r = \sqrt{2}} \quad \text{--- (1)}$$

$$\alpha = \tan^{-1} \left| \frac{\frac{\sqrt{3} + 1}{2}}{\frac{\sqrt{3} - 1}{2}} \right|$$

$$\alpha = \tan^{-1} \left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1} \right) = 75^\circ \times \frac{\pi}{180} = \boxed{\frac{5\pi}{12}}$$

For I Quadrant, the principle argument
 $\theta = \alpha$

$$\boxed{\theta = \frac{5\pi}{12}} \quad \text{--- (2)}$$

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from ① and ②

Polar form $\Rightarrow$

$$z = \sqrt{2} \left[\cos\left(\frac{5\pi}{12}\right) + i \sin\left(\frac{5\pi}{12}\right) \right]$$

$$z = \sqrt{2} \left[\cos\left(\frac{5\pi}{12} + 2k\pi\right) + i \sin\left(\frac{5\pi}{12} + 2k\pi\right) \right]$$

Q2: Write the complex numbers in rectangular form?

(i) $\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$

Sol:

$$\Rightarrow \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$\Rightarrow \text{Let } z = (\cos 30^\circ + i \sin 30^\circ) (\cos 60^\circ + i \sin 60^\circ)$$

$$\Rightarrow z = \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)$$

$$z = \left(\frac{\sqrt{3} + i}{2} \right) \left(\frac{1 + \sqrt{3}i}{2} \right)$$

$$z = \frac{\sqrt{3}}{4} (1 + \sqrt{3}i) + i \frac{\sqrt{3}}{4} (1 + \sqrt{3}i)$$

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$$z = \frac{\sqrt{3} + (\sqrt{3})^2 i + i + \sqrt{3} i^2}{4}$$

$$z = \frac{\sqrt{3} + 3i + i + \sqrt{3}}{4}$$

$$z = \frac{4i}{4}$$

$$\boxed{z = 0 + i} \text{ Ans}$$

$$(ii) \frac{\cos \frac{\pi}{6} - i \sin \frac{\pi}{6}}{2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})}$$

Sol:

$$\frac{\cos \frac{\pi}{6} - i \sin \frac{\pi}{6}}{2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})}$$

$$z \Rightarrow \frac{\cos 30^\circ - i \sin 30^\circ}{2(\cos 60^\circ + i \sin 60^\circ)}$$

$$\Rightarrow \frac{\frac{\sqrt{3}}{2} - i \frac{1}{2}}{2(\frac{1}{2} + i \frac{\sqrt{3}}{2})}$$

$$\Rightarrow \frac{\frac{\sqrt{3} - i}{2}}{2(\frac{1 + \sqrt{3}i}{2})}$$

$$\Rightarrow \frac{\sqrt{3} - i}{2(1 + \sqrt{3}i)} \times \frac{1 - \sqrt{3}i}{1 - \sqrt{3}i}$$

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$$\Rightarrow \frac{1}{2} \left[\frac{\sqrt{3} - (\sqrt{3})^2 i - i + \sqrt{3} i^2}{(1)^2 - (\sqrt{3} i)^2} \right]$$

$$\Rightarrow \frac{1}{2} \left[\frac{\sqrt{3} - 3i - i - \sqrt{3}}{1 + 3} \right]$$

$$\Rightarrow \frac{1}{2} \left[\frac{-4i}{4} \right]$$

$$\Rightarrow \frac{-i}{2}$$

$$\boxed{z = \frac{-i}{2}} \quad \text{Ans}$$

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Q3: If $(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) \dots$
 $\dots (x_n + iy_n) = a + ib$, Show that

$$(i) (x_1^2 + y_1^2)(x_2^2 + y_2^2)(x_3^2 + y_3^2) \dots$$

$$\dots (x_n^2 + y_n^2) = a^2 + b^2$$

Sol:

$$\Rightarrow (x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) \dots (x_n + iy_n) = a + ib$$

Taking Modulus on both

$$\Rightarrow |(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) \dots (x_n + iy_n)| = |a + ib|$$

$$\Rightarrow |x_1 + iy_1| |x_2 + iy_2| |x_3 + iy_3| \dots |x_n + iy_n| = |a + ib|$$

$$\sqrt{x_1^2 + y_1^2} \sqrt{x_2^2 + y_2^2} \sqrt{x_3^2 + y_3^2} \dots \sqrt{x_n^2 + y_n^2} = \sqrt{a^2 + b^2}$$

$$\Rightarrow \sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)(x_3^2 + y_3^2) \dots (x_n^2 + y_n^2)} = \sqrt{a^2 + b^2}$$

Taking square on b/s

$$(x_1^2 + y_1^2)(x_2^2 + y_2^2)(x_3^2 + y_3^2) \dots (x_n^2 + y_n^2) = a^2 + b^2$$

Hence Proved.

