Q. No. 2 Part (i)

$$5x - \frac{8}{x} + 6 = 0$$

Solution:

$$5x - \frac{8}{x} + 6 = 0$$

Multiply L.C.M $= x$ on both sides

$$x(5x) - \cancel{x}\left(\frac{8}{\cancel{x}}\right) + x(6) = x(0)$$

$$5x^2 - 8 + 6x = 0$$

$$\boxed{5x^2 + 6x - 8 = 0}$$

↳ Standard Form of Quadratic Equation

$\Rightarrow$ By FACTORIZATION,

$$5x^2 + 10x - 4x - 8 = 0$$

$$5x(x+2) - 4(x+2) = 0$$

$$(x+2)(5x-4) = 0$$

$\Rightarrow (x+2) = 0$, $\Rightarrow (5x-4) = 0$

$\Rightarrow \boxed{x = -2}$

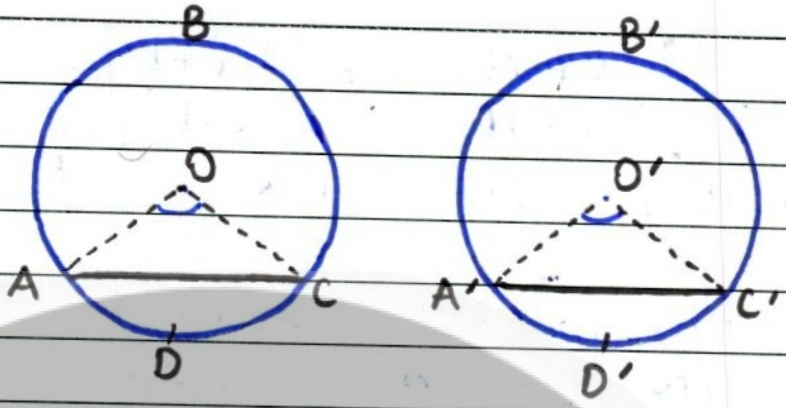
$\Rightarrow 5x = 4$

$\Rightarrow \boxed{x = \frac{4}{5}}$

$\therefore$ Solution Set $= \left\{-2, \frac{4}{5}\right\}$

Q. No. 4 (Page 1)

Figure:



Given: ABCD and A'B'C'D' are TWO congruent circles with centres O and O' respectively. Such that

$$m\overset{\frown}{ADC} = m\overset{\frown}{A'D'C'}$$

Arcs ADC and A'D'C' of circles with centres O and O' respectively subtend chords AC and A'C' respectively.

To Prove: The chords corresponding to $\overset{\frown}{ADC}$ and $\overset{\frown}{A'D'C'}$ of circles with centres O and O' respectively are EQUAL i.e.

$$m\overline{AC} = m\overline{A'C'}$$

Construction: Join O with A, O with C, O' with A' and O' with C'. So that we can form Δs AOC and A'O'C', in first and second circle respectively.

Proof:

Statements	Reasons
$m\overset{\frown}{ADC} = m\overset{\frown}{A'D'C'}$	Given
$\therefore m\angle AOC = m\angle A'O'C'$ —— (1)	Central Angles subtended by equal arcs of CONGRUENT circles.

Q. No. 4 (Page 2)

In $\triangle AOC \longleftrightarrow \triangle A'O'C'$

$m\overline{OA} = m\overline{O'A'}$	Radii of congruent circles
$m\angle AOC = m\angle A'O'C'$	Already proved in (1)
$m\overline{OC} = m\overline{O'C'}$	Radii of congruent circles
$\therefore \boxed{\triangle AOC \cong \triangle A'O'C'}$	S.A.S postulate

Therefore,

$m\overline{AC} = m\overline{A'C'}$	Corresponding sides of CONGRUENT triangles.

