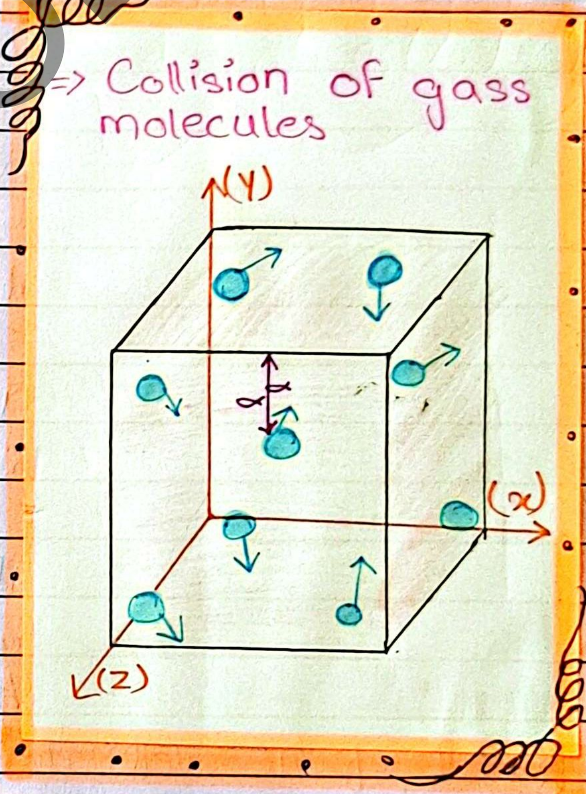
~08[CHAPTER #16]80~

Date: 25-July-25

Statistical Mechanics And Thermodynamics

Statistical Mechanics connects microscopic behavior of individual atoms and molecules to microscopic properties (like temperature, pressure and entropy) of materials.

PRESSURE EXERTED BY GAS MOLECULES



Pressure :-

$$P = \frac{F}{A}$$

$$P = \left(\frac{\Delta \vec{P}}{\Delta t}\right) \times \frac{1}{A} \quad \text{— i} \qquad \because F = \frac{\Delta P}{\Delta t}$$

Change in momentum :-

$$\Delta P = P_f - P_i$$

$$= -mv_{x1} - (+mv_{x1})$$

$$= -2mv_{x1} \quad \text{— ii}$$

Time :-

$$S = vt$$

$$\Delta t = \frac{S}{v} \qquad \because S = 2l \; (\xrightarrow{l}], S = 2l) \qquad v = v_{x1}$$

$$\Delta t = \frac{2l}{v_{x1}} \quad \text{— iii}$$

reverse momentum :- Wall se takra kr wapis aa raha.

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As, $F_{x_1} = -\frac{\Delta P'}{\Delta t} \rightarrow (a)$

Putting eq (ii) and (iii) in equation (a)

$$F_{x_1} = -\left(\frac{-2mv_{x_1}}{2l/v_{x_1}}\right)$$

$$F_{x_1} = \cancel{2}mv_{x_1} \times \frac{v_{x_1}}{\cancel{2}l}$$

$$\boxed{F_{x_1} = \frac{mv_{x_1}^2}{l}}$$

In equation 'a' we r taking force negative this is because, the force is applied on the particle by wall.

Similarly for $F_{x_2}, F_{x_3}, \ldots, F_{x_n}$,

$$\boxed{F_{x_2} = \frac{mv_{x_2}^2}{l}} \rightarrow \text{for 2nd Particle}$$

$$\boxed{F_{x_3} = \frac{mv_{x_3}^2}{l}} \rightarrow \text{for 3rd Particle}$$

$\vdots$

$$\boxed{F_{x_n} = \frac{mv_{x_n}^2}{l}} \rightarrow \text{for } n^{th} \text{ Particle}$$

Reverse Momentum

Wall se takra ker wapis aa raha.