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$$(ii) \sum_{r=1}^n \tan^{-1} \left(\frac{y_r}{x_r} \right) = \tan^{-1} \left(\frac{b}{a} \right) + 2K\pi, K \in \mathbb{Z}$$

Sol:

$$(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) \dots (x_n + iy_n) = a + ib$$

Taking arg on b/s

$$\arg [(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) \dots (x_n + iy_n)] = \arg (a + ib)$$

$$\because \arg (z_1 z_2) = \arg z_1 + \arg z_2$$

$$\Rightarrow \arg (x_1 + iy_1) + \arg (x_2 + iy_2) + \arg (x_3 + iy_3) \dots + \arg (x_n + iy_n) = \arg (a + ib)$$

$$\Rightarrow \tan^{-1} \left| \frac{y_1}{x_1} \right| + \tan^{-1} \left| \frac{y_2}{x_2} \right| + \tan^{-1} \left| \frac{y_3}{x_3} \right| + \dots + \tan^{-1} \left| \frac{y_n}{x_n} \right| = \tan^{-1} \left| \frac{b}{a} \right| + 2K\pi$$

$$\Rightarrow \sum_{r=1}^n \tan^{-1} \left(\frac{y_r}{x_r} \right) = \tan^{-1} \left(\frac{b}{a} \right) + 2K\pi, K \in \mathbb{Z}$$

Hence Proved.

Q4: If $\frac{1+z}{1-z} = \cos 2\theta + i \sin 2\theta$, show that

$$z = i \tan \theta$$

Sol:

By componendo-Dividendo Theorem

$$\Rightarrow \frac{(1+z) + (1-z)}{(1+z) - (1-z)} = \frac{(\cos 2\theta + i \sin 2\theta) + 1}{(\cos 2\theta + i \sin 2\theta) - 1}$$

$$\Rightarrow \frac{1+z+1-z}{1+z-1-z} = \frac{\cos 2\theta + i \sin 2\theta + 1}{\cos 2\theta + i \sin 2\theta - 1}$$

$$\Rightarrow \frac{2}{2z} = \frac{\cos 2\theta + i \sin 2\theta + 1}{\cos 2\theta + i \sin 2\theta - 1}$$

$$\Rightarrow \frac{1}{z} = \frac{\cos 2\theta + i \sin 2\theta + 1}{\cos 2\theta + i \sin 2\theta - 1}$$

$$\Rightarrow z = \frac{\cos 2\theta + i \sin 2\theta - 1}{\cos 2\theta + i \sin 2\theta + 1}$$

$$\Rightarrow z = \frac{1 - 2\sin^2\theta + i 2\sin\theta \cos\theta - 1}{2\cos^2\theta - 1 + i 2\sin\theta \cos\theta + 1}$$

$$\Rightarrow z = \frac{2\sin\theta (-\sin\theta + i \cos\theta)}{2\cos\theta (\cos\theta + i \sin\theta)}$$

$$z = \tan \theta \left(\frac{i 2\sin\theta + i \cos\theta}{\cos\theta + i \sin\theta} \right)$$

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$$z = i \tan \theta \frac{(i \sin \theta + \cos \theta)}{(i \sin \theta + \cos \theta)}$$

$$\boxed{z = i \tan \theta}$$

Hence proved.

Q5: If $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$, show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos (\alpha + \beta + \gamma)$

Sol:

Euler Identity

$$\cos \theta + i \sin \theta = e^{i\theta}$$

$$\Rightarrow e^{i\alpha} = a, \quad e^{i\beta} = b, \quad e^{i\gamma} = c$$

$$a + b + c = e^{i\alpha} + e^{i\beta} + e^{i\gamma}$$

$$a + b + c = (\cos \alpha + i \sin \alpha) + (\cos \beta + i \sin \beta) + (\cos \gamma + i \sin \gamma)$$

$$\begin{aligned} a + b + c &= \cos \alpha + i \sin \alpha + \cos \beta + i \sin \beta + \cos \gamma + i \sin \gamma \\ &= \cos \alpha + \cos \beta + \cos \gamma + i (\sin \alpha + \sin \beta + \sin \gamma) \end{aligned}$$

As given

$$\begin{aligned} \cos \alpha + \cos \beta + \cos \gamma &= \sin \alpha + \sin \beta + \sin \gamma = 0 \\ &= 0 + i0 \end{aligned}$$

$$a + b + c = 0$$

Taking cube on b/s

$$(a + b + c)^3 = 0$$

$$a^3 + b^3 + c^3 - 3abc = 0$$

$$a^3 + b^3 + c^3 = 3abc$$

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$$\Rightarrow (e^{i\alpha})^3 + (e^{i\beta})^3 + (e^{i\gamma})^3 = 3(e^{i\alpha})(e^{i\beta})(e^{i\gamma})$$

$$\Rightarrow e^{i3\alpha} + e^{i3\beta} + e^{i3\gamma} = 3e^{i(\alpha+\beta+\gamma)}$$

$$\Rightarrow (\cos 3\alpha + i\sin 3\alpha) + (\cos 3\beta + i\sin 3\beta) + (\cos 3\gamma + i\sin 3\gamma) = 3[\cos(\alpha+\beta+\gamma) + i\sin(\alpha+\beta+\gamma)]$$

$$\Rightarrow \cos 3\alpha + i\sin 3\alpha + \cos 3\beta + i\sin 3\beta + \cos 3\gamma + i\sin 3\gamma = 3\cos(\alpha+\beta+\gamma) + 3i\sin(\alpha+\beta+\gamma)$$

