19-4-2024 Math

Unit 1

Ex 1.2

Q1] By using the poperty of product and quotient rule for radicals, write each expression as a single radical and simplify?

i) $\sqrt[3]{6} \cdot \sqrt[3]{6}$

$= \sqrt[3]{36}$ Ans.

ii) $\sqrt[5]{4} \cdot \sqrt[5]{8}$

$= \sqrt[5]{32}$ Ans.

iii) $\sqrt[4]{x} \cdot \sqrt[4]{x^3}$

$= \sqrt[4]{x^{1+3}}$

$= \sqrt[4]{x^4}$ Ans

iv) $\sqrt{10} \cdot \sqrt[3]{11}$

$= (10)^{\frac{1}{2}} \cdot (11)^{\frac{1}{3}}$

$= (110)^{\frac{1}{2}+\frac{1}{3}}$ (x^3, x^2)

$= (110)^{\frac{3+2}{6}} = \frac{5}{6}$

$= (110)^{\frac{5}{6}}$ Ans.

v) $\frac{\sqrt[4]{x^7}}{\sqrt[4]{x^5}}$

$= \sqrt[4]{\frac{x^7}{x^5}}$

$= \sqrt[4]{x^{7-5}}$

$= \sqrt[4]{x^2}$ Ans.

vi) $\frac{\sqrt[3]{5000}}{\sqrt[3]{5}}$

$= \sqrt[3]{\frac{5000}{5}}^{1000}$

$= \sqrt[3]{1000}$ Ans.

vii) $\frac{\sqrt[2]{500}}{\sqrt[2]{5}}$

$= \sqrt[2]{\frac{500}{5}}^{100}$

$= \sqrt[2]{100}$ Ans

viii) $\sqrt[2]{10} \cdot \sqrt[3]{7}$

As Index is not same so product rule is not applied.

iv) $\sqrt{10} \cdot \sqrt[3]{11}$

= As Index is not same so product rule is not applied.

Q² Write each exponential expression as an equivalent radical expression and simplify if possible.

i) $(216)^{\frac{2}{3}}$

$= (\sqrt[3]{216})^2$

$= \sqrt[3]{46656}$ Ans.

ii) $(29)^{\frac{1}{2}}$

$= \sqrt{29}$ Ans.

iii) $\left[\frac{1}{32}\right]^{\frac{1}{5}}$

$= \left[\frac{1}{\sqrt[5]{32}}\right]$ Ans.

iv) $(216)^{-\frac{2}{3}}$

$= \frac{1}{(216)^{\frac{2}{3}}}$

$= \frac{1}{\sqrt[3]{216^2}}$

$= \frac{1}{\sqrt[3]{46656}}$

$\frac{1}{36}$

v) $(1000)^{\frac{1}{3}}$

$= \sqrt[3]{1000}$

$= 10$ Ans.

vi) $\left[\frac{1}{39}\right]^{\frac{1}{2}}$

$= \frac{1}{\sqrt{39}}$ Ans.

Q3 Write each radical expression as an equivalent exponential expression and simplify if possible?

i) $(\sqrt[3]{5})^2$

$= 5^{\frac{2}{3}}$ Ans.

ii) $(\sqrt[4]{10})^8$

$= 10^{\frac{8}{4}^2}$

$= 10^2$

$= 100$ Ans.

iii) $-(\sqrt[3]{6})^6$

$= -(6)^{\frac{6}{3}^2}$

$= -6^2$

$= -36$ Ans.

iv) $(\sqrt[3]{6})^6$

$= 16^{\frac{6}{3}^2}$

$= 6^2$

$= 36$ Ans.

v) $-(\sqrt[3]{5})^2$

$= -5^{\frac{2}{3}}$ Ans.

vi) $-(\sqrt[4]{10})^8$ vi) $-(\sqrt[4]{10})^8$

$= -10^{\frac{8}{4}^2}$

$= 10^2$

$= -100$ Ans.

Q4 Use the properties of exponents to simplify each of the following. Assume that all variables represent positive numbers. (write all results with positive exponents).

i) $\frac{16^{\frac{1}{5}} \cdot 16^{\frac{1}{4}}}{16^{\frac{-3}{10}}}$

$= \frac{16^{\frac{1}{5}+\frac{1}{4}}}{16^{\frac{-3}{10}}}$ (×4, ×5)

$= \frac{16^{\frac{4+5}{20}} = \frac{9}{20}}{16^{\frac{-3}{10}}}$

$= 16^{\frac{9}{20}} \cdot 16^{\frac{3}{10}}$ (+3)

$= 16^{\frac{9}{20}+\frac{3}{10}}$ (×1, ×2)

