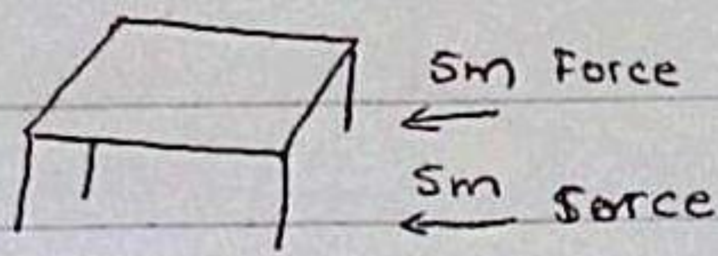
Chapter no:- 02

VECTOR

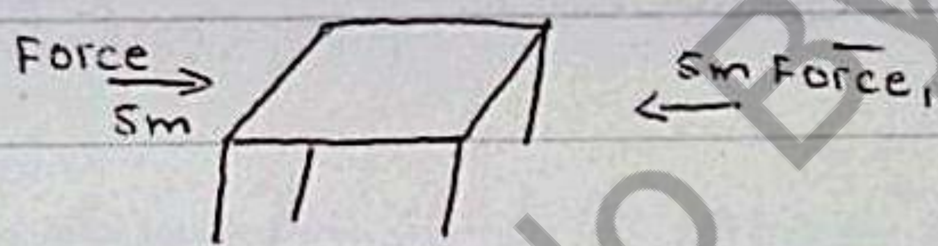
Vector → Magnitude
↳ Direction

"Vector are those quantities which have both Magnitude and Direction"

Example:-

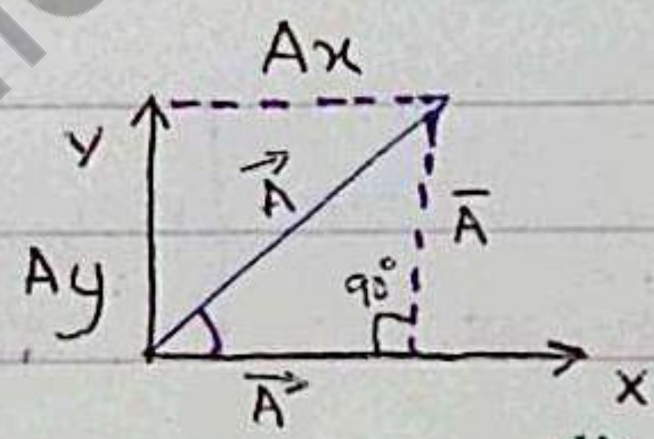


Different answer Because of vector



Resolution of Vector:-

To break something into parts



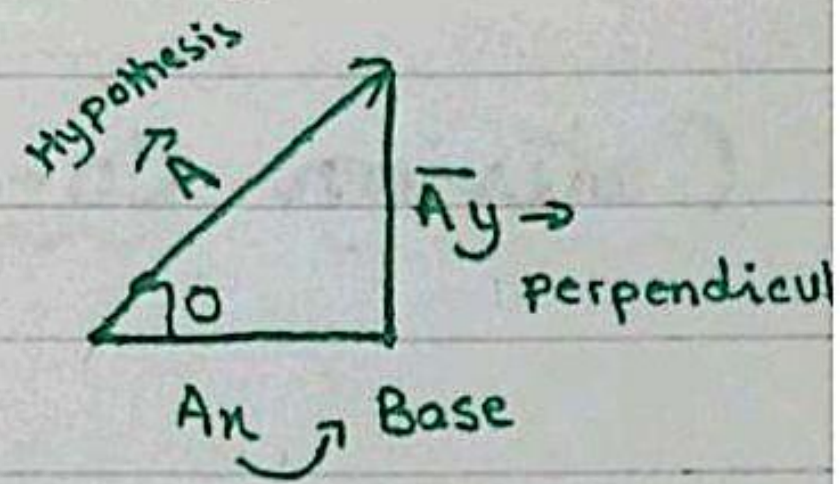
$\vec{A} = \vec{A_x} + \vec{A_y}$

Some People Have
Curly Brown Hair
Through Proper Brush

$Sin = \frac{p}{h}$

$cos = \frac{b}{h}$

$Tan = \frac{p}{b}$



$Sin\theta = \frac{Perp}{Hyp} = \frac{\vec{A_y}}{\vec{A}} \Rightarrow \boxed{Sin\theta = \frac{\vec{A_y}}{\vec{A_x}}}$ — (i)

$Cos\theta = \frac{Base}{Hyp} = \frac{\vec{A_x}}{\vec{A}} \Rightarrow \boxed{cos\theta = \frac{\vec{A_x}}{\vec{A}}}$ — (ii)

$$\tan\theta = \frac{\text{Prep}}{\text{Base}} = \boxed{\frac{\vec{A_y}}{\vec{A_x}}} \quad \text{—— (iii)}$$

SIO Based Topic $\vec{A} = A_x\hat{i} + A_y\hat{j}$

Unit Vector :—

"A Vector having magnitude "one" but it is only used to show direction."

"A vector divided by its own magnitude is called (Scaler) $\times\,\hat{n}$ Unit Vector"