Comparing for Real Parts

$$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$$

Hence Proved

$$(ii) \sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$$

Sol:

Euler Identity

$$\cos \theta + i\sin \theta = e^{i\theta}$$

$$\Rightarrow e^{i\alpha} = a, e^{i\beta} = b, e^{i\gamma} = c$$

$$a+b+c = e^{i\alpha} + e^{i\beta} + e^{i\gamma}$$

$$a+b+c = (\cos \alpha + i\sin \alpha) + (\cos \beta + i\sin \beta) + (\cos \gamma + i\sin \gamma)$$

$$a+b+c = \cos \alpha + i\sin \alpha + \cos \beta + i\sin \beta + \cos \gamma + i\sin \gamma$$

As given

$$\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$$

Taking cube on both sides

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$$(a+b+c)^3 = 0$$

$$a^3 + b^3 + c^3 - 3abc = 0$$

$$a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow (e^{i\alpha})^3 + (e^{i\beta})^3 + (e^{i\gamma})^3 = 3(e^{i\alpha})(e^{i\beta})(e^{i\gamma})$$

$$\Rightarrow e^{i3\alpha} + e^{i3\beta} + e^{i3\gamma} = 3e^{i(\alpha+\beta+\gamma)}$$

$$\Rightarrow (\cos 3\alpha + i\sin 3\alpha) + (\cos 3\beta + i\sin 3\beta) + (\cos 3\gamma + i\sin 3\gamma) = 3[\cos(\alpha+\beta+\gamma) + i\sin(\alpha+\beta+\gamma)]$$

$$\Rightarrow (\cos 3\alpha + i\sin 3\alpha + \cos 3\beta + i\sin 3\beta + \cos 3\gamma + i\sin 3\gamma) = 3\cos(\alpha+\beta+\gamma) + i3\sin(\alpha+\beta+\gamma)$$

Comparing for imaginary parts

$$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$$

Hence Proved.

Q6: Write a given complex number in the algebraic form:

$$(i) \sqrt{2} (\cos 315^\circ + i\sin 315^\circ)$$

Sol:

$$\sqrt{2} (\cos 315^\circ) + i \left(\sin \frac{315^\circ}{2} \right)$$

$$\Rightarrow \sqrt{2} \left(\frac{\sqrt{2}}{2} \right) + i \left(-\frac{\sqrt{2}}{2} \right)$$

$$\Rightarrow \frac{(\sqrt{2})^2}{2} - \frac{(\sqrt{2})^2}{2} i$$

$$\Rightarrow \frac{2}{2} - \frac{2}{2} i$$

$$\Rightarrow \boxed{1 - i} \text{ Ans}$$

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$$(ii) 5 (\cos 210^\circ + i \sin 210^\circ)$$

Sol:

$$\Rightarrow 5 (\cos 210^\circ + i \sin 210^\circ)$$

$$\Rightarrow 5 \left[\frac{-\sqrt{3}}{2} + i \left(\frac{-1}{2} \right) \right]$$

$$\Rightarrow \boxed{-\frac{5\sqrt{3}}{2} - \frac{5}{2}i} \quad \text{Ans}$$

$$(iii) 2 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$$

Sol:

$$\Rightarrow 2 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$$

$$\Rightarrow \frac{3\pi}{2} = \frac{3(180^\circ)}{2} = 270^\circ$$

$$\Rightarrow 2 (\cos 270^\circ + i \sin 270^\circ)$$

$$\Rightarrow 2 [0 + i(-1)]$$

$$\Rightarrow 2(0 - i)$$

$$\Rightarrow \boxed{0 - 2i} \quad \text{Ans}$$

$$(iv) 4 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

$$\Rightarrow 4 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

$$\Rightarrow \frac{5\pi}{6} = \frac{5(180^\circ)}{6} = 150^\circ$$

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$$\Rightarrow 4 (\cos 150^\circ + i \sin 150^\circ)$$

$$\Rightarrow 4 \left[-\frac{\sqrt{3}}{2} + i \left(\frac{1}{2} \right) \right]$$

$$\Rightarrow \frac{-4\sqrt{3}}{2} + 4 \left(\frac{1}{2} \right) i$$

$$\Rightarrow \boxed{-2\sqrt{3} + 2i} \text{ Ans}$$

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$$(vi) 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

