Chapter 16:- STATISTICAL MECHANICS and THERMODYNAMICS (25-7-25)

Statistical Mechanics:-

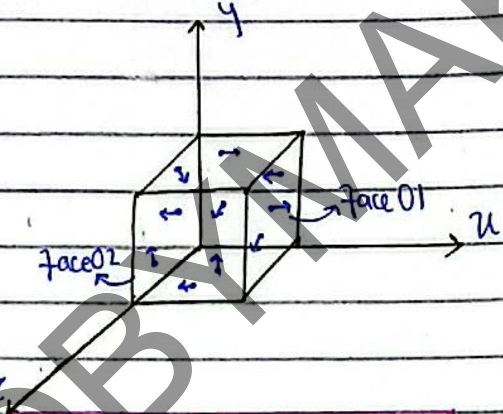
A Branch of Physics that applies statistical mechanics or Statistical method to understand the behaviour of Physical Systems Composed of a large number of Particles

16.1: Pressure Exerted By Gas Molecules.

Pressure $= P = \frac{F}{A}$

$P = F \times \frac{1}{A}$ $\quad \because \frac{\Delta \vec{P}}{\Delta t}$

$\Rightarrow \boxed{P = \left[\frac{\Delta \vec{P}}{\Delta t}\right] \cdot \frac{1}{A}}$ ——— (i)



Now, Change in the momentum

$= \Delta P = P_f - P_i$

$\hookrightarrow \Delta P = (-mv_{x_1}) - (+mv_{x_1})$

$\hookrightarrow \Delta P = -mv_{x_1} - mv_{x_1}$

$\Rightarrow \boxed{\Delta P = -2mv_{x_1}}$ ——— (ii)

Possible Qs (LRQ)
- Derive $P = \frac{mN}{3V}\langle v^2 \rangle$
[OR] - How can we get the pressure exerted by an ideal gas on a Container?

Now, $S = v\Delta t$

So, time taken $\therefore \Delta t = \frac{S}{v}$

$\Rightarrow \boxed{\Delta t = \frac{2l}{v_{x_1}}}$ ——— (iii) $\quad \because S = 2l$

we know that, $F_{x_1} = -\frac{\Delta P}{\Delta t}$

minus sign due to Newtons 3rd law of Motion

putting values from eq (ii) & (iii), we get:-

$F_{x_1} = \frac{-(-2mv_{x_1})}{(2l/v_{x_1})}$

$\hookrightarrow F_{x_1} = \frac{\cancel{2}mv_{x_1}}{\cancel{2}l} \times v_{x_1}$

$\hookrightarrow F_{x_1} = \frac{mv_{x_1}^2}{l}$

Similarly for other particles, we get:-

SOCHBADLOBYMAK

GET ADMISSION IN OUR ONLINE INSTITUTE
SOCH BADLO BY MAK
Contact WhatsApp Number: +92 331 5014353

CamScanner

$$\Rightarrow F_{x_1} = \frac{mv_{x_1}^2}{L},\ F_{x_2} = \frac{mv_{x_2}^2}{L},\ F_{x_3} = \frac{mv_{x_3}^2}{L},\ ----,\ F_{x_n} = \frac{mv_{x_n}^2}{L}$$

Now Calculating Total force along x-axis:

$$F_{x} = F_{x_1} + F_{x_2} + F_{x_3} + ---- + F_{x_n}$$

$$\hookrightarrow F_x = \frac{mv_{x_1}^2}{L} + \frac{mv_{x_2}^2}{L} + \frac{mv_{x_3}^2}{L} + --- + \frac{mv_{x_n}^2}{L}$$

$$\hookrightarrow F_x = \frac{m}{L}\left(v_{x_1}^2 + v_{x_2}^2 + v_{x_3}^2 + --- + v_{x_n}^2\right)$$

$$\hookrightarrow F_x = \frac{mN}{L}\left[\frac{v_{x_1}^2 + v_{x_2}^2 + v_{x_3}^2 + --- + v_{x_n}^2}{N}\right] \quad \because \times \frac{N}{N}$$

$$\Rightarrow \boxed{F_x = \frac{mN}{L}\langle v_x^2\rangle} \text{ —— (iv)}$$

Here, N = Total number of Particles. This equation can be written as =

$$F_x = N\left[\frac{m}{L}\langle v_x^2\rangle\right]$$

Here $\langle v_x^2\rangle$ in the mean Square velocity of the gas molecules which are moving along x-axis direction.

