Unit 6

Important

Ex 6.1

1) Trigonmetic Identites:-

Reciprocal Identites:-

$\sin\theta = \frac{1}{\operatorname{Cosec}\theta}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$

$\operatorname{Tan}\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\operatorname{Tan}\theta}$

2) Quotient Identites:-

$\operatorname{Tan}\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocal

$\therefore \operatorname{Tan} = \frac{\sin\theta}{\cos}$, $\cot\theta$

3) Pythagorean Identites

3 figures Used In questions

$\sin^2\theta + \cos^2\theta = 1$	$\sec^2\theta + \operatorname{Tan}^2\theta = 1$	$\operatorname{cosec}^2\theta - \cot^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$	$\sec^2\theta = 1 - \operatorname{Tan}^2\theta$	
$\cos^2\theta = 1 - \sin^2\theta$	$\operatorname{Tan}^2\theta = 1 - \sec^2\theta$	$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

Q1 i) $(1-\sin^2\theta)\sec^2\theta = 1$ $\quad \because \cos^2\theta = 1-\sin^2\theta$

$\cos^2\theta \cdot \sec^2\theta = 1$

$\cancel{\sec^2\theta = 1}$

$\dfrac{1}{\sec^2\theta} \times \sec^2\theta = 1$ $\quad \because \cos^2\theta = \dfrac{1}{\sec^2\theta}$

$\boxed{1 = 1 \ \boxed{\text{Ans}}}$

$\dfrac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

$= \dfrac{1-\sin\theta}{1+\sin\theta} \times \dfrac{1-\sin\theta}{1-\sin\theta}$

$= \dfrac{(1-\sin\theta)^2}{(1^2-\sin^2\theta)}$

$= \dfrac{(1-\sin\theta)^2}{1-\sin^2\theta}$

$= \dfrac{(1-\sin\theta)^2}{\cos^2\theta}$ $\quad \because \cos^2\theta = 1-\sin^2\theta$

$= \left(\dfrac{1-\sin\theta}{\cos\theta}\right)^2$

$= \left(\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\right)^2$

$\sec\theta - \tan\theta = \sec\theta - \tan\theta$ Ans.

iii) $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{\sin\theta}{1+\cos\theta}$

$= \sqrt{\dfrac{1-\cos\theta}{1+\cos\theta} \times \dfrac{1+\cos\theta}{1+\cos\theta}}$

$= \sqrt{\dfrac{(1^2-\cos^2\theta)}{(1+\cos\theta)^2}}$

$= \sqrt{\dfrac{1-\cos^2\theta}{(1+\cos\theta)^2}}$

$= \sqrt{\dfrac{\sin^2\theta}{(1+\cos\theta)^2}}$

$= \sqrt{\left(\dfrac{\sin\theta}{1+\cos\theta}\right)^2}$

$= \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{\sin\theta}{1+\cos\theta}$ Ans.

iv) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

L.H.S

$\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \dfrac{\sin\theta(1-\sin^2\theta-\sin^2\theta)}{\cos\theta(\cos^2\theta+\cos^2\theta-1)}$

$= \dfrac{\sin\theta(\cos^2\theta-\sin^2\theta)}{\cos\theta(\cos^2\theta-\sin^2\theta)}$

$\because \sin^2\theta = \cos^2\theta - 1$

$\because \sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta - 1 = \sin^2\theta$

$= \dfrac{\sin\theta}{\cos\theta}$ $\quad \because \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$\tan\theta$ Ans.

v) $\sqrt{\sec^2 + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

L.H.S

$\sqrt{1+\tan^2\theta+1+\cot^2\theta}$

$\sqrt{\tan^2\theta + 2 + \dfrac{1}{\tan^2\theta}}$

$\sqrt{\left(\tan^2\theta + \dfrac{1}{\tan^2\theta}\right)^2}$

$\tan^2\theta + \dfrac{1}{\tan^2\theta}$ $\quad \because \cos\theta = \dfrac{1}{\tan\theta}$

$\tan\theta + \cot\theta$ Ans.

vi) $\frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$

L.H.S.

$\frac{1}{\sec\theta - \tan\theta} \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$

$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$

$\because \sec^2\theta - \tan^2\theta = 1$

$= \frac{\sec\theta + \tan\theta}{1}$

$= \sec\theta + \tan\theta$ Ans

vii) $\sin^2\theta + \cos^4\theta = \sin^4\theta + \cos^2\theta$

L.H.S

$= \sin^2\theta + \cos^2\theta \cdot \cos^2\theta$

$= \sin^2\theta + (1 - \sin^2\theta)(1 - \sin^2\theta)$

$= \sin^2\theta + 1 - \sin^2\theta - \sin^2\theta + \sin^4\theta$

$= \cancel{\sin^2\theta} + 1 - \sin^2\theta - \cancel{\sin^2\theta} + \sin^4\theta$

$= 1 - \sin^2\theta + \sin^4\theta$

$= \cos^2\theta + \sin^4\theta$ Ans.

viii) $\dfrac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \dfrac{\sec\theta + 1}{\sec\theta - 1}$