Sol:

$$\Rightarrow 2 [\cos 30^\circ + i \sin 30^\circ]$$

$$\Rightarrow 2 \left[\frac{\sqrt{3}}{2} + i \left(\frac{1}{2} \right) \right]$$

$$\Rightarrow 2 \left(\frac{\sqrt{3}}{2} \right) + 2 \left(\frac{1}{2} \right) i$$

$$\Rightarrow \boxed{\sqrt{3} + i} \text{ Ans.}$$

$$(vii) \cos 135^\circ + i \sin 135^\circ$$

Sol:

$$\Rightarrow \cos 135^\circ + i \sin 135^\circ$$

$$\Rightarrow -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$$

$$\Rightarrow \boxed{-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i} \text{ Ans}$$

$$(vii) 10 (\cos 50^\circ + i \sin 50^\circ)$$

Sol:

$$\Rightarrow 10 (\cos 50^\circ + i \sin 50^\circ)$$

$$\Rightarrow 10 (0.643 + 0.766i)$$

$$\Rightarrow \boxed{6.43 + 7.66i} \text{ Ans}$$

$$(viii) \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

Sol:

$$\frac{3\pi}{4} = \frac{3(180^\circ)}{4} = 135^\circ$$

$$\Rightarrow \sqrt{2} [\cos 135^\circ + i \sin 135^\circ]$$

$$\Rightarrow \sqrt{2} \left[-\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right]$$

$$\Rightarrow \frac{-(\sqrt{2})^2}{2} + \frac{(\sqrt{2})^2}{2} i$$

$$\Rightarrow -\frac{2}{2} + \frac{2}{2} i$$

$$\Rightarrow \boxed{-1 + i} \text{ Ans}$$

$$(ix) 4 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

Sol:

$$\frac{2\pi}{3} = \frac{2(180^\circ)}{3} = 120^\circ$$

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$$\Rightarrow 4 \left[\cos 120^\circ + i \sin 120^\circ \right]$$

$$\Rightarrow 4 \left[-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right]$$

$$\Rightarrow -4 \left(\frac{1}{2} \right) + 4 \left(\frac{\sqrt{3}}{2} \right) i$$

$$\Rightarrow \boxed{-2 + 2\sqrt{3}i} \text{ Ans}$$

$$(x) 7\sqrt{2} \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right)$$

Sol:

$$\frac{5\pi}{4} = \frac{5(180)}{4} = 225^\circ$$

$$\Rightarrow 7\sqrt{2} \left(\cos 225^\circ + i \sin 225^\circ \right)$$

$$\Rightarrow 7\sqrt{2} \left[-\frac{1}{\sqrt{2}} + i \left(-\frac{1}{\sqrt{2}} \right) \right]$$

$$\Rightarrow 7\sqrt{2} \left(\frac{1}{\sqrt{2}} \right) - 7\sqrt{2} \left(\frac{1}{\sqrt{2}} \right) i$$

$$\Rightarrow \boxed{-7 - 7i} \text{ Ans}$$

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$$(xi) 10\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

Sol:

$$\frac{7\pi}{4} = \frac{7(180)}{4} = 315^\circ$$

$$\Rightarrow 10\sqrt{2} (\cos 315^\circ + i \sin 315^\circ)$$

$$\Rightarrow 10\sqrt{2} \left[\frac{1}{\sqrt{2}} + i \left(-\frac{1}{\sqrt{2}} \right) \right]$$

$$\Rightarrow 10\sqrt{2} \left(\frac{1}{\sqrt{2}} \right) - 10\sqrt{2} \left(\frac{1}{\sqrt{2}} \right) i$$

$$\Rightarrow \boxed{10 - 10i} \text{ Ans}$$

$$(xii) 2 \left(\cos \frac{5\pi}{2} + i \sin \frac{5\pi}{2} \right)$$

Sol:

$$\frac{5\pi}{2} = \frac{5(180^\circ)}{2} = 450^\circ$$

$$\Rightarrow 2 [\cos 450^\circ + i \sin 450^\circ]$$

$$\Rightarrow 2 [0 + i(1)]$$

$$\Rightarrow 2(0 + i)$$

$$\Rightarrow 2(i)$$

$$\Rightarrow \boxed{0 + 2i} \text{ Ans}$$

$$(xiii) \frac{1}{\sqrt{2}} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

Sol:

$$\frac{\pi}{4} = \frac{180}{4} = 45^\circ$$

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$$\Rightarrow \frac{1}{\sqrt{2}} [\cos 45^\circ + i \sin 45^\circ]$$

$$\Rightarrow \frac{1}{\sqrt{2}} \left[\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right]$$

$$\Rightarrow \frac{1}{\sqrt{2}} \left(\frac{\sqrt{2}}{2} \right) + \frac{1}{\sqrt{2}} \left(\frac{\sqrt{2}}{2} \right) i$$

$$\Rightarrow \boxed{\frac{1}{2} + \frac{1}{2}i} \text{ Ans}$$

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$$(xiv) 7 (\cos 180^\circ + i \sin 180^\circ)$$