$$\hookrightarrow \langle v^2\rangle = \langle v_x^2\rangle + \langle v_y^2\rangle + \langle v_z\rangle^2$$

Here, $\langle v^2\rangle$ is the total mean Square velocity along all three dimensions (x, y, z). Since

$$\langle v_x^2\rangle = \langle v_y^2\rangle = \langle v_z\rangle^2$$

$$\hookrightarrow \langle v^2\rangle = \langle v_x^2\rangle + \langle v_x^2\rangle + \langle v_x^2\rangle$$

$$\hookrightarrow \langle v^2\rangle = 3\langle v_x^2\rangle$$

$$\Rightarrow \frac{\langle v^2\rangle}{3} = \langle v_x^2\rangle$$

$$\Rightarrow \boxed{\langle v_x^2\rangle = \frac{\langle v^2\rangle}{3}} \text{ —— (v)}$$

putting eq. (v) in eq. (iv), we get

$$F_x = \frac{Nm}{l}\left[\frac{\langle v^2\rangle}{3}\right]$$

Hence total force is given by:-

$$\boxed{F = \frac{Nm}{L}\left[\frac{\langle v^2\rangle}{3}\right]} \text{ —— (vi)}$$

SOCHBADLOBYMAK

GET ADMISSION IN OUR ONLINE INSTITUTE
SOCH BADLO BY MAK
Contact WhatsApp Number: +92 331 5014353

CamScanner

Since, Pressure = $\frac{\text{Force}}{\text{Area}}$ and Area = length x length = L^2

Pascal laws

$P = \frac{F}{A}$ or $P = F \times \frac{1}{A}$

$\hookrightarrow P = \frac{Nm}{l}\left[\frac{\langle v^2 \rangle}{3}\right] \times \frac{1}{l^2}$

$\hookrightarrow P = \frac{Nm}{3l^2}\langle v^2 \rangle$

GET ADMISSION IN OUR ONLINE INSTITUTE
SOCH BADLO BY MAK
Contact WhatsApp Number: +92 331 5014353

Since Volume = $l \times l \times l \implies V = l^3$

$\implies \boxed{P = \frac{Nm}{3V}\langle v^2 \rangle}$

This expression gives the equation for Pressure exerted by an ideal gas.

Q Relation b/w Pressure and Average Translational Kinetic Energy.

Since $P = \frac{mN}{3V}\langle v^2 \rangle$

multiplying & dividing by '2', we get

$P = \frac{mN}{3V}\langle v^2 \rangle \times \frac{2}{2}$

by re-arranging, we get

$P = \frac{2N}{3V} \times \frac{1}{2} m\langle v^2 \rangle$

We know that

$\langle K.E \rangle = \frac{1}{2} m\langle v^2 \rangle$

So pressure can be written as

$P = \frac{2N}{3V} \times \langle K.E \rangle$

Possible Questions (SRQ)
- Show that Pressure & $\langle K.E \rangle$ are directly proportional
- what is the relationship b/w P & $\langle K.E \rangle$
- Express P in terms of $\langle K.E \rangle$
- Prove/show that $P \propto \langle K.E \rangle$

$\frac{N}{V}$ = No, representing the number of molecules per unit volume

$\hookrightarrow P = \frac{2}{3} N_o \times \langle K.E \rangle$

Here 2/3 No are all Constant Values so,

$P = \text{Constant} \times \langle K.E \rangle$

$\implies \boxed{P \propto \langle K.E \rangle}$

SOCHBADLOBYMAK

CS CamScanner

The pressure exerted by the gas molecules on the walls of container is directly proportional to the average Translational Kinetic energy of the gas molecules.

② Relation of Pressure and Density

GET ADMISSION IN OUR ONLINE INSTITUTE
SOCH BADLO BY MAK
Contact WhatsApp Number: +92 331 5014353

Since, $P = \frac{mN}{3V} \langle v^2 \rangle$

by rearranging, we get :- $P = \frac{Nm}{V} \times \frac{\langle v^2 \rangle}{3}$

Now density is given by $\rho = \frac{Nm}{V}$

So, Pressure can be written as:-

$$P = \rho \times \frac{\langle v^2 \rangle}{3}$$

or $\boxed{P = \frac{1}{3} \rho \times \langle v^2 \rangle}$

Possible Questions (SRQ)
- Derive a relation b/w P & ρ
- Derive $P = \frac{1}{3} \rho \langle v^2 \rangle$
- Prove/show that $P = 1/3 \, \rho \langle v^2 \rangle$