L.H.S

$= \dfrac{\tan\theta + \sin\theta}{\tan\theta \mp \sin\theta} \times \dfrac{\tan\theta + \sin\theta}{\tan\theta + \sin\theta}$

$= \tan^2\theta + \sin^2\theta$

L.H.S

$\dfrac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta}$ $\quad \because \tan = \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{\frac{\sin\theta}{\cos\theta} - \sin\theta}$

$\dfrac{\cancel{\sin\theta}\left(\frac{1}{\cos\theta} + 1\right)}{\cancel{\sin\theta}\left(\frac{1}{\cos\theta} - 1\right)}$ $\quad \because \sec = \dfrac{1}{\cos}$

$\boxed{\dfrac{\sec\theta + 1}{\sec\theta - 1}}$ Ans.

ix) $\sec^2\theta + \operatorname{cosec}^2\theta$

L.H.S.

$\sec^2\theta + \operatorname{cosec}^2\theta$

$\dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$

$\dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta} = \dfrac{1}{\cos^2\theta \sin^2\theta}$

$\boxed{\sec^2\theta . \operatorname{cosec}^2\theta}$ Ans.

$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

L.H.S

$= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)\left[(\sin^2\theta)^2 - (\sin^2\theta)(\cos^2\theta) + (\cos^2\theta)^2\right]$

$= 1\left[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 3\sin^2\theta\cos^2\theta\right]$

$= 1\left[(\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta\right.$

$= 1\,(1)^2 - 3\sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$ Ans.

Q2

$x\cos\theta + y\sin\theta = m$ —i) and $x\sin\theta - y\cos\theta = n$ — ii

Taking square of eq(i) and (ii).

$(x\cos\theta + y\sin\theta)^2 = (m)^2$, $(x\sin\theta - y\cos\theta)^2 = (n)^2$

$(x\cos\theta + y\sin\theta)^2 + (x\sin\theta - y\cos\theta)^2 = m^2 + n^2$

$(x\cos\theta)^2 + (y\sin\theta)^2 + \cancel{2(x\cos\theta)(y\sin\theta)} + (x\sin\theta)^2 + (y\cos\theta)^2 - 2(x\sin\theta)(y\cos\theta)$

$x^2\cos^2\theta + y^2\sin^2\theta + x^2\sin^2\theta + y^2\cos^2\theta = m^2 + n^2$

$\cancel{x^2\cos^2\theta + \sin^2\theta}$

$x^2\cos^2\theta + x^2\sin^2\theta + y^2\sin^2\theta + y^2\cos^2\theta) = m^2 + n^2$

$x^2(\cos^2\theta + \sin^2\theta) + y^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$

$x^2\,(1) + y^2\,(1) = m^2 + n^2$

$x^2 + y^2 = m^2 + n^2$ Ans.

Q3 i) $2\sin^2\theta = \frac{1}{2}$

$$\sin^2\theta = \frac{1}{2 \times 2}$$

$$\sin^2\theta = \frac{1}{4}$$

$$\sqrt{\sin^2\theta} = \sqrt{\frac{1}{4}}$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$\theta = 30^\circ$ Ans.

$$\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1$$

$$\tan\theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45^\circ$$

iii) $\sec^2\theta - 2\tan^2\theta = 0$

$$1 + \tan^2\theta - 2\tan^2\theta$$

$$1 - \tan^2\theta = 0$$

$$1 = \tan^2\theta$$

$$\sqrt{1} = \sqrt{\tan^2\theta}$$

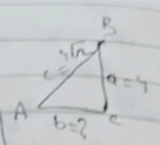
$$1 = \tan\theta$$

$$\tan\theta = 1$$

$$\theta = \tan^{-1}(1)$$

$\boxed{\theta = 45^\circ \text{ Ans.}}$

Q4 Solve the following rigth angled triangles?



i) $\Delta ABC, \angle C = 90^\circ, a = 4cm, c = 4\sqrt{2}$ cm

$h^2 = b^2 + p^2$

$\cancel{(4)^2 = b^2 + C}$

$(4\sqrt{2})^2 = b^2 + (4)^2$

$16 \times 2 = b^2 + 16$

$32 = b^2 + 16$

$32 - 16 = b^2$

$16 = b^2$

$\sqrt{16} = \sqrt{b^2}$

$4 = b$

$b = 4$

$\sin\angle A = \frac{P}{h}$

$= \frac{4}{4\sqrt{2}}$

$\sin\angle A = \frac{1}{2}$

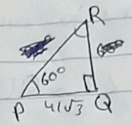
$\angle A = \sin^{-1}\frac{1}{2}$

$\boxed{\angle A = 45^\circ}$

$\angle B = 180^\circ - 90^\circ - 45^\circ$

$\boxed{\angle B = 45^\circ}$ Ans

ii) ΔPQR, $\angle Q = 90^\circ$, $\angle P = 60^\circ$, $PQ = 4\sqrt{3}$



$\angle R = 180^\circ - \angle Q - \angle P$

$\angle R = 180^\circ - 90^\circ - 60^\circ$

$\boxed{\angle R = 30^\circ}$

$\cos\angle P = \frac{B}{H}$

$\cos 60 = \frac{4\sqrt{3}}{h}$

$h = \frac{4\sqrt{3}}{\cos 60} = \frac{4\sqrt{3}}{0.5}$

$= 8\sqrt{3}$

$H^2 = B^2 + P^2$

$(8\sqrt{3})^2 = (4\sqrt{3})^2 + (P)^2$

$= 64 \times 3 = 16 \times 3 + (P)^2$

$= 192 - 48 = (P)^2$

$= 144 = (P)^2$

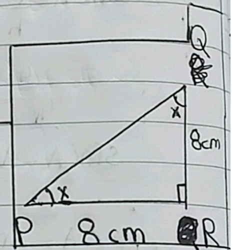
Taking square root on b/s.