Sol:

$$\Rightarrow 7 [-1 + i(0)]$$

$$\Rightarrow 7(-1)$$

$$\Rightarrow \boxed{-7 + 0i}$$

$$(xv) 2e^{i\frac{\pi}{4}}$$

Sol:

$$\Rightarrow 2e^{i\frac{\pi}{4}}$$

$$\Rightarrow 2 \left[\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right]$$

$$\Rightarrow \frac{\pi}{4} = \frac{180}{4} = 45^\circ$$

$$\Rightarrow 2 \left[\cos 45^\circ + i \sin 45^\circ \right]$$

$$\Rightarrow 2 \left[\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right]$$

$$\Rightarrow 2 \left(\frac{\sqrt{2}}{2} \right) + i 2 \left(\frac{\sqrt{2}}{2} \right)$$

$$\Rightarrow \boxed{\sqrt{2} + \sqrt{2}i} \text{ Ans}$$

(xvi) $3e^{i\frac{\pi}{2}}$

Sol:

$$3e^{i\frac{\pi}{2}} = 3 \left[\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right]$$

$$\Rightarrow 3 \left[\cos 90^\circ + i \sin 90^\circ \right]$$

$$\Rightarrow 3 [0 + i(1)]$$

$$\Rightarrow 3(i)$$

$$\Rightarrow \boxed{0 + 3i} \text{ Ans}$$

(xvii) $5e^{i\frac{\pi}{3}}$

Sol:

$$5 \left[\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right]$$

$$\Rightarrow 5 \left[\cos 60^\circ + i \sin 60^\circ \right]$$

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$$\Rightarrow 5 \left[\frac{1}{2} + i \frac{\sqrt{3}}{2} \right]$$

$$\Rightarrow 5 \left(\frac{1}{2} \right) + 5 \left(\frac{\sqrt{3}}{2} \right) i$$

$$\Rightarrow \boxed{\frac{5}{2} + \frac{5\sqrt{3}}{2} i}$$

Q7: Convert the following equations and inequations in Cartesian form:

(ii) $\arg(z-1) = -\frac{\pi}{4}$

Sol:

Let $z = x + yi$

$$\Rightarrow \arg(x + yi - 1) = -\frac{\pi}{4}$$

$$\Rightarrow \arg(x-1 + yi) = -\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left| \frac{y}{x-1} \right| = -\frac{\pi}{4}$$

$$\Rightarrow \frac{y}{x-1} = \tan \left(-\frac{\pi}{4} \right)$$

$$\Rightarrow \frac{y}{x-1} = -\tan \left(\frac{\pi}{4} \right)$$

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$$\frac{\pi}{4} = \frac{180}{4} = 45^\circ$$

$$\Rightarrow \frac{y}{x-1} = -1$$

$$\Rightarrow y = -1(x-1)$$

$$\Rightarrow \boxed{y = -x + 1}$$

$$\boxed{x + y = 1} \text{ Answer}$$

$$(ii) z \bar{z} = 4 |e^{i\theta}|$$

Sol:

$$\Rightarrow z \bar{z} = 4 |e^{i\theta}|$$

$$\Rightarrow \text{Let } z = x + yi$$

$$\bar{z} = x - yi$$

$$\Rightarrow (x + yi)(x - yi) = 4 | \cos \theta + i \sin \theta |$$

$$\Rightarrow (x)^2 + (yi)^2 = 4 \sqrt{\cos^2 \theta + \sin^2 \theta}$$

$$\Rightarrow x^2 - y^2 i^2 = 4 \sqrt{1}$$

$$\Rightarrow x^2 + y^2 = 4(1)$$

$$\because \sin^2 \theta + \cos^2 \theta = 1$$

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$$\Rightarrow \boxed{x^2 + y^2 = 4} \text{ Ans}$$

$$(iii) -\frac{\pi}{3} \leq \arg(z-4) \leq \frac{\pi}{3}$$

Sol:

$$\text{Let } z = x+yi$$

$$-\frac{\pi}{3} \leq \arg(x+yi-4) \leq \frac{\pi}{3}$$

$$\Rightarrow -\frac{\pi}{3} \leq \arg((x-4)+yi) \leq \frac{\pi}{3}$$

$$\Rightarrow -\frac{\pi}{3} \leq \tan^{-1} \left| \frac{y}{x-4} \right| \leq \frac{\pi}{3}$$

$$\Rightarrow -\frac{\pi}{3} \leq \tan^{-1} \left(\frac{y}{x-4} \right) \leq \frac{\pi}{3}$$

$$\Rightarrow -\frac{\pi}{3} \leq \tan^{-1} \left(\frac{y}{x-4} \right) \leq \frac{\pi}{3}$$

Applying tan with all

$$\tan\left(-\frac{\pi}{3}\right) \leq \left(\frac{y}{x-4}\right) \leq \tan\left(\frac{\pi}{3}\right)$$

$$\Rightarrow -\tan\left(\frac{\pi}{3}\right) \leq \frac{y}{x-4} \leq \tan\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \boxed{\sqrt{3} \leq \frac{y}{x-4} \leq \sqrt{3}} \text{ Ans}$$

$$(iv) 0 \leq \arg\left(\frac{z-4}{1+i}\right) \leq \frac{\pi}{6}$$

Sol:-

$$\text{Let } z = x+yi$$

$$0 \leq \arg\left(\frac{x+yi-4}{1+i}\right) \leq \frac{\pi}{6}$$

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$$\Rightarrow 0 \leq \arg \left(\frac{(x-4)+yi}{1+i} \right) \leq \frac{\pi}{6}$$