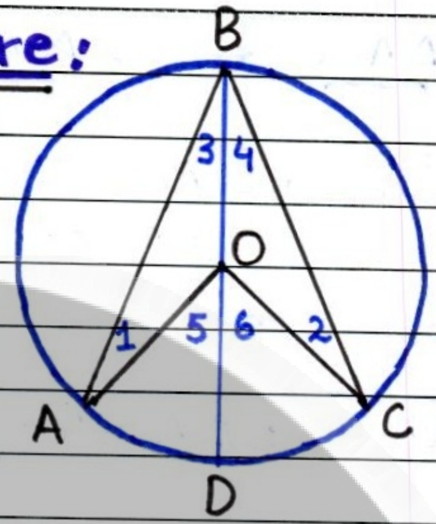
Hence,

Congruent ARCS of congruent circles subtend EQUAL chords. — PROVED

Similarly,
This theorem can also be proved with EQUAL/CONGRUENT arcs in the same circle.

Q. No. 5 (Page 1)

Figure:



Given: In a circle ABCD with centre O,

$\angle AOC$ is the CENTRAL angle.
whereas,
$\angle ABC$ is the CIRCUM angle.

To Prove: CENTRAL angle is DOUBLE in measure than CIRCUM angle i.e.

$$m\angle AOC = 2\, m\angle ABC$$

Construction: Join B to O and produce it to meet the circle at point D.

Label $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$ as shown in figure.

Proof:

Statements	Reasons
In ΔAOB,	
$m\angle 1 = m\angle 3$ —— (1)	Angles opposite to congruent sides i.e. RADII of the same circle $(m\overline{OA} = m\overline{OB})$
Similarly, In ΔBOC,	
$m\angle 2 = m\angle 4$ —— (2)	As in (1) $(m\overline{OB} = m\overline{OC})$

Q. No. 5 (Page 2)

Now; $m\angle 5 = m\angle 1 + m\angle 3$ — (3)	External angle is the sum of internal opposite angles.
Also, $m\angle 6 = m\angle 2 + m\angle 4$ — (4)	As in (3)
Again; $m\angle 5 = m\angle 1 + m\angle 3$	From (3)
$m\angle 5 = \boxed{m\angle 3} + m\angle 3$	Using (1); $\boxed{m\angle 1 = m\angle 3}$
$\therefore \boxed{m\angle 5 = 2m\angle 3}$ — (5)	
Similarly; $m\angle 6 = m\angle 2 + m\angle 4$	From (4)
$m\angle 6 = \boxed{m\angle 4} + m\angle 4$	Using (2); $\boxed{m\angle 2 = m\angle 4}$
$\therefore \boxed{m\angle 6 = 2m\angle 4}$ — (6)	
$m\angle 5 + m\angle 6 = 2m\angle 3 + 2m\angle 4$	Adding (5) and (6)
$m\angle 5 + m\angle 6 = 2(m\angle 3 + m\angle 4)$	
$\boxed{m\angle AOC = 2m\angle ABC}$	From figure $m\angle AOC = m\angle 5 + m\angle 6$ and $m\angle ABC = m\angle 3 + m\angle 4$
Therefore, Central angle AOC is DOUBLE in measure than circum angle ABC.	PROVED.

[illegible]

Rough Work

Q. No. 2 Part (v)

Given: $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 4, 6, 8\}$
$C = \{1, 4, 7, 8\}$

To Verify: $A \cup (B \cup C) = (A \cup B) \cup C$

PROOF:

L.H.S $= A \cup (B \cup C)$
$= \{1,2,3,4,5,6,7\} \cup (\{2,4,6,8\} \cup \{1,4,7,8\})$
$= \{1,2,3,4,5,6,7\} \cup (\{1,2,4,6,7,8\})$
$= \{1,2,3,4,5,6,7\} \cup \{1,2,4,6,7,8\}$
$= \{1,2,3,4,5,6,7,8\}$ —— (1)

R.H.S $= (A \cup B) \cup C$
$= (\{1,2,3,4,5,6,7\} \cup \{2,4,6,8\}) \cup \{1,4,7,8\}$
$= (\{1,2,3,4,5,6,7,8\}) \cup \{1,4,7,8\}$
$= \{1,2,3,4,5,6,7,8\} \cup \{1,4,7,8\}$
$= \{1,2,3,4,5,6,7,8\}$ —— (2)

Result:
From (1) and (2);