Area, $\hat{n}$

$\boxed{\vec{A} = |A|\,\hat{n}}$ Magnitude

$\vec{x} = |x| \times \hat{n}$

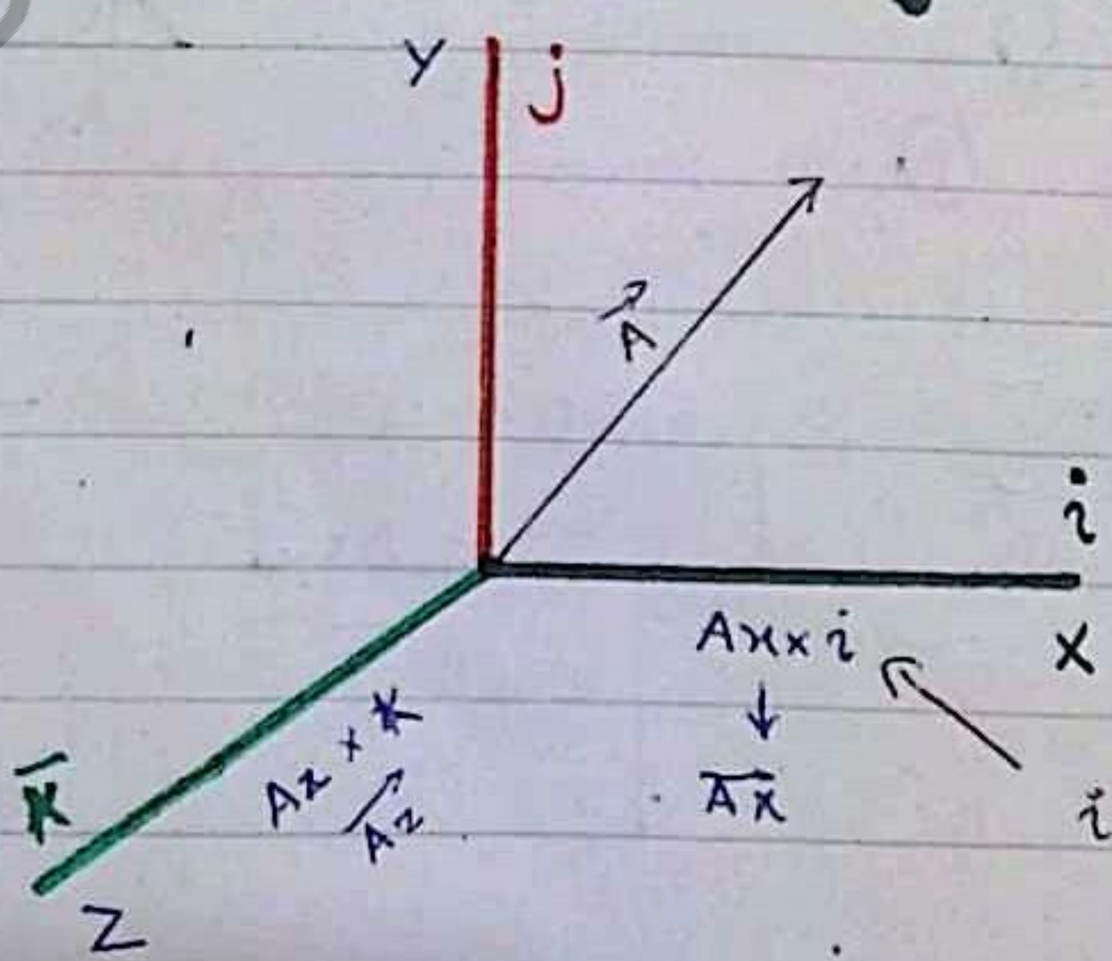
$\because \vec{A} = |A|\,\hat{n} \qquad \frac{\vec{A}}{|A|} = \hat{n}$

Example:- if we want to make a person magnitude to vector then we could $\Rightarrow$ $\boxed{\hat{n} = \frac{\vec{A}}{|A|}}$ attached something with that person which is 90°

E.g:- Gilgit cap with straight fur

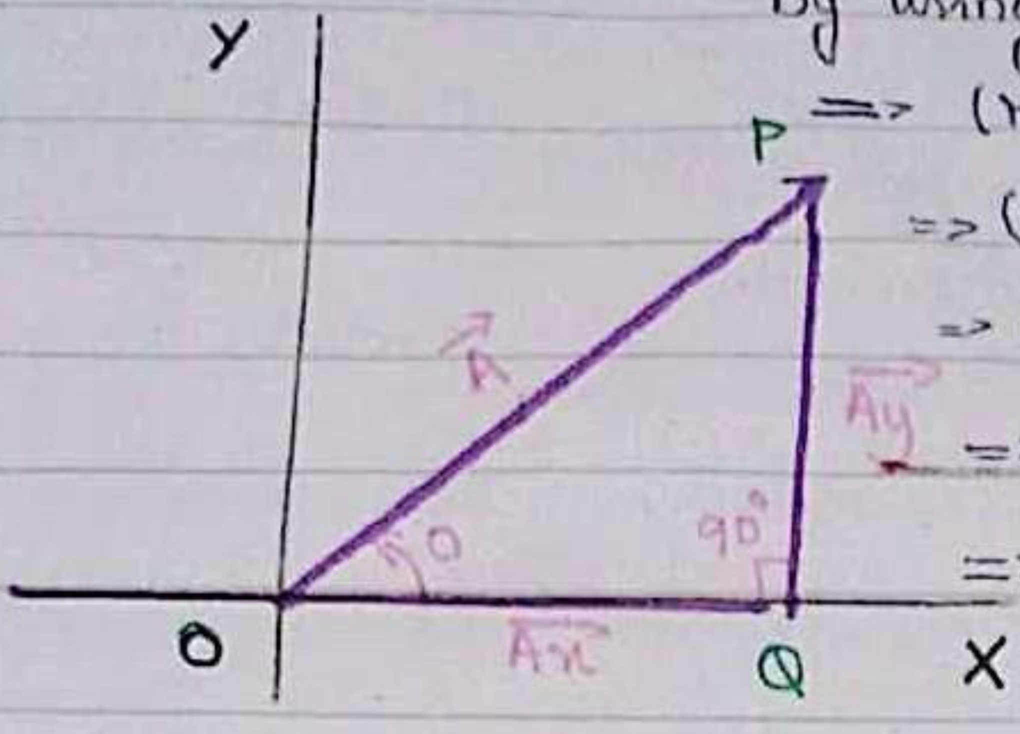
Catision Co-ordinate System :—

$$\boxed{\begin{aligned} \vec{A_x} &= A_x \cdot i \\ \vec{A_y} &= A_y \cdot j \\ \vec{A_z} &= A_z \cdot k \end{aligned}}$$



i is for vector without i A_x is only magnitude

Determination of Vector from its components :-



By using pathagoras Theorem

$\Rightarrow (Hyp)^2 = (Prp)^2 + (Base)^2$

$\Rightarrow (OP)^2 = (OQ)^2 + (QP)^2$

$\Rightarrow (A)^2 = (Ax)^2 + (Ay)^2$

$\Rightarrow (A)^2 = (Ax^2 + Ay^2)$

$\Rightarrow A^2 = Ax^2 + Ay^2$

$\Rightarrow \sqrt{A^2} = \sqrt{Ax^2 + Ay^2}$

$$\boxed{\Rightarrow A = \sqrt{Ax^2 + Ay^2}}$$

Jab kisi vector ka Square liya jata ha to jo scaler mam covert, ho jata ha

E.g

$(\vec{Ax})^2 = Ax^2$

Angle :-

$\tan\theta = \frac{Perp}{Base} = \tan\theta = \frac{Ay}{Ax}$

$$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{Ay}{Ax}\right)}$$

$$\boxed{F_x = F\cos\theta}$$

DOT PRODUCT :-

"When you multiply vector with vector by using • then we get scaler"