$$\Rightarrow 0 \leq \arg \left[\frac{(x-4)+yi}{1+i} \times \frac{1-i}{1-i} \right] \leq \frac{\pi}{6} \Rightarrow$$

$$\Rightarrow 0 \leq \arg \left[\frac{(1-i)(x-4+yi)}{1^2 - (i)^2} \right] \leq \frac{\pi}{6} \Rightarrow$$

$$\Rightarrow 0 \leq \arg \left[\frac{x-4+yi-xi+4i-yi^2}{1+1} \right] \leq \frac{\pi}{6}$$

$$\Rightarrow 0 \leq \arg \left[\frac{x-4+yi-xi+4i+y}{1+1} \right] \leq \frac{\pi}{6}$$

$$\Rightarrow 0 \leq \arg \left[\frac{(x+y-4) + i(y-x+4)}{2} \right] \leq \frac{\pi}{6} \Rightarrow$$

$$\Rightarrow 0 \leq \arg \left[\frac{x+y-4}{2} + i \frac{y-x+4}{2} \right] \leq \frac{\pi}{6} \Rightarrow$$

$$\Rightarrow 0 \leq \tan^{-1} \left| \frac{\frac{y-x+4}{2}}{\frac{x+y-4}{2}} \right| \leq \frac{\pi}{6}$$

$$\Rightarrow 0 \leq \tan^{-1} \left(\frac{y-x+4}{x+y-4} \right) \leq \frac{\pi}{6}$$

Applying (tan) on b/s

$$\tan(0) \leq \left(\frac{y-x+4}{x+y-4} \right) \leq \tan\left(\frac{\pi}{6}\right)$$

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$$\Rightarrow 0 \leq \frac{y-x+4}{x+y-4} \leq \frac{1}{\sqrt{3}}$$

$$\Rightarrow 0 \leq \sqrt{3} \left(\frac{y-x+4}{x+y-4} \right) \leq 1 \quad \text{Ans}$$

$$(v) \arg \left(\frac{1-z}{1-\bar{z}} \right) = \frac{\pi}{4}; z \neq i$$

Sol:

$$\text{Let } z = x+yi$$

$$\Rightarrow \arg \left[\frac{1-(x+yi)}{1-(x-yi)} \right] = \frac{\pi}{4}$$

$$\Rightarrow \arg \left[\frac{1-xi+yi}{1-x-yi} \right] = \frac{\pi}{4}$$

$$\Rightarrow \arg \left[\frac{(1+y)-xi}{(1-x)-yi} \times \frac{(1-x)+yi}{(1-x)+yi} \right] = \frac{\pi}{4}$$

$$\Rightarrow \arg \left[\frac{(1+y)[1-x+yi] - xi[1-x+yi]}{(1-x)^2 - (y^2)} \right] = \frac{\pi}{4}$$

$$\Rightarrow \arg \left[\frac{(1+y)(1-x) + (1+y)yi - xi(1-x) - xi(yi)}{1+x^2-2x-y^2i^2} \right] = \frac{\pi}{4}$$

$$\Rightarrow \arg \left[\frac{1-x+y-xy+yi+y^2i-xi+x^2i-xyi^2}{1+x^2-2x+y^2} \right] = \frac{\pi}{4}$$

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$$\Rightarrow \arg \left[\frac{1-x+y-xy+yi+y^2i-xi+x^2i+xy}{1+x^2+y^2-2x} \right] = \frac{\pi}{4}$$

$$\Rightarrow \arg \left[\frac{1-x+y}{1+x^2+y^2-2x} + \frac{(y-x+y^2+x^2)i}{1+x^2+y^2-2x} \right] = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{y-x+y^2+x^2}{1+x^2+y^2-2x}}{\frac{1-x+y}{1+x^2+y^2-2x}} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{y-x+y^2+x^2}{1-x+y} \right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{y-x+y^2+x^2}{1-x+y} = \tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \frac{y-x+y^2+x^2}{1-x+y} = 1$$

$$\Rightarrow y-x+y^2+x^2 = 1-x+y$$

$$\Rightarrow y-x+y^2+x^2+y-x-y = 1$$

$$\Rightarrow \boxed{x^2+y^2=1}$$

$$(vi) \frac{1}{2} \arg(z-i) = \frac{\pi}{3} - \frac{1}{2} \arg(z+i)$$

$$\text{Sol: } \frac{1}{2} \arg(z-i) + \frac{1}{2} \arg(z+i) = \frac{\pi}{3}$$

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$$\Rightarrow \frac{1}{2} \left[\arg(z-i) + \arg(z+i) \right] = \frac{\pi}{3}$$

$$\Rightarrow \frac{1}{2} \left[\arg(z-i) + \arg(z+i) \right] = \frac{\pi}{3} \quad \because \arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$$

$$\Rightarrow \frac{1}{2} \left[\arg(z-i)(z+i) \right] = \frac{\pi}{3}$$

$$\Rightarrow \frac{1}{2} \left[\arg(z^2 - i^2) \right] = \frac{\pi}{3}$$

$$\Rightarrow \arg(z^2 + 1) = \frac{2\pi}{3}$$

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$$\Rightarrow \text{Let } z = x + yi$$