L.H.S = R.H.S

i.e. $A \cup (B \cup C) = (A \cup B) \cup C$

Q. No. 2 Part (vi)

Class Intervals	Frequency (f)	Mid-point (X)	fX
1 – 9	6	$\frac{1+9}{2} = 5$	30
10 – 18	4	$\frac{10+18}{2} = 14$	56
19 – 27	1	$\frac{19+27}{2} = 23$	23
28 – 36	2	$\frac{28+36}{2} = 32$	64
37 – 45	2	$\frac{37+45}{2} = 41$	82
	$\Sigma f = 15$		$\Sigma fX = 255$

$\Sigma f = 6+4+1+2+2$

$\boxed{\Sigma f = 15}$

$\Sigma fX = 30+56+23+64+82$

$\boxed{\Sigma fX = 255}$

⇒ As we know that,

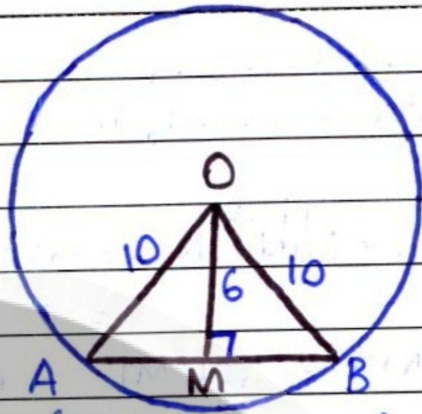
$\because \bar{X} = \frac{\Sigma fX}{\Sigma f}$

$\bar{X} = \frac{255}{15}$

$\boxed{\bar{X} = 17}$

RESULT: Thus, arithmetic mean of the given grouped data is 17.

Q. No. 2 Part (vii)



Given Data:

- Distance of chord $\overline{AB}$ from centre O $= m\overline{OM} = 6$ cm
- Radius of circle $= m\overline{OA}$
  $= m\overline{OB}$
  $= 10$ cm
- $m\angle OMA = m\angle OMB = 90°$ (From figure)

Required Data:

Length of chord $= \boxed{m\overline{AB}} = ?$

Solution:

In $\angle$rt. $\triangle OMB$,

$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$(m\overline{OB})^2 = (m\overline{OM})^2 + (m\overline{MB})^2$ [PYTHAGORAS' THEOREM]

$(10)^2 = (6)^2 + (m\overline{MB})^2$

$100 = 36 + (m\overline{MB})^2$

$(m\overline{MB})^2 = 100 - 36$

$(m\overline{MB})^2 = 64$

Taking SQUARE ROOT on both sides

$\sqrt{(m\overline{MB})^2} = \sqrt{64}$

$m\overline{MB} = 8$ cm

⇒ Similarly, $m\overline{MA} = 8$ cm ($\overline{OM} \perp \overline{AB}$)

(Perpendicular from centre of circle on a chord BISECTS it.)

As, $m\overline{AB} = m\overline{MA} + m\overline{MB}$

$m\overline{AB} = 8\text{ cm} + 8\text{ cm}$

$$\boxed{m\overline{AB} = 16 \text{ cm}}$$

Result:

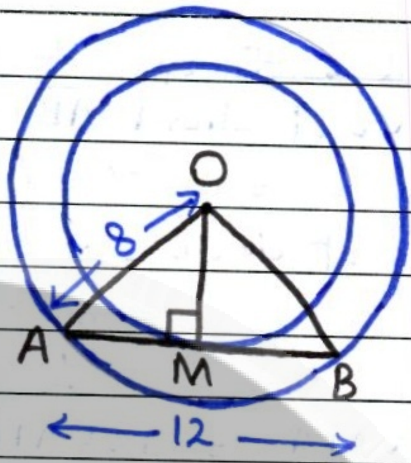
The length of required chord $\overline{AB}$ is 16 cm.

Q. No. 2 Part (viii)

Given Data:

Length of chord, $\overline{AB} = m\overline{AB}$

$= 12\,cm$



Radius of first circle $= m\overline{OA} = m\overline{OB}$

$= 8\,cm$

$m\angle OMA = m\angle OMB = 90^\circ$

$(\overline{OM} \perp \overline{AB}$, From figure)

Required Data:

Radius of SECOND circle cocentric with first circle $= m\overline{OM} = ?$

Solution:

$\because\ m\overline{AB} = 12\,cm$ (GIVEN)

$\therefore\ m\overline{AM} = m\overline{BM} = \frac{1}{2}(m\overline{AB})$

$m\overline{AM} = m\overline{BM} = \frac{1}{\cancel{2}} \times \cancel{12}^{\,6}\,cm$

$\boxed{m\overline{AM} = m\overline{BM} = 6\,cm}$ $(\overline{OM} \perp \overline{AB})$

(Perpendicular from centre of a circle on a chord BISECTS it.)