(Vector)·(Vector) = Scaler

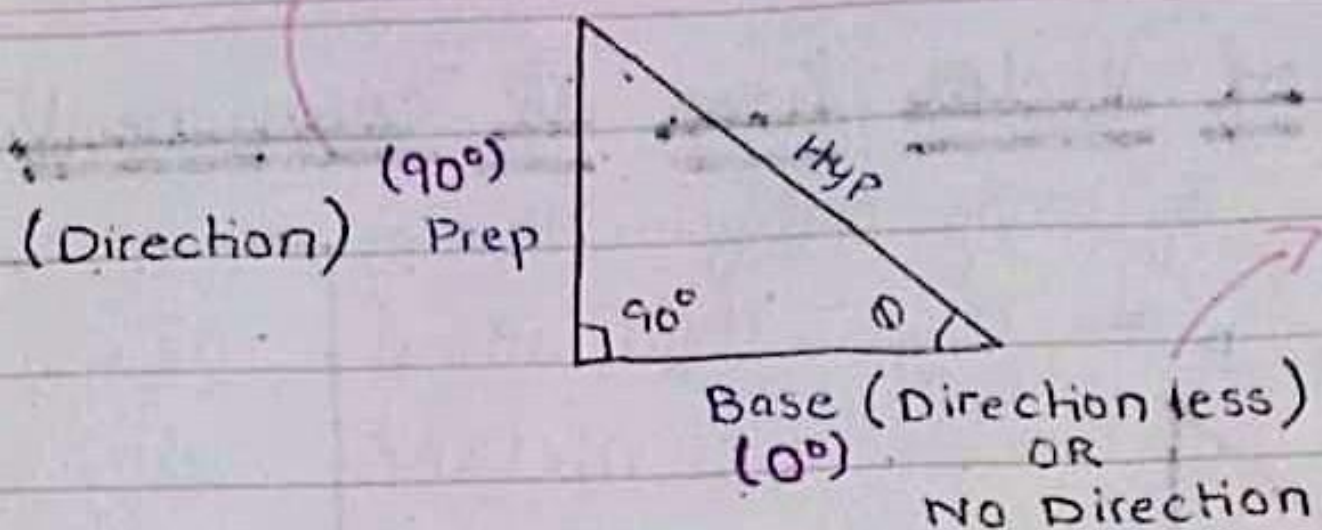
$\vec{A} \cdot \vec{B} = C$

Vector — vector — Scaler

• → Dot product / Scaler Product

X → Cross product / Vector Product

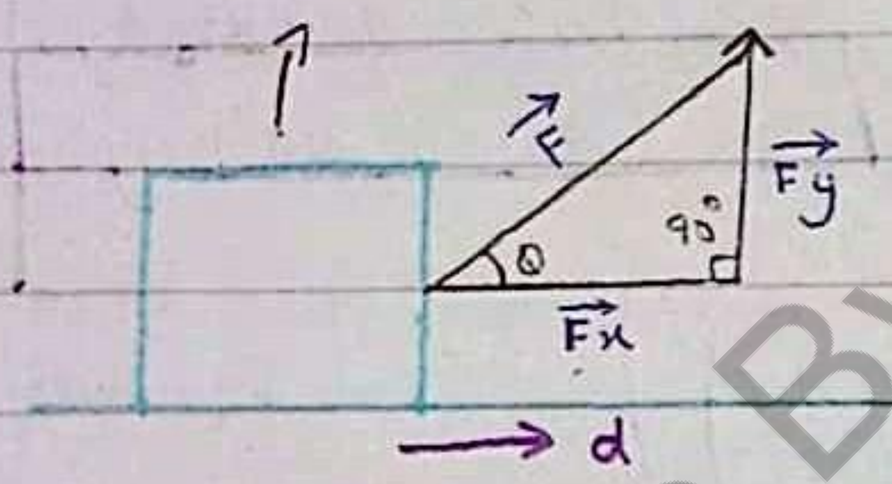
Deal with
Cross Product



Deal with dot
Product

Parallel

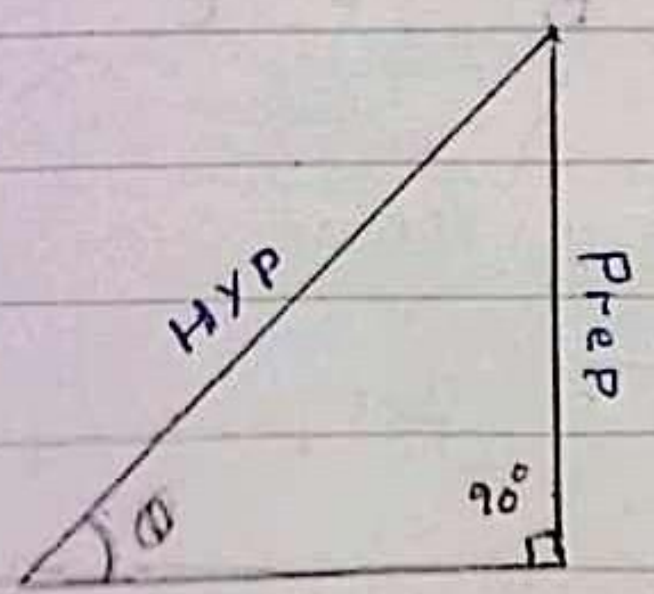
Base:– Base do not have
any direction like



$$\boxed{F_x = F\cos Q}$$

Workdone $= \vec{F} \cdot \vec{d}$
" $= F_x \cdot \vec{d}$
" $= F\cos Q \cdot d$
workdone $= Fd\cos Q$