Now Calculating total force along x-axis

$$F_x = F_{x_1} + F_{x_2} + F_{x_3} \cdots F_{x_n}$$

$$= \frac{mv_{x_1}^2}{l} + \frac{mv_{x_2}^2}{l} + \frac{mv_{x_3}^2}{l} + \cdots + \frac{mv_{x_n}^2}{l}$$

$$= \frac{m}{l} \times \left[V_{x_1}^2 + V_{x_2}^2 + V_{x_3}^2 + \cdots + V_{x_n}^2\right]$$

[Multiplying & Dividing by N]
[N = total no. of Particles]

$$= \frac{m}{l} \times N \times \left[\frac{V_{x_1}^2 + V_{x_2}^2 + V_{x_3}^2 + \cdots + V_{x_n}^2}{N}\right] \Rightarrow \boxed{F_x = \frac{mN}{l}\langle V_x^2 \rangle} \text{ (iv)}$$

Date: 27^{th} July

$$F_x = N\left[\frac{m}{l}\langle V_x^2\rangle\right]$$ ....force exerted by all molecules in x-axis direction

here, $\langle V_x^2\rangle$ is the mean sq. velocity of gas molecules moving along x-axis direction.

$$\Rightarrow \langle V^2\rangle = \langle V_x^2\rangle + \langle V_y^2\rangle + \langle V_z^2\rangle$$

here, $\langle V^2\rangle$ is the total mean sq. velocity along 3 dimensions (x, y, z)

Since,

$$\langle V_x^2\rangle = \langle V_y^2\rangle = \langle V_z^2\rangle$$

$$\langle V^2\rangle = \langle V_x^2\rangle + \langle V_x^2\rangle + \langle V_x^2\rangle$$

$$\langle V^2\rangle = 3\langle V_x^2\rangle$$

$$\boxed{\langle V_x^2\rangle = \frac{\langle V^2\rangle}{3}} \quad (v)$$

∵ (Putting eq (v) in eq (iv))

$$(vi) - \quad \boxed{F = N\left[\frac{m}{l}\left(\frac{\langle V^2\rangle}{3}\right)\right]}$$

[Since total force is given by 'F', so we'll substitute 'F' in the place of 'F_x']

$(\because A = l^2)$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \text{Force} \times \frac{1}{\text{Area}}$$

$$= \frac{Nm}{l}\left[\frac{\langle V^2\rangle}{3}\right] \times \frac{1}{l^2}$$

$$= \frac{Nm}{3l^3} \times \langle V^2\rangle$$

result :- 18 Aug ± 4

Date: 28/July

(As Volume = $l \times l \times l$, therefore, $V = l^3$)

$$\boxed{P = \frac{Nm}{3V}\langle v^2\rangle}$$

Pressure exerted by a gas molecule

RELATION B/W PRESSURE AND (i) TRANSITION -AL KINETIC ENERGY

As,

$$P = \frac{mN}{3V}\langle v^2\rangle$$

(Divide and Multiply by 2)

$$P = \frac{2N}{3V} \times \frac{m\langle v^2\rangle}{2}$$

$\because \langle K.E\rangle = \frac{1}{2} m\langle v^2\rangle$

$$P = \frac{2N}{3V} \times \langle K.E\rangle$$

$\because N_o = \frac{N}{V}$

$$P = \frac{2}{3} N_o \times \langle K.E\rangle$$

$$P = (\text{constant}) \times \langle K.E\rangle$$

$$P \propto \langle K.E\rangle$$

As Volume 'V' and N are constant. and 2, 3 are also constant, so we'll consider 2N/3V, constant.

$\therefore$ Therefore, the Av. kinetic energy is directly proportional to Pressure.