⇒ Now;

In $\angle$rt. Δ OMA, By Pythagoras Theorem,

$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$(m\overline{OA})^2 = (m\overline{OM})^2 + (m\overline{AM})^2$

$(8)^2 = (m\overline{OM})^2 + (6)^2$

$64 = (m\overline{OM})^2 + 36$

$(m\overline{OM})^2 = 64 - 36$

$(m\overline{OM})^2 = 28$ (Taking SQUARE ROOT on both sides)

$\sqrt{(m\overline{OM})^{\cancel{2}}} = \sqrt{28}$

(Radius of SECOND circle)

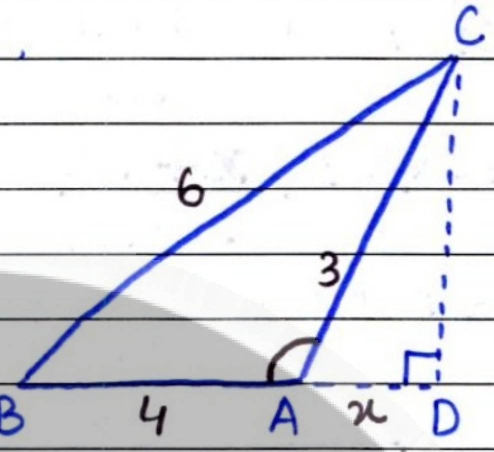
$\therefore \boxed{m\overline{OM} = \sqrt{28}\,cm = 2\sqrt{7}\,cm = 5.29\,cm}$

Result: Thus, radius of SECOND circle is $2\sqrt{7}$ cm or 5.29 cm.

Q. No. 2 Part (ix)

Given Data: In ΔABC,

1. $m\overline{BC} = 6\,cm$
2. $m\overline{AC} = 3\,cm$
3. $m\overline{BA} = 4\,cm$
4. $\overline{CD} \perp \overline{BD}$ (From figure)



Required Data:

Projection Length of $\overline{AC}$ on $\overline{BA}$ produced $= m\overline{AD} = x = ?$

Solution:

$\Rightarrow$ Since In ΔABC, $\boxed{\angle BAC}$ is OBTUSE;

$$\therefore (\overline{BC})^2 = (\overline{AC})^2 + (\overline{BA})^2 + 2(m\overline{BA})(m\overline{AD})$$

$(6)^2 = (3)^2 + (4)^2 + 2(4)(x)$ (In an obtuse-angled triangle, the square on the side opposite to the obtuse angle is EQUAL to the sum of the SQUARES on the sides containing the obtuse angle together with twice the rectangle contained by one of the sides and the projection on it of the other).

$$36 = 9 + 16 + 8x$$
$$36 = 25 + 8x$$
$$8x + 25 = 36$$
$$8x = 36 - 25$$
$$8x = 11$$
$$x = \frac{11}{8}\ cm$$

$$\therefore \boxed{m\overline{AD} = x = 1.375\,cm}$$

RESULT:

Therefore, the projection length of x of $\overline{AC}$ on $\overline{BA}$ produced is $\boxed{1.375\ cm}$ or $\boxed{\frac{11}{8}\ cm}$.