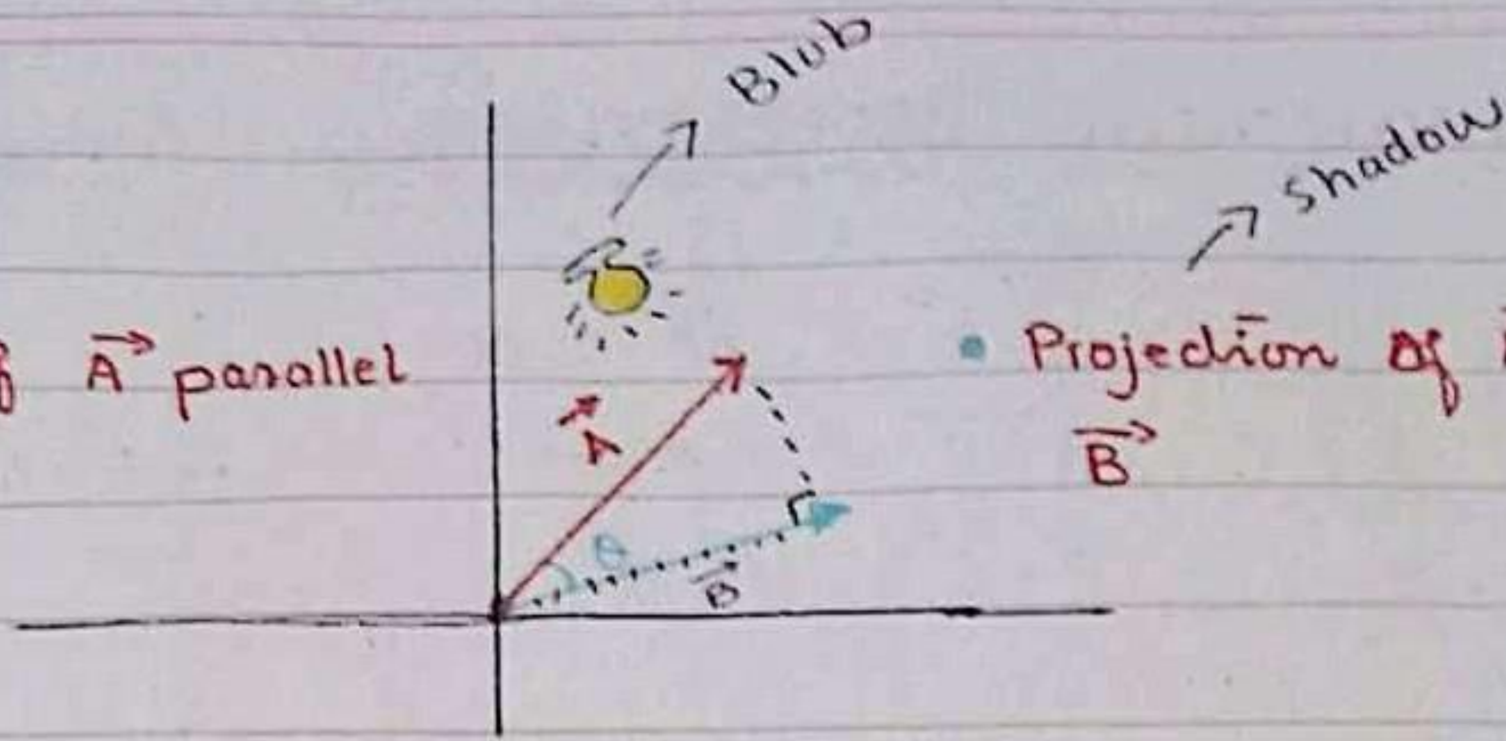
That's why work done
is zero because Fx,
cos likned with Base
So workdone is
Scaler quantity



Prep = Sin Q = 90° = Ø Cross Product

Base => Cos Q = 0° = Dot Product

- Component of $\vec{A}$ parallel to $\vec{B}$



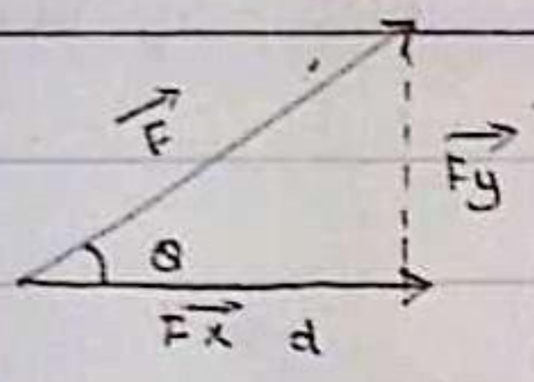
- Projection of $\vec{A}$ on $\vec{B}$

$\vec{A}\cdot\vec{B} = (\text{component of } \vec{A} \text{ parallel to } \vec{B})\cdot(\text{Magnitude of } B)$

$\vec{A}\cdot\vec{B} = \vec{Ax}\cdot\vec{B}$

$\vec{A}\cdot\vec{B} = (A\cos\theta)\cdot B$

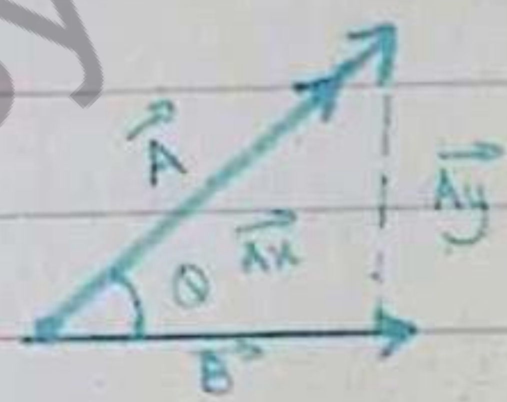
$\boxed{\vec{A}\cdot\vec{B} = AB\cos\theta}$