③ Temperature in terms of Average translational K.E of Gas molecules:-

Since, $\boxed{PV = nRT}$ —— (i)

Now, Number of moles = $\frac{\text{No of molecules}}{\text{Avagadro's number}}$

$\hookrightarrow \boxed{n = \frac{N}{N_A}}$ —— (ii)

$PV = \frac{n}{N_A} \times R \times T$ $\quad \because R =$ universal gas Constant

$PV = N \times \frac{R}{N_A} \times T$ $\quad \because \frac{R}{N_A} = k$ (Boltzmann's Constant)

$\boxed{PV = NkT}$ —— (iii)

Possible Question (SRQ)
- Show the relation b/w T & $\langle K.E \rangle$
- Show/Prove that $T \propto \langle K.E \rangle$

Since, we have:-

$$P = \frac{2N}{3V} \times \frac{1}{2} m \langle v^2 \rangle$$

$$PV = \frac{2N}{3\cancel{V}} \times \left[\frac{1}{2} m \langle v^2 \rangle\right] \times \cancel{V}$$

$\because \frac{1}{2} m \langle v^2 \rangle = \langle K.E \rangle$

$\Rightarrow \boxed{PV = \frac{2N}{3} \times \langle K.E \rangle}$ —— (iv)

SOCHBADLOBYMAK

CS CamScanner

Comparing eq: (iii) and (iv), we get

$$\cancel{N}KT = \frac{2\cancel{N}}{3} \times \langle K.E \rangle$$

$$KT = \frac{2}{3} \langle K.E \rangle$$

$$T = \frac{2}{3}K \langle K.E \rangle$$

Since $\frac{2}{3}k$ is Constant, So, we get

$$T = \text{Constant} \times \langle K.E \rangle$$

$$\Rightarrow \boxed{T \propto \langle K.E \rangle}$$

GET ADMISSION IN OUR ONLINE INSTITUTE
SOCH BADLO BY MAK
Contact WhatsApp Number: +92 331 5014353

- The Absolute Temperature of an ideal gas is directly proportional to the average translational Kinetic energy of the gas molecule.

16.2. Root Mean Square of an Ideal gas

RMS:-

R = Root
M = Mean
S = Square
→ ① Square root of mean square value
② Effective Velocity
③ It is to avoid negative velocities and to get a suitable value.

↳ root mean square Speed

Square – because the value are negative
Mean – to take average
Root – Root to make value Small in range of original value

⟹ RMS – we use it to find the effective velocity
– Tells us the useful Power.

★ We Use Root Mean Square velocity in Calculations because gas molecules move randomly in all directions with different Speed, and its not Practical to describe their motion using a single velocity value.

★ RMS Velocity provides a meaningful average speed that relates directly to the kinetic Energy of the gas particles.

⟹ The root mean square speed of gas molecules is the square root of the average of the Squares of molecular Speeds.

SOCHBADLOBYMAK

CamScanner

Example:-

Hydrogen molecules have a 4 times greater RMS Speed than oxygen molecules at the Same Temperature due to lower molar mass.

$\Longrightarrow$ Derivation:-

$$V^2 = V_1^2 + V_2^2 + V_3^2 + \cdots + V_4^2$$

Step② Taking ~~Mean~~, square root, we get

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + \cdots + V_n^2}{N}}$$

Step① ~~Since~~, Taking mean, we get:-

$$\langle V^2 \rangle = \frac{V_1^2 + V_2^2 + V_3^2 + \cdots + V_n^2}{N}$$

Since, the Pressure of gas molecules is given by:-

$$P = \frac{1}{3} \frac{mN}{V} \langle v^2 \rangle$$

$$\Longrightarrow \boxed{PV = \frac{1}{3} mN \langle v^2 \rangle} \quad \text{—(i)}$$

from the Ideal gas equation,

$$PV = nRT$$

$$PV = \left(\frac{N}{N_A}\right) RT \qquad \because n = \frac{N}{N_A}$$

$$PV = N\left(\frac{R}{N_A}\right) T \qquad \because \frac{R}{N_A} = K \text{ (Boltzmanns Constant)}$$

$$\boxed{PV = NKT} \quad \text{—(ii)}$$

Comparing eq. (i) and (ii)

$$\frac{1}{3} m\cancel{N} \langle v^2 \rangle = \cancel{N} KT$$

$$\langle v^2 \rangle = \frac{3KT}{m}$$

Taking Square root on both Sides, we get

$$\boxed{V_{r.m.s} = \sqrt{\frac{3KT}{m}}} \quad \text{—(iii)}$$

Now, $\sqrt{\frac{3K}{m}}$ = Constant, So

$$V_{rms} = \sqrt{\frac{3K}{m}} \cdot \sqrt{T}$$

GET ADMISSION IN OUR ONLINE INSTITUTE
SOCH BADLO BY MAK
Contact WhatsApp Number: +92 331 5014353
SOCHBADLOBYMAK
CamScanner