Q. No. 3 (Page 1)

Given : $x = \dfrac{14ab}{a+b}$ —— (1)

To Prove : $\dfrac{x+7a}{x-7a} + \dfrac{x+7b}{x-7b} = 2$

Proof : From eq. (1);

$$x = \frac{(7a)(2b)}{a+b} \quad \text{—— (A)}$$

$$\boxed{\frac{x}{7a} = \frac{2b}{a+b}} \quad \text{—— (2)}$$

By using COMPONENDO-DIVIDENDO Theorem, eq. (2) becomes,

L.H.S

$$\frac{x+7a}{x-7a} = \frac{2b+(a+b)}{2b-(a-b)}$$

$$\frac{x+7a}{x-7a} = \frac{2b+a+b}{2b-a-b}$$

$$\boxed{\frac{x+7a}{x-7a} = \frac{a+3b}{b-a}} \quad \text{—— (3)}$$

Now; From eq. (A);

$$x = \frac{(7b)(2a)}{a+b}$$

$$\boxed{\frac{x}{7b} = \frac{2a}{a+b}} \quad \text{—— (4)}$$

By COMPONENDO-DIVIDENDO Theorem, eq (4) becomes;-

$$\frac{x+7b}{x-7b} = \frac{2a+(a+b)}{2a-(a+b)}$$

$$\frac{x+7b}{x-7b} = \frac{2a+a+b}{2a-a-b}$$

$$\boxed{\frac{x+7b}{x-7b} = \frac{3a+b}{a-b}} \quad — \ (5)$$

$\Rightarrow$ Adding eq. (3) and (5) :-

$$\frac{x+7a}{x-7a} + \frac{x+7b}{x-7b} = \frac{a+3b}{b-a} + \frac{3a+b}{a-b}$$

$$\frac{x+7a}{x-7a} + \frac{x+7b}{x-7b} = \frac{a+3b}{-(a-b)} + \frac{3a+b}{a-b}$$

$$\frac{x+7a}{x-7a} + \frac{x+7b}{x-7b} = \frac{-(a+3b)}{a-b} + \frac{3a+b}{a-b}$$

$$\frac{x+7a}{x-7a} + \frac{x+7b}{x-7b} = \frac{-a-3b+3a+b}{a-b}$$

$$\frac{x+7a}{x-7a} + \frac{x+7b}{x-7b} = \frac{2a-2b}{a-b}$$

$$\frac{x+7a}{x-7a} + \frac{x+7b}{x-7b} = \frac{2\cancel{(a-b)}}{\cancel{a-b}}$$

$$\boxed{\frac{x+7a}{x-7a} + \frac{x+7b}{x-7b} = 2}$$

R.H.S = 2

Result: Hence, it is proved that;

$$\boxed{\frac{x+7a}{x-7a} + \frac{x+7b}{x-7b} = 2}$$

i.e. L.H.S = R.H.S

Q. No. 2 Part (ii)

$$\sqrt{x-3}+5 = x$$

Solution:

$$\sqrt{x-3}+5 = x$$

$$\sqrt{x-3} = x-5$$

Taking SQUARE on both sides

$$(\sqrt{x-3})^2 = (x-5)^2$$

$$x-3 = (x)^2-2(x)(5)+(5)^2$$

$$x-3 = x^2-10x+25 \quad \{\because (a-b)^2=a^2-2ab+b^2\}$$

$$0 = x^2-10x+25-x+3$$

$$0 = x^2-10x-x+25+3$$

$$0 = x^2-11x+28$$

$$x^2-11x+28 = 0$$

$\Rightarrow$ By FACTORIZATION,

$$x^2-7x-4x+28 = 0$$

$$x(x-7)-4(x-7) = 0$$

$$(x-7)(x-4) = 0$$

$\Rightarrow (x-7)=0$, $\Rightarrow (x-4)=0$

$\Rightarrow \boxed{x=7}$ $\Rightarrow \boxed{x=4}$

Check: when $\boxed{x=7}$

$$\sqrt{7-3}+5 = 7$$

$$\sqrt{4}+5 = 7$$

$$2+5 = 7$$

$$\boxed{7=7} \text{ (TRUE)}$$

when $\boxed{x=4}$,

$$\sqrt{4-3}+5 = 4$$

$$\sqrt{1}+5 = 4$$

$$1+5 = 4$$

$$\boxed{6 \neq 4} \text{ (FALSE)}$$

$\Rightarrow \boxed{x=4}$ is an extraneous root

$\therefore$ Solution Set $= \{7\}$

Q. No. 2 Part (iii)

Given: $\frac{x}{p} = \frac{y}{q} = \frac{z}{r}$

To Prove: $\frac{x^3+y^3+z^3}{p^3+q^3+r^3} = \frac{xyz}{pqr}$

LET:

$$\frac{x}{p} = \frac{y}{q} = \frac{z}{r} = k$$

$$\therefore \boxed{x=pk}, \boxed{y=qk}, \boxed{z=rk}$$

PROOF:

$$L.H.S = \frac{x^3+y^3+z^3}{p^3+q^3+r^3}$$

$$= \frac{(pk)^3+(qk)^3+(rk)^3}{p^3+q^3+r^3}$$

$$= \frac{p^3k^3+q^3k^3+r^3k^3}{p^3+q^3+r^3}$$

$$= \frac{k^3\cancel{(p^3+q^3+r^3)}}{\cancel{p^3+q^3+r^3}}$$

$$\boxed{L.H.S = k^3} \text{ --- } (1)$$

$$R.H.S = \frac{xyz}{pqr}$$

$$= \frac{(pk)(qk)(rk)}{pqr}$$

$$= \frac{\cancel{pqr}\,k^3}{\cancel{pqr}}$$

$$\boxed{R.H.S = k^3} \text{ --- } (2)$$

Result:

From (1) and (2);

L.H.S = R.H.S

i.e.

$$\frac{x^3+y^3+z^3}{p^3+q^3+r^3} = \frac{xyz}{pqr}.$$

Q. No. 2 Part (iv)

• **Let:** $$\boxed{\frac{x^2-2}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}} \quad \text{— (A)}$$

Multiplying both sides by L.C.M $= (x-1)(x+1)^2$; we get :-

$$\cancel{(x-1)(x+1)^2} \times \frac{x^2-2}{\cancel{(x-1)(x+1)^2}} = (\cancel{x-1})(x+1)^2 \times \frac{A}{\cancel{x-1}} + (x-1)(\cancel{x+1})^{\cancel{2}} \times \frac{B}{\cancel{x+1}} + (x-1)\cancel{(x+1)^2} \times \frac{C}{\cancel{(x+1)^2}}$$

$$x^2-2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1) \quad \text{— (1)}$$

Eq. (1) is an IDENTITY and holds true for all values of "x"; so,

• Put: $(x-1)=0 \Rightarrow \boxed{x=1}$ in eq. (1):

$$(1)^2-2 = A(1+1)^2 + B(1-1)(1+1) + C(1-1)$$
$$1-2 = A(2)^2 + B(0)(2) + C(0)$$
$$-1 = A(4) + B(0) + 0$$
$$-1 = 4A + 0 + 0$$
$$4A = -1 \Rightarrow \boxed{A = -1/4}$$

• Put: $(x+1)=0 \Rightarrow \boxed{x=-1}$ in eq. (1):

$$(-1)^2-2 = A(-1+1)^2 + B(-1-1)(-1+1) + C(-1-1)$$
$$1-2 = A(0)^2 + B(-2)(0) + C(-2)$$
$$-1 = 0 + 0 - 2C$$
$$-2C = -1 \Rightarrow C = \frac{\cancel{-}1}{\cancel{-}2} \Rightarrow \boxed{C = 1/2}$$

⇒ From eq. (1), $x^2-2 = A(x^2+1+2x) + B(x^2-1) + Cx - C$ $\left\{ \because (a+b)(a-b) = a^2-b^2 \right\}$

$$x^2-2 = Ax^2 + A + 2Ax + Bx^2 - B + Cx - C$$
$$x^2-2 = (A+B)x^2 + (2A+C)x + (A-B-C)$$

Comparing coefficients of x^2 on both sides; we get :-

$$A+B = 1 \Rightarrow -1/4 + B = 1 \Rightarrow B = 1 + 1/4 \Rightarrow \boxed{B = 5/4}$$

⇒ Substituting the values of A, B and C, eq. (A) becomes;

$$\boxed{\frac{x^2-2}{(x-1)(x+1)^2} = \frac{-1}{4(x-1)} + \frac{5}{4(x+1)} + \frac{1}{2(x+1)^2}}$$

where $\frac{-1}{4(x-1)}$, $\frac{5}{4(x+1)}$ and $\frac{1}{2(x+1)^2}$ are required partial fractions. Answer.