$\vec{F}\cdot\vec{d} = \vec{F_x}\cdot\vec{d}$

$\vec{F}\cdot\vec{d} = (F\cos\theta)\cdot d$

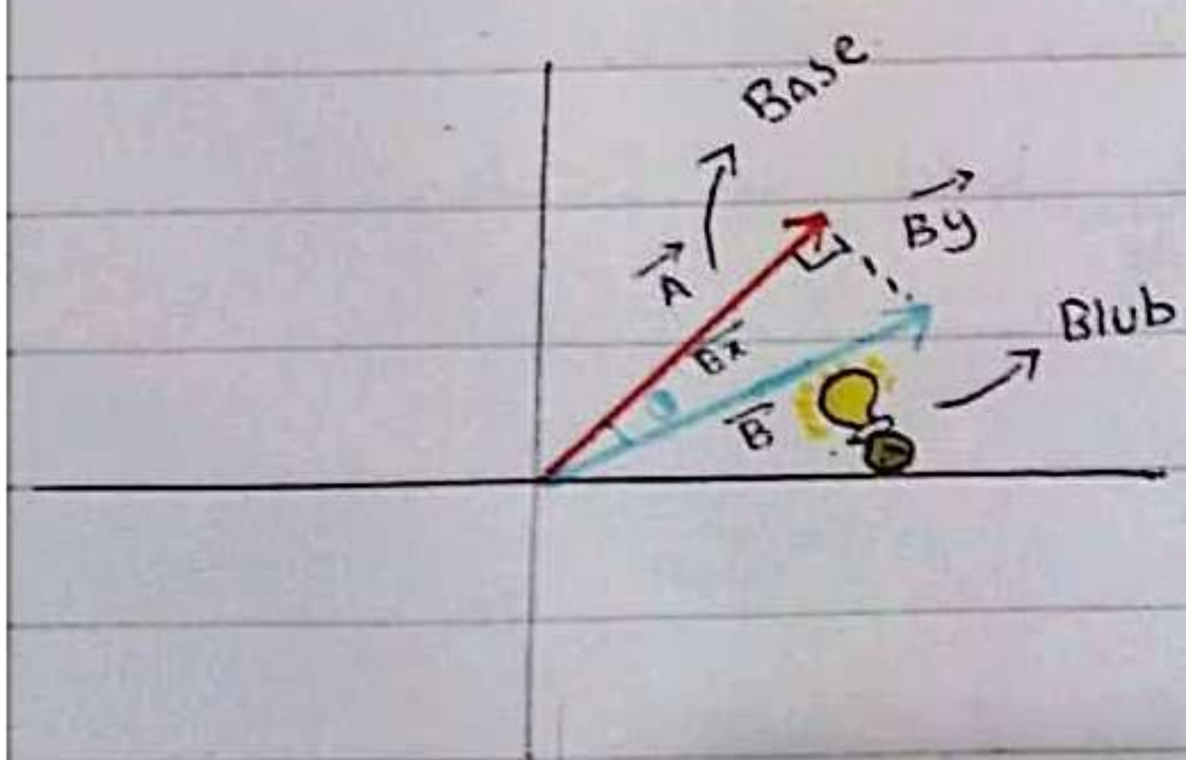
$\therefore \boxed{\vec{F}\cdot\vec{d} = Fd\cos\theta}$



$\vec{A}\cdot\vec{B} = \vec{Ax}\cdot\vec{B}$

$\vec{A}\cdot\vec{B} = (A\cos\theta)\cdot\vec{B}$

$\therefore \boxed{\vec{A}\cdot\vec{B} = AB\cos\theta}$



$\vec{B}\cdot\vec{A} = (\text{component of } \vec{B} \text{ parallel to } \vec{A})\cdot(\text{Magnitude of } \vec{A})$

$\vec{B}\cdot\vec{A} = \vec{Bx}\cdot\vec{A}$

$\vec{B}\cdot\vec{A} = (B\cos\theta)\cdot A$

$\boxed{\vec{B}\cdot\vec{A} = BA\cos\theta}$

COMMUTATIVE PROPERTY:–

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$
$$\vec{A}\cdot\vec{B} = BA\cos\theta \qquad \therefore \vec{B}\cdot\vec{A} = BA\cos\theta$$
$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

Hence dot product is Commutative

S10 Based Question:–

Q1. Show that commutative law of multiply hold in scaler product of vectors

Q2- Prove that :-

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

PROPERTIES OF SCALER PRODUCT

(1) Commutative property :-

$$\boxed{\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}}$$

Commutative property hold on Scaler.

(2) Orthogonal / Perpendicular :— ($\theta = 90^\circ$)

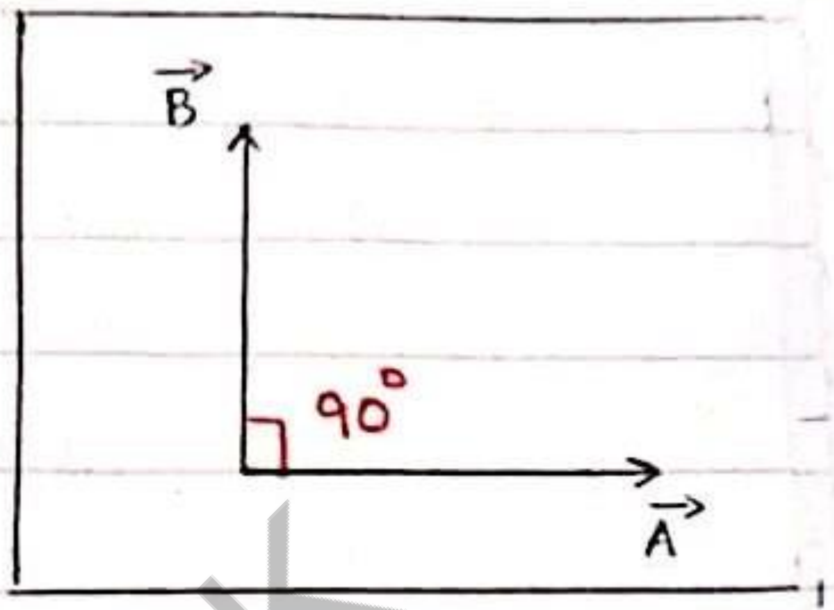
if $\vec{A}$ is perpendicular with $\vec{B}$ then