$\sqrt{144} = \sqrt{(P)^2}$

$12 = P$ Ans.

iii) ΔPQR

$\angle R = 90^\circ$, $PR = 8m$, RQ $8m$



$H^2 = B^2 + P^2$

$H^2 = (8)^2 + (8)^2$

$H^2 = 64 + 64$

$H^2 = 128$

Taking square root on b/s.

$\sqrt{H^2} = \sqrt{128}$

$\boxed{H = 8\sqrt{2}}$ or $\boxed{PQ = 8\sqrt{2}}$

Find angles of both P and Q

$x + x + 90^\circ = 180^\circ$

$2x = 180^\circ - 90^\circ$

$2x = 90^\circ$

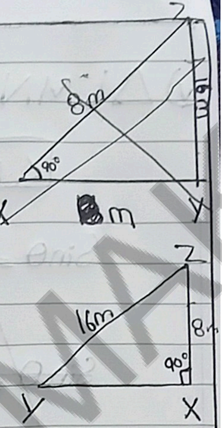
$x = \frac{\cancel{90^\circ}\,45}{\cancel{2}\,1}$

$x = 45^\circ$

$\boxed{\angle P = 45^\circ}$
$\boxed{\angle Q = 45^\circ}$ Ans.

(iv) ΔXYZ

$\angle X = 90^\circ$, $YZ = 16m$, $XZ = 8m$



$H^2 = B^2 + P^2$

$(16)^2 = (B)^2 + (8)^2$

$256 = B^2 + 64$

$256 - 64 = B^2$

$192 = B^2$

Taking square root on b/s.

$\sqrt{192} = \sqrt{B^2}$

$8\sqrt{3} = B$

or

$\boxed{XY = 8\sqrt{3}}$

$x + x + 90^\circ = 180^\circ$

$2x = 180^\circ - 90^\circ$

$2x = 90^\circ$

$x = \frac{90^\circ}{2}$ 45

$\boxed{x = 45^\circ}$

$\angle Y = 45^\circ$

$\angle Z = 45^\circ$

$\sin\theta = \frac{P}{H}$

$\sin\theta = \frac{8m}{16m}$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\frac{1}{2}$

$\boxed{\angle Y = 30^\circ}$

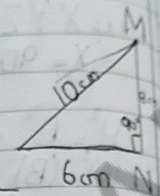
$\angle X + \angle Y + \angle Z = 180^\circ$

$90^\circ + 30^\circ + \angle Z$

$\angle Z = 180^\circ - 120^\circ$

$\boxed{\angle Z = 60^\circ}$ Ans.

v) $\triangle LMN$, $LM = 10cm$, $MN = 8cm$, $NL = 6cm$



$\boxed{\angle N = 90^\circ}$

$\sin\theta = \frac{P}{H}$

~~$\sin\theta = \frac{8}{10}$~~

~~$\sin\theta =$~~

$\sin\theta = \frac{P}{H}$

$\sin\angle M = \frac{MN}{LM}$

$\sin\angle M = \frac{8}{10}$

$\sin\angle M = 0.8$

$\angle M = \sin^{-1}(0.8)$

$\boxed{\angle M = 53.13}$

$\angle L + \angle M + \angle N = 180^\circ$

$\angle L + 53.13^\circ + 90^\circ = 180^\circ$

$\angle L + 143.13^\circ = 180^\circ$

$\angle L = 180^\circ - 143.13^\circ$

$\boxed{\angle L = 36.87}$

Q5) $x = (r\cos\alpha\sin\beta)$, $y = (r\cos\alpha\cos\beta)$

$z = r\sin\alpha$ then find of $x^2+y^2+z^2$

$x^2+y^2+z^2 = (r\cos\alpha\sin\beta)^2 + (r\cos\alpha\cos\beta)^2 + (r\sin\alpha)^2$

$= r^2\cos^2\alpha\sin^2\beta + r^2\cos^2\alpha\cos^2\beta + r^2\sin^2\alpha$

$= r^2[\cos^2\alpha\sin^2\beta + \cos^2\alpha\cos^2\beta + \sin^2\alpha]$

$= r^2[\cos^2\alpha(\sin^2\beta + \cos^2\beta) + \sin^2\alpha]$

$= r^2[\cos^2\alpha(1) + \sin^2\alpha]$

$= r^2(\cos^2\alpha + \sin^2\alpha)$

$= r^2(1)$

$= \boxed{r^2}$ Ans.