~~Vrmm~~

$$V_{rms} = (\text{Constant})\sqrt{T}$$

$$\boxed{V_{rms} \propto \sqrt{T}}$$

↳ This relation shows that if the value of temperature increases, then the value of Vrms will also increase i.e. there is a direct relation b/w Root Mean Square Speed of an Ideal gas molecules and the temperature.

Possible Questions

- Derive a relation for Vrms & T
- Show that Vrms $\sqrt{T}$ are directly Proportional
- Derive a formula for vrms involving k, T, & m.

GET ADMISSION IN OUR ONLINE INSTITUTE
SOCH BADLO BY MAK
Contact WhatsApp Number: +92 331 5014353

Now, as we Know that:-

$$V_{rms} = \sqrt{\frac{3KT}{m}}$$

$$V_{rms} = \sqrt{\frac{3T \times K}{m}}$$

$$V_{rms} = \sqrt{\frac{3T}{m} \times \frac{R}{N_A}} \qquad \because K = R/N_A$$

$$\boxed{V_{rms} = \sqrt{\frac{3RT}{mN_A}}} \quad \text{———} (iv)$$

Since, the mass of Single molecule (m) x Avagadros Number (N_A) = Molar Mass

$$\boxed{mN_A = M} \quad \text{—} (v)$$

we Can write that:-

$$\boxed{V_{rms} = \sqrt{\frac{3RT}{M}}} \quad \text{———} (vi)$$

Expected Questions:-

- show that $\sqrt{M}$ & Vrms have an inverse relation
- Derive a relation b/w $\sqrt{M}$ & Vrms
- Derive an equation for Vrms involving R, T, & M.

Now $\sqrt{3RT}$ = Constant, so

$$V_{rms} = \frac{\sqrt{3RT}}{\sqrt{M}}$$

$$\rightarrow V_{rms} = \text{Constant} \times \frac{1}{\sqrt{M}}$$

$$V_{rms} \propto \frac{1}{\sqrt{M}}$$

This relation, shows that if the value of molar mass is increased, then the value of Vrms will decrease, ie, there is an inverse

SOCH BADLO BY MAK

CS CamScanner

relation b/w Root Mean Square Speed of an ideal gas and Molar mass at Constant temperature.

★ Possible Questions ★
- Derive a relation b/w Pressure & Vrms.
- Show that $\sqrt{P}$ & Vrms have a direct relation.
- Derive an equation for Vrms involving P and ρ.

Since $P = \frac{1}{3} \rho \langle v^2 \rangle$

$$3P = \rho \langle v^2 \rangle$$

$$\frac{3P}{\rho} = \langle v^2 \rangle$$

$$\langle v^2 \rangle = \frac{3P}{\rho}$$

Taking Square-root on both sides

$$\langle v^2 \rangle = \sqrt{\frac{3P}{\rho}}$$

$$\boxed{V_{rms} = \sqrt{\frac{3P}{\rho}}} \quad \text{—(vii)}$$

Now, if the density is Constant, then-

$$\sqrt{\frac{3}{\rho}} = \text{Constant}$$

So, $V_{rms} = \sqrt{\frac{3}{\rho}} \times \sqrt{P}$

$$V_{rms} = (\text{Constant}) \times \sqrt{P}$$

$$\boxed{V_{rms} \propto \sqrt{P}}$$

This equation shows that increasing Pressure will increase the Vrms as well, i.e. Pressure and Root Mean Square Speed of an Ideal gas have a direct relationship.