$\Rightarrow \vec{A}\cdot\vec{B} = AB\cos\theta$

$\Rightarrow \vec{A}\cdot\vec{B} = AB\cos 90^\circ$

$\Rightarrow \vec{A}\cdot\vec{B} = AB(0)$

$\boxed{\vec{A}\cdot\vec{B} = 0}$



$\because \cos 90^\circ = 0^\circ$

(3) Parallel ($\theta = 0^\circ$)

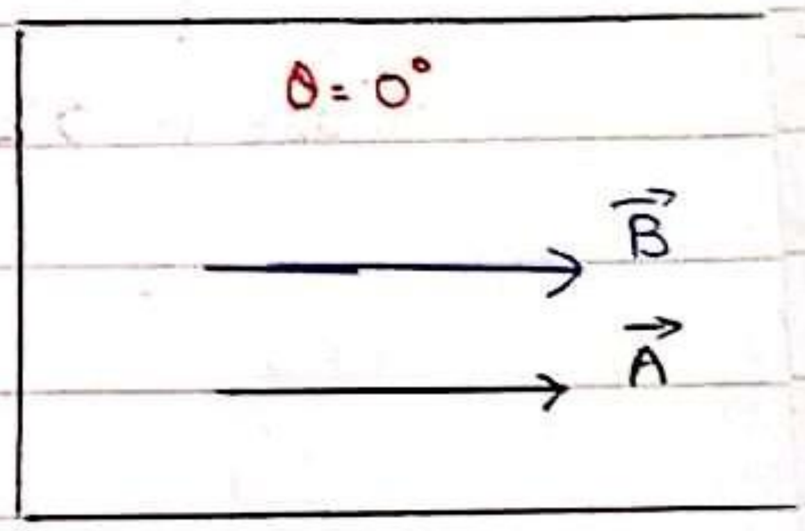
if $\vec{A}$ is parallel to $\vec{B}$ then

$\Rightarrow \vec{A}\cdot\vec{B} = AB\cos\theta$

$\Rightarrow \vec{A}\cdot\vec{B} = AB\cos(0)^\circ$

$\Rightarrow \vec{A}\cdot\vec{B} = AB(1)$

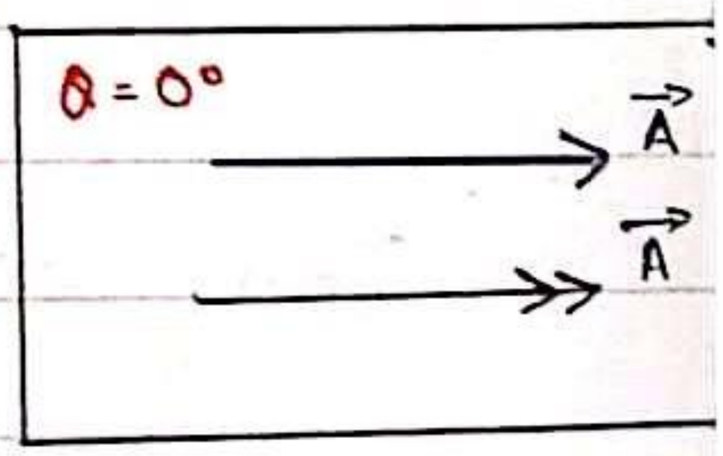
$\boxed{\vec{A}\cdot\vec{B} = AB}$



$\because \cos\theta = 1$

(4) Self Product :—

When $\vec{A}$ is multiply with $\vec{A}$ or when we take a self product of $\vec{A}$



$\Rightarrow \vec{A}\cdot\vec{A} = AA\cos\theta$

$\Rightarrow \vec{A}\cdot\vec{A} = AA\cos(0)$

$\because \cos\theta = 1$

$\Rightarrow \vec{A}\cdot\vec{A} = AA(1)$

$\Rightarrow \boxed{\vec{A}\cdot\vec{A} = A^2}$

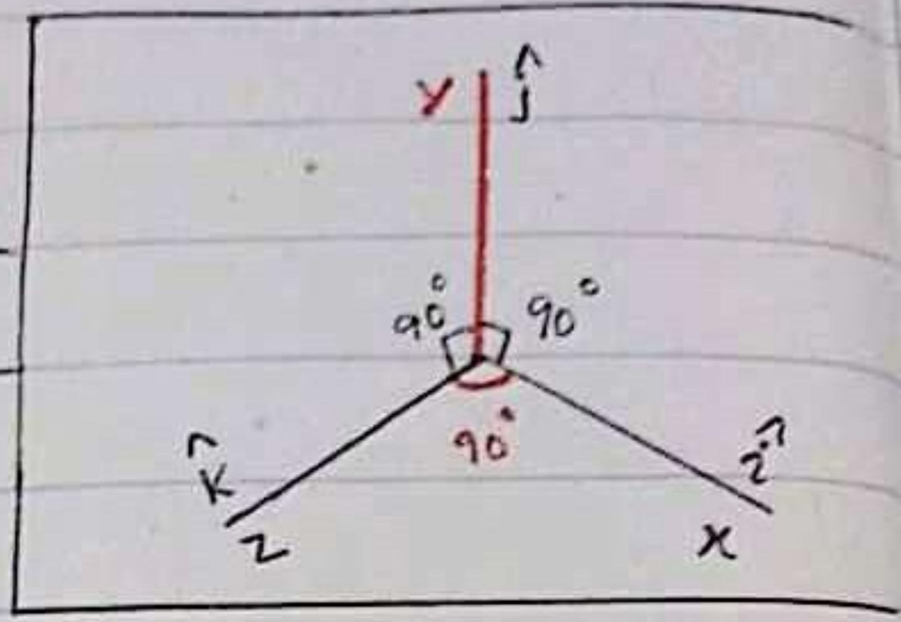
5) Scaler product of Unit Vector :-

Case :- 01

(Similar unit vector)

When we multiply Similar unit vector.