25 m³ 25 m³

50,000 19000

Date:

ii-) DENSITY!

$$P = \frac{Nm}{3V} \cdot \langle v^2 \rangle$$

$$P = \frac{1}{3} \cdot \langle v^2 \rangle \cdot \frac{Nm}{V}$$

$$\because \rho = \frac{Nm}{V}$$

$$P = \frac{1}{3} \cdot \langle v^2 \rangle \cdot \rho$$

$$P = \frac{1}{3} \rho \langle v^2 \rangle$$

iii-) RELATION BETWEEN TEMPERATURE AND K.E of GAS

Since,

$$PV = nRT \longrightarrow (i)$$

$$\because n = \frac{N}{N_A}$$

$$PV = \frac{N}{N_A} \times R \times T$$

$$PV = N \times (R/N_A) \times T$$

$$PV = NKT$$

$\because$ R = Universal Gas Constant

K = Boltzman Constant

$$\because \text{As } P = \frac{Nm}{3V} \langle v^2 \rangle$$

$$\cancel{V} \times \left(\langle v^2 \rangle \frac{2 \cancel{N} m}{2 \times 3 \cancel{V}} \right) = \cancel{N} KT$$

$$\frac{1}{2} \cdot m \langle v^2 \rangle \times \frac{2}{3} = KT$$

$$T = \langle K.E \rangle \cdot \frac{2}{3K}$$

$$(2/3K = \text{constant})$$

$$T \propto \langle K.E \rangle$$

Root Mean Square Speed of Gas Molecule

Ideal Gas

Basic Steps:-

#01 :- $(\vec{V})^2$

#02 :- $V^2/N = \langle V^2 \rangle$

#03 :- $\sqrt{\langle V^2 \rangle}$

Mechanism:-

i- Initially, we take square of velocities of the particles present.

ii- Then, we take Average or mean of these values.

iii- At the end, we take square root, so that the final value we recieve is under our range.

Reason:-

So that our value is neither negative, nor beyond or below our required value, and the value recieved is under our effective value.

Mathematically:-

i) (Square) $V^2 = V_1^2 + V_2^2 + V_3^2 + \ldots\ldots + V_n^2$

ii) (Mean) $\langle V^2 \rangle = \dfrac{V_1^2 + V_2^2 + V_3^2 + \ldots\ldots + V_n^2}{N}$

iii) (Root) $\sqrt{\langle V^2 \rangle} = \sqrt{\dfrac{V_1^2 + V_2^2 + V_3^2 + \ldots + V_n^2}{N}}$

PKMZ SEGMENT

R.M.S

Further Reduce to Bring it into our range	Reduce value	To remove -ve factor
$\sqrt{\langle V^2 \rangle}$	$\langle V^2 \rangle$	$(\vec{V})^2$

In Simple Words,

Through RMS we recieve effective value, this value is neither -ve, nor much Alternating or beyond our limit.

And the best Part is, that, this value is applicable for all the particles present in the container (∵ Subject)

$\sqrt{\langle V^2 \rangle} = V_{R.m.s}$ (Effective Value)

Relation of $V_{r.m.s}$ w/ Temp

Date: 30th July 25

Since, the Pressure of gas molecule is given by;

$$P = \frac{1}{3}\left(\frac{mN}{V}\right)\langle v^2\rangle$$

$$\Rightarrow PV = \frac{1}{3}(mN)\langle v^2\rangle$$

$$\Rightarrow \cancel{N}KT = \frac{1}{3}(m\cancel{N})\langle v^2\rangle$$

$$\Rightarrow \langle v^2\rangle = \frac{3KT}{m}$$

$\because PV = nRT$

$\left(\because n = \frac{N}{N_A}\right)$

$\Rightarrow PV = \left(\frac{N}{N_A}\right)RT$

$\Rightarrow PV = N\left(\frac{R}{N_A}\right)T$

$\Rightarrow PV = NKT$ $\quad \because k = \frac{R}{N_A}$

By taking sq. root on both sides.