⟹ by Combining eqs. (iii), (vi) & (vii), we get.

$$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}$$

16.3. MODIFICATION OF THE IDEAL GAS MODEL TO DISCUSS BEHAVIOUR OF NON-IDEAL GASES

↳ Behaviour of Ideal Gases

An Ideal gas has the properties listed below:-

GET ADMISSION IN OUR ONLINE INSTITUTE
SOCH BADLO BY MAK
Contact WhatsApp Number: +92 331 5014353
SOCHBADLOBYMAK
CamScanner

① No Attractive forces b/w ~~gases~~ gas molecules

② Gas molecules have negligible volume

③ Collisions b/w molecules and walls of the Container are elastic

④ An ideal gas obeys the law $PV = nRT$ (ideal gas law)

→ **Statistical mechanics:-**

▫ A branch of Physics that Connects the microscopic details of a System, Such as motion, energy and the interaction of individual particles, with macroscopic observables we measure, Such as temperature, pressure, volume & entropy. It Provides a mathematical foundation for thermodynamics.

↳ At low temperature and high temperatures a real gas can behave as an ideal gas.

GET ADMISSION IN OUR ONLINE INSTITUTE
SOCH BADLO BY MAK
Contact WhatsApp Number: +92 331 5014353

★ **Behaviour of Real Gases:-**

Ideal gas law (or equation) $PV = nRT$, is not Valid for real gases. This formula can be Corrected for real gas, which is

$$\boxed{\left[P + \frac{an^2}{V^2}\right](V - nb) = nRT}$$

⇒ POSSIBLE QUESTIONS ⇐

1. Why do real gases deviate from ideal gas behaviour?

Under normal Conditions, they do not behave as ideal gas. This is due to the reasons listed below:-

① Finite Molecular Volume (ideal gas have negligible Volume)

② Intermolecular forces (ideal gases are assumed to have no force b/w particles)

③ Elastic Collisions (no energy is lost b/w the collisions of ideal gases)

SOCHBADLOBYMAK

CamScanner

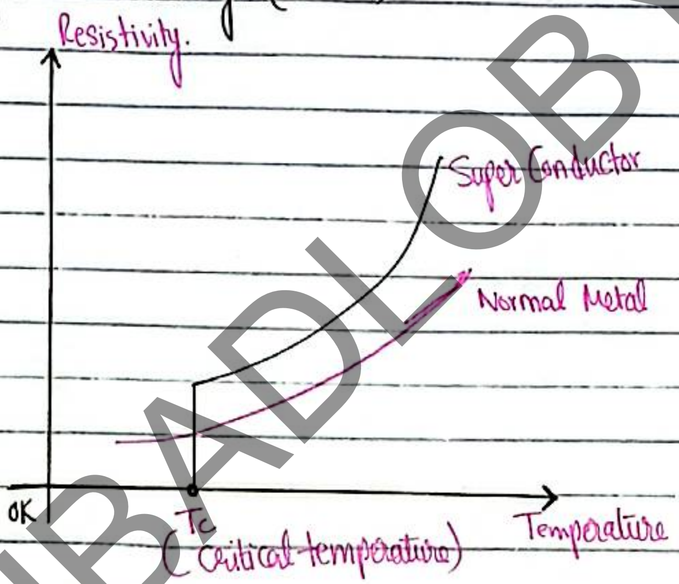
2. Under what Conditions ~~doe~~ do real gases behave as ideal gases?

A. Real gases behave as ideal gases under certain conditions. Following are some of the main conditions where real gases behave as ideal gases:-

GET ADMISSION IN OUR ONLINE INSTITUTE
SOCH BADLO BY MAK
Contact WhatsApp Number: +92 331 5014353

1. Low pressure
2. High temperature
3. low Density
4. Non-Polar or Small molecules
5. far from liquification Conditions.

⟹ Super Conductivity (1.7.25)



↳ Super Conductors:-

are Substances whose resistivity becomes zero at very low Specified Temperatures.

↳ Resistivity:-

Defined as the measure of opposition of flow of Charges in a Substance.

mostly,

Temperature & resistivity have direct Relation

$T \propto \rho$

SOCHBADLOBYMAK

CS CamScanner

GET ADMISSION IN OUR ONLINE INSTITUTE
SOCH BADLO BY MAK
Contact WhatsApp Number: +92 331 5014353

From the diagram ~~Resistivity~~ its
↳ Critical Temperature:-
The Temperature at ~~#~~ which the resistivity of a Substance drops to zero, making it a Super Conductor.

Also, Super Conductivity:-
Defined as the property of certain materials to Conduct Current without energy loss when they are Cooled below a Critical temperature (Specified) or transition Temperature (at which it changes)

SOCHBADLOBYMAK

CS CamScanner