$\boxed{\therefore \hat{i} = 1}$



$\Rightarrow \hat{i}\cdot\hat{i} = i\,i\cos\theta$

$\Rightarrow \hat{i}\cdot\hat{i} = i\cdot i\,(1)$

$\Rightarrow \hat{i}\cdot\hat{i} = (1)(1)(1)$ $\quad\therefore \cos 0^\circ := 1$

$\boxed{\Rightarrow \hat{i}\cdot\hat{i} = 1}$

Thus we can say $\Rightarrow \hat{j}\cdot\hat{j} = 1 \Rightarrow \hat{k}\cdot\hat{k} = 1$

So,

$$\boxed{\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1}$$

Case :- 02

(Different unit vector)

When we multiply different unit vectors

let,

$\Rightarrow \hat{i}\cdot\hat{j} = i\cdot j\cos\theta$ $\quad\therefore \cos 90^\circ = 0$

$\Rightarrow \hat{i}\cdot\hat{j} = i\cdot j\cos(90^\circ)$

$\Rightarrow \hat{i}\cdot\hat{j} = i\cdot j\,(0)$

$\boxed{\hat{i}\cdot\hat{j} = 0}$

Then $\Rightarrow \hat{i}\cdot\hat{k} = 0 \Rightarrow \hat{j}\cdot\hat{k} = 0$

So,

$$\boxed{\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0}$$

6) Scaler Product of two vector in term of their rectangular Component :-

let .

$$\vec{A} = (A_x\hat{i} + A_y\cdot\hat{j} + A_z\hat{k}),\ \vec{B} = (B_x\hat{i} + B_y\cdot\hat{j} + B_z\hat{k})$$

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\cdot\hat{j} + B_z\cdot\hat{k})$$

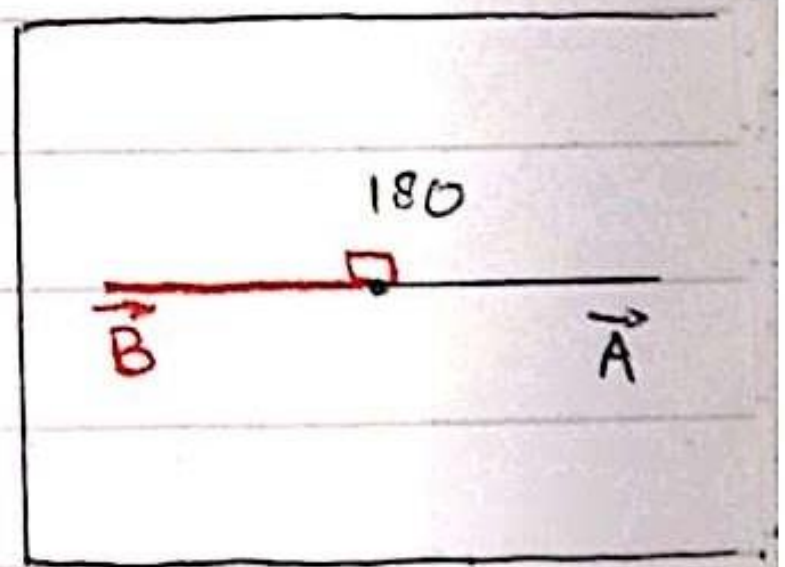
$$\vec{A}\cdot\vec{B} = (A_x B_x(\hat{i}\cdot\hat{i}) + A_y B_y)(\hat{j}\cdot\hat{j}) + B_z + A_z(\hat{k}\cdot\hat{k}) +$$

$$\boxed{\vec{A}\cdot\vec{B} = (A_x B_x + A_y B_y + A_z B_z)} \quad \text{— (i)}$$

$$\because \vec{A}\cdot\vec{B} = AB\cos\theta \Rightarrow \boxed{\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}} \quad \text{— (iii)}$$

) Anti parallel :- $\theta = 180^\circ$

if $\vec{A}$ is antiparallel with $\vec{B}$ then.



$$\Rightarrow \vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\Rightarrow \vec{A}\cdot\vec{B} = AB\cos 180^\circ$$

$$\Rightarrow \vec{A}\cdot\vec{B} = AB(-1)$$

$$\Rightarrow \boxed{\vec{A}\cdot\vec{B} = -AB}$$

$\therefore \cos 180^\circ = -1$

Perpendicular

Sin θ

3 dimensional

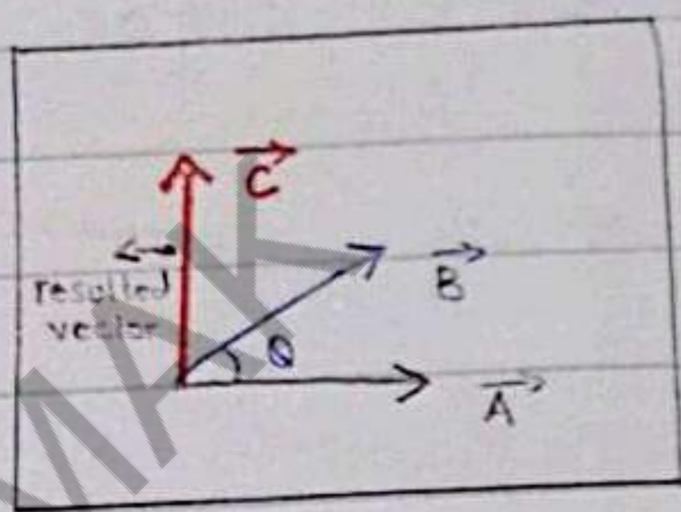
Vector product or Cross product:-

"When the product of two vector quantities gives a vector quantity then the product is called vector product"