$= 16^{\frac{9+6}{20}} = \frac{15}{20}$

$= 16^{\frac{3}{4}}$ Ans

$= (\sqrt[4]{16})^3$

ii) $7^{\frac{-1}{3}}(7^{\frac{5}{3}} - 7^{\frac{1}{3}})$

$= 7^{\frac{-1}{3}+\frac{5}{3}} - 7^{\frac{-1}{3}+\frac{1}{3}}$

$= 7^{\frac{4}{3}} - 7^{\frac{3}{3}}$

$= 7^{\frac{4}{3}} - 7$ Ans

iii) $\frac{2^{\frac{2}{3}} \cdot 2^{\frac{1}{7}}}{2^{\frac{1}{2}}}$

$= \frac{2^{\frac{2}{3}+\frac{1}{7}}}{2^{\frac{1}{2}}}$ (×7, ×3)

$= \frac{2^{\frac{14+3}{21}} = \frac{17}{21}}{2^{\frac{1}{2}}}$

$= 2^{\frac{17}{21}} \cdot 2^{\frac{-1}{2}}$

$= 2^{\frac{17}{21}-\frac{1}{2}}$ (×2, ×21)

$= 2^{\frac{34-21}{42}} = 2^{\frac{13}{42}}$ Ans

iv) $\frac{3^{-\frac{1}{2}} \cdot 3^{\frac{1}{2}}}{3^{\frac{1}{2}}}$

$= \frac{3^{-\frac{1}{2}+\frac{1}{2}}}{3^{\frac{1}{2}}}$

$= \frac{3^{0}}{3^{\frac{1}{2}}}$

$= \frac{1}{3^{\frac{1}{2}}}$ Ans.

vi) $\left[\frac{36^{\frac{1}{2}}}{8^{\frac{1}{2}}} \cdot \frac{6^{\frac{1}{2}}}{27^{\frac{1}{2}}}\right]^3$

$= \left[\left[\frac{\overset{4}{\cancel{36}} \times \overset{3}{\cancel{6}}}{\underset{4}{\cancel{8}} \times \underset{3}{\cancel{27}}}\right]^{\frac{1}{2}}\right]^3$ $\left[\left(\frac{\overset{4}{\cancel{36}} \times 6}{8 \times \underset{3}{\cancel{27}}}\right)^{\frac{1}{2}}\right]^3$

$= \left[\left(\frac{\cancel{4 \times 3}}{\cancel{4 \times 3}}\right)\right]$ $\left[\left(\frac{\overset{1}{\cancel{4}} \times \overset{2}{\cancel{6}}}{\underset{2}{\cancel{8}} \times \underset{1}{\cancel{3}}}\right)^{\frac{1}{2}}\right]^3$

$= \left[\left(\frac{1}{2}\right)^{\frac{1}{2}}\right]^3$

$= \left[(1)^{\frac{1}{2}}\right]^3$

$= 1$ Ans

vi) $\left[\frac{2187\, a^5\, b^{17}}{a^{12}\, b^3}\right]^{\frac{1}{7}}$

$= \left[3^7 a^{5-12}\, b^{17-3}\right]^{\frac{1}{7}}$

$= \left[3^7 a^{-7}\, b^{14}\right]$

$= \left[3^{7 \times \frac{1}{7}} a^{-7 \times \frac{1}{7}} b^{14 \times \frac{1}{7}}\right]$

$= \frac{3b^2}{a}$ Ans.

vii) $\sqrt[4]{\frac{a^3}{b^3}} \times \sqrt[4]{\frac{b^3}{c^3}} \times \sqrt[4]{\frac{c^3}{a^3}}$

$= \sqrt[4]{\frac{a^3 \times b^3 \times c^3}{b^3 \times c^3 \times a^3}}$

$= \sqrt[4]{1}$

$= (1)^{\frac{1}{4}} = 1$ Ans.

Q5 Use suitable laws of exponents to show that:

$$\left[\frac{x^p}{x^q}\right]^{p+q} \times \left[\frac{y^q}{y^r}\right]^{q+r} \times \left[\frac{z^r}{z^p}\right]^{r+p} \times x^{a^2} \times y^{r^2} \times z^{p^2} = x^{p^2} \times y^{a^2} \times z^{r^2}$$

$$= \left[x^{p-q}\right]^{p+q} \times \left[y^{q-r}\right]^{q+r} \times \left[z^{r-p}\right]^{r+p} \times x^{a^2} \times y^{r^2} \times z^{p^2} = x^{p^2} \times y^{a^2} \times z^{r^2}$$

$$= x^{p^2-q^2} \times y^{q^2-r^2} \times z^{r^2-p^2} \times x^{a^2} \times y^{r^2} \times z^{p^2} = x^{p^2} \times y^{a^2} \times z^{r^2}$$

$$= x^{p^2-q^2+q^2} \times y^{q^2-r^2+r^2} \times z^{r^2-p^2+p^2} = x^{p^2} \times y^{a^2} \times z^{r^2}$$

$$= x^{p^2} \times y^{a^2} \times z^{r^2} = x^{p^2} \times y^{a^2} \times z^{r^2} \text{ Ans.}$$