$$\Rightarrow \arg[(x+yi)^2 + 1] = \frac{2\pi}{3}$$

$$\Rightarrow \arg[x^2 - y^2 + 1 + 2xyi] = \frac{2\pi}{3}$$

$$\Rightarrow \arg[x^2 - y^2 + 1 + 2xyi] = \frac{2\pi}{3}$$

$$\Rightarrow \arg[(x^2 - y^2 + 1) + 2xyi] = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1} \left(\frac{2xy}{x^2 - y^2 + 1} \right) = \frac{2\pi}{3}$$

$$\Rightarrow \frac{2xy}{x^2 - y^2 + 1} = \tan \left(\frac{2\pi}{3} \right)$$

$$\Rightarrow \frac{2xy}{x^2-y^2+1} = -\sqrt{3}$$

$$\Rightarrow 2xy = -\sqrt{3} (x^2-y^2+1)$$

$$\Rightarrow \boxed{\sqrt{3} (x^2-y^2+1) + 2xy = 0} \quad \text{Ans}$$

Q8. Calculate the position of a particle from mean position when amplitude is 0.004 mm and angle is:

$$(i) \theta = \frac{\pi}{4}$$

Sol:

$$\theta = \frac{\pi}{4}, \quad x_{\max} = 0.004 \text{ mm}, \quad x = ?$$

$$x = x_{\max} e^{i\theta}$$

$$x = 0.004 e^{i\frac{\pi}{4}}$$

$$x = 0.004 \left[\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right]$$

$$x = \frac{4 \times 10^{-3}}{1000} \left[\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right]$$

$$x = \frac{2}{500} \times \frac{\sqrt{2}}{2} (1+i)$$

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$$x = \frac{\sqrt{2}}{500} (1+i) \quad \text{Ans}$$

(ii) $\frac{\pi}{3}$

Sol:

$$\alpha = \frac{\pi}{3}, \quad x_{\max} = 0.004$$

$$x = x_{\max} e^{i\alpha}$$

$$x = 0.004 e^{i\frac{\pi}{3}}$$

$$x = 0.004 \left[\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right]$$

$$x = 0.004 \left[\cos 60^\circ + i \sin 60^\circ \right]$$

$$x = \frac{4}{1000} \left[\frac{1}{2} + i \frac{\sqrt{3}}{2} \right]$$

$$x = \frac{2 \times 1}{500} [1 + \sqrt{3}i]$$

$$x = \frac{1}{500} (1 + \sqrt{3}i) \quad \text{Ans}$$

(iii) $\frac{\pi}{6}$

Sol: $\alpha = \frac{\pi}{6}, \quad x_{\max} = 0.004 \text{ mm}$

$$x = x_{\max} e^{i\alpha}$$

$$x = 0.004 e^{i\frac{\pi}{6}}$$

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$$x = 0.004 \left[\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right]$$

$$x = \frac{24}{1000 \times 500} \left[\frac{\sqrt{3}}{2} + i \frac{1}{2} \right]$$

$$x = \frac{2}{500} \times \frac{1}{2} \left[\sqrt{3} + i \right]$$

$$x = \frac{1}{500} (\sqrt{3} + i)$$

Ans

$$x = 2 + 3i$$

Q9: When particle is at position $\downarrow$ from its mean position and $x_{\max} = 1 + 4i$ is the position at maximum distance from mean position as it can be seen under microscope at the point.

$$x = 2 + 3i, x_{\max} = 1 + 4i$$

(i) Calculate the angle at time $t=0$ and find the position of the particle:

Sol: Q = ? , x = ? , t = 0

For Q:

$$x = x_{\max} e^{i\phi}$$

$$\Rightarrow 2 + 3i = 1 + 4i e^{i\phi}$$

$$\Rightarrow \frac{2 + 3i}{1 + 4i} = e^{i\phi}$$

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$$\Rightarrow e^{i\alpha} = \frac{2+3i}{1+4i} \times \frac{1-4i}{1-4i}$$

$$\Rightarrow e^{i\alpha} = \frac{2-8i+3i-12i^2}{1-4i^2}$$

$$\Rightarrow e^{i\alpha} = \frac{2-5i+12}{1+16}$$

$$\Rightarrow e^{i\alpha} = \frac{14-5i}{17}$$

$$\Rightarrow \cos\alpha + i\sin\alpha = \frac{14}{17} - \frac{5}{17}i$$

(Comparing the Real and Imag)

$$\cos\alpha = \frac{14}{17} \quad \text{--- (1)}$$

$$\sin\alpha = \frac{-5}{17} \quad \text{--- (2)}$$

Divide eq (2) by (1)

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{-5/17}{14/17}$$

$$\Rightarrow \tan\alpha = \frac{-5}{14}$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{-5}{14}\right)} \quad \text{Ans of } \alpha$$

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For x : $t=0$ (Rest)