$$\sqrt{\langle v^2\rangle} = \sqrt{\frac{3KT}{m}}$$

$\because \sqrt{\langle v^2\rangle} = V_{r.m.s}$

$$V_{r.m.s} = \sqrt{\frac{3K}{m}} \cdot \sqrt{T}$$

$\because \sqrt{3K/m} = \text{constant}$

$$V_{r.m.s} = (\text{constant})\sqrt{T}$$

$$\boxed{V_{r.m.s} \propto \sqrt{T}}$$

This relation tells that if value of Temp inc. then $V_{r.m.s}$ value will also increase

Relation between V_{rms} & Molar Mass

Since,

$$V_{rms} = \sqrt{\frac{3kT}{m}}$$

$$V_{rms} = \sqrt{\frac{3T}{m} \cdot k}$$

$$V_{r.m.s} = \sqrt{\frac{3T}{m} \cdot \frac{R}{N_A}}$$

$$V_{r.m.s} = \sqrt{\frac{3TR}{M_r}}$$

$$V_{r.m.s} = \sqrt{3TR} \times \frac{1}{\sqrt{M_r}}$$

$\because$ $3TR$ = Constant

$$V_{r.m.s} \propto \frac{1}{\sqrt{M_r}}$$

Side by Side

$\because k = R/N_A$

$$mol = \frac{mass}{M_r}$$

$$mol = \frac{n}{N_A}$$

$$\frac{mass}{M_r} = \frac{n}{N_A}$$

$$\frac{m}{M_r} = \frac{n}{N_A}$$

$$m \cdot N_A = M_r \cdot n$$

as, n = no. of mol. So, we can say that,

$$M_r = m \cdot N_A$$

$V_{r.m.s}$ with Pressure

Since,

$$P = \frac{1}{3} \rho \langle v^2 \rangle$$

$$3P = \rho \langle v^2 \rangle$$

$$\sqrt{\langle v^2 \rangle} = \sqrt{\frac{3P}{\rho}}$$

$$V_{r.m.s} = \sqrt{\frac{3P}{\rho}}$$

ASSIGNMENT'S

Date: 2/08

16.1

DATA:-

K (Boltzmann's constant) = 1.38×10^{-23} J/K

T (Temperature in kelvin) = 320 k

Required:-

Average translational kinetic energy?

Calculation:-

$$\langle K.E \rangle = \frac{3}{2} \times T \times K$$

$$= \frac{3}{2}(320)(1.38 \times 10^{-23})$$

$$\langle K.E \rangle = 6.624 \times 10^{-21} \approx \boxed{6.6 \times 10^{-21}\ J}$$

16.2

DATA:-

R (Ideal gas constant):- 8.314 J/mol.K

T (Temperature in kelvin):- 273 K

M (Molar Mass of Nitrogen):- 0.028 Kg/mol

Required:-

Root mean square speed value:- ?

Calculation:-

$$V_{r.m.s} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3 \cdot (8.314) \cdot (273)}{0.028}}$$

$$\boxed{= 493.14\ m/s}$$

10^{th} DCC — Boards

IDEAL GAS → NON IDEAL

Date: 31/7-25

Acc. to kinetic molecular theory :-

$PV = nRT$

→ No attractive forces among gas molecules
→ Gas molecules have negligable volume

• But at high pressure :-
→ gas converts into liquid (so first point of KMT ✗)
→ It have volume, that is why it's converted to liq. (2^{nd} ✗)

• And at low temperature :-
→ gas molecule will lose energy & will be converted to liq.
(again first point rejected of KMT)
→ Condensation agr ho rhi, tou volume hona zaroori he
(and second one too)

Q Why real gas deviates from ideal behavior?
Q When the " " behave like ideal gas?
↳ Low Pressure ↳ high temperature.

Person who figured out the problem b/w real & Ideal gas was Van Der Waals ~~Equation~~.
He modified $PV = nRT$ equation and converted it to,

$$\left[P + \frac{an^2}{V^2}\right](V - nb) = nRT$$