Option 1: $x = 1+4i$ could be answer

Option 2: Position of the particle at $t=0$ rest

$$x = 2+3i$$

OR

$$x = x_{\max} e^{i\theta}$$

$$x = (1+4i) [\cos \theta + i \sin \theta]$$

$$x = (1+4i) \left[\frac{14}{17} + i \left(\frac{-5}{17} \right) \right]$$

$$x = (1+4i) \left(\frac{14}{17} - \frac{5}{17}i \right)$$

$$x = \frac{14}{17} - \frac{5}{17}i + \frac{56i}{17} - \frac{20i^2}{17}$$

$$x = \frac{14}{17} - \frac{5}{17}i + \frac{56i}{17} + \frac{20}{17}$$

$$x = \frac{34}{17} + \frac{51i}{17}$$

$$x = 2+3i$$

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(ii) If $x = 2 + 3i$ and $x_{\max} = 2 + 4i$. Calculate the frequency when $t = 2$.

Sol:

$$x = 2 + 3i, x_{\max} = 2 + 4i, f = ?, t = 2$$

$$\Rightarrow \omega = \frac{\phi}{t}$$

$$\Rightarrow \phi = \omega t$$

$$\Rightarrow \phi = 2\pi f t$$

$$\Rightarrow \frac{\phi}{2\pi t} = f$$

$$\Rightarrow f = \frac{\phi}{2\pi t}$$

$$\Rightarrow f = \frac{1}{2\pi(2)} \tan^{-1}\left(\frac{-5}{14}\right)$$

$$\Rightarrow \boxed{f = \frac{1}{4\pi} \tan^{-1}\left(\frac{-5}{14}\right)}$$

OR

$$x = x_{\max} e^{i\phi}$$

$$\phi = \omega t$$

$$\phi = 2\pi f t$$

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$$\Rightarrow 2+3i = 1+4i e^{i2\pi f t}$$

$$\Rightarrow \frac{2+3i}{1+4i} = e^{i2\pi f(2)}$$

$$\Rightarrow \frac{2+3i}{1+4i} = e^{i4\pi f}$$

$$\Rightarrow e^{i4\pi f} = \frac{2+3i}{1+4i} \times \frac{1-4i}{1-4i}$$

$$\Rightarrow e^{i4\pi f} = \frac{2-8i+3i-12i^2}{1-4i^2}$$

$$\Rightarrow e^{i4\pi f} = \frac{2-5i+12}{1+16}$$

$$\Rightarrow e^{i4\pi f} = \frac{14-5i}{17}$$

$$\Rightarrow \cos 4\pi f + i \sin 4\pi f = \frac{14}{17} - \frac{5}{17}i$$

Comparing for Real and Imaginary

$$\cos 4\pi f = \frac{14}{17} \quad \text{--- (1)}$$

$$\sin 4\pi f = -\frac{5}{17} \quad \text{--- (2)}$$

Divide (2) by (1)

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$$\Rightarrow \frac{\sin 4\pi f}{\cos 4\pi f} = \frac{-5/14}{14/14}$$

$$\Rightarrow \tan 4\pi f = \frac{-5}{14}$$

$$\Rightarrow \tan 4\pi f = \tan^{-1} \left(\frac{-5}{14} \right)$$

$$\Rightarrow \boxed{f = \frac{1}{4\pi} \tan^{-1} \left(\frac{-5}{14} \right)} \text{ Hz Ans}$$

Q10. Find the impedance Z for the following values:

(i) $E = (-50 + 100i)$ volts, $I = (-6 - 2i)$ amp

Sol:

$$Z = \frac{E}{I}$$

$$Z = \frac{-50 + 100i}{-6 - 2i}$$

$$Z = \frac{25}{+2} \frac{1 - 2i}{3 + i}$$

$$Z = 25 \left(\frac{1 - 2i}{3 + i} \times \frac{3 - i}{3 - i} \right)$$

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$$Z = 25 \left(\frac{3 - i - 6i + 2i^2}{3^2 - i^2} \right)$$

$$Z = 25 \left(\frac{3 - 7i - 2}{9 + 1} \right)$$

$$Z = \frac{5}{2} \left(\frac{1 - 7i}{10} \right)$$

$$Z = \frac{5}{2} (1 - 7i)$$

(ii) $E = (100 + 10i)$ volts, $I = (-8 + 3i)$ amps
Sol:

$$Z = \frac{E}{I}$$

$$Z = \frac{100 + 10i}{-8 + 3i}$$

$$Z = \frac{10(10 + i)}{-8 + 3i}$$

$$Z = 10 \left[\frac{10(-8 - 3i) + i(-8 - 3i)}{(-8)^2 - (3i)^2} \right]$$

$$Z = 10 \left[\frac{-80 - 30i - 8i - 3i^2}{64 - 9} \right]$$

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$$z = 10 \left[\frac{-80 - 38i - 13}{64 + 9} \right]$$

$$z = 10 \left[\frac{-77 - 38i}{73} \right]$$

$$z = -10 \left[\frac{77 + 38i}{73} \right]$$

$$z = \frac{-10}{73} (77 + 38i)$$

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CHAPTER

ONE

LOSING