$\vec{A} \times \vec{B} = \vec{C}$

vector

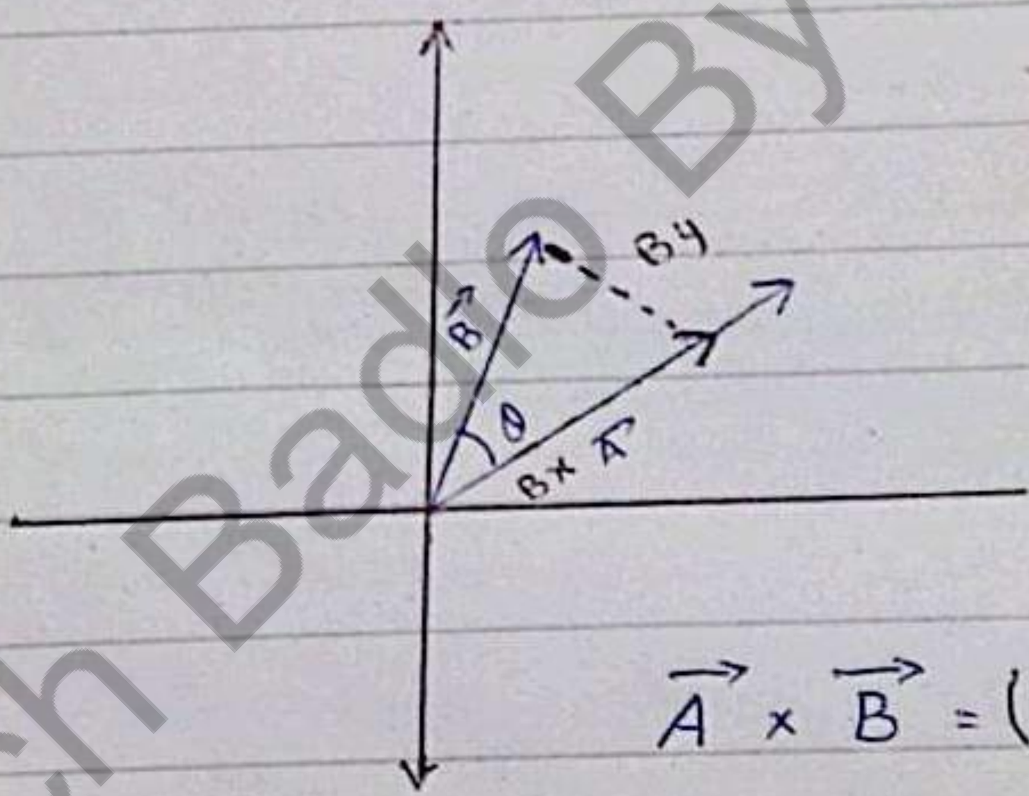
vector



- It is denoted by "x".

$\hat{n}$

$\hat{n}$ is always Perpendicular to the plane Because $\hat{n}$ is at 3rd dimension are at perpendicular to each other:



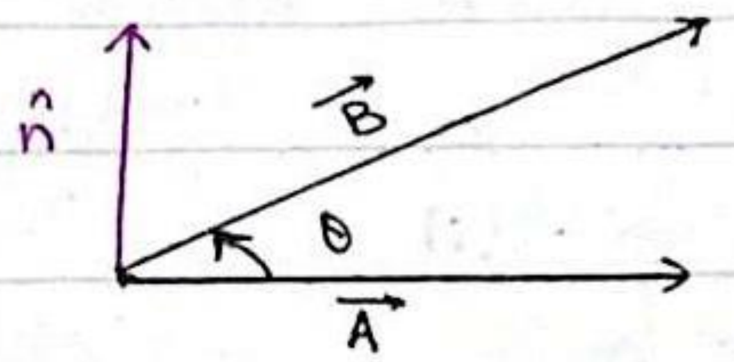
Take y component because of vector product (sin θ)

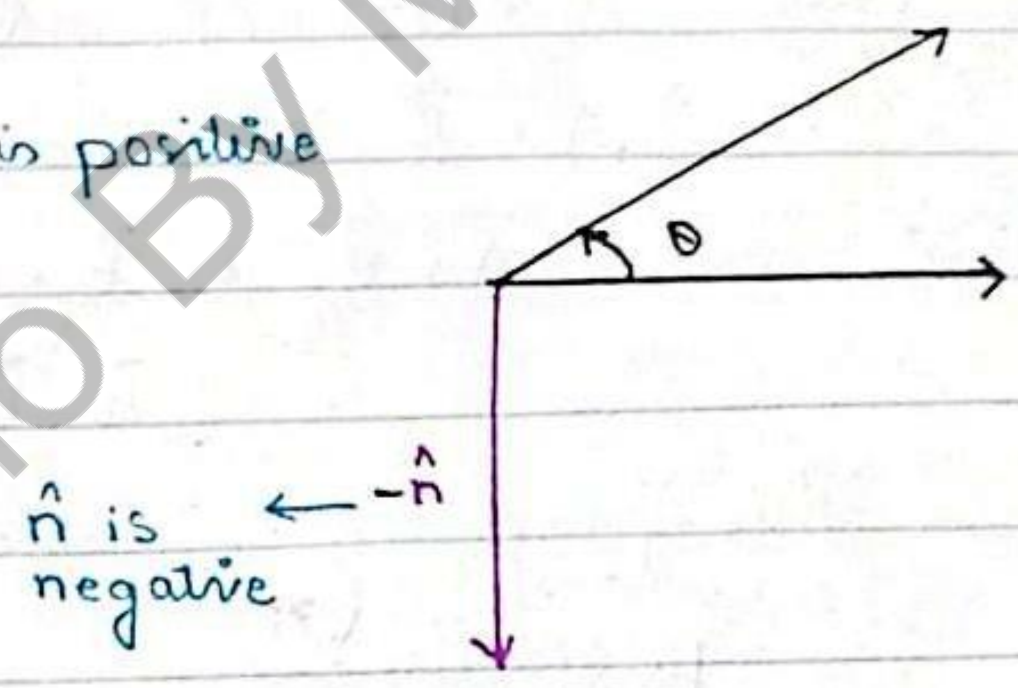
$\vec{A} \times \vec{B} =$ (Magnitude of

$$\vec{A} \times \vec{B} = AB\sin\theta \, \hat{n}$$

Direction of $\hat{n}$ could be found by using right hand rule.

Put your right hand at the vector from plane (A) and Curl your finger in the direction of θ your thumb point vector $\vec{C}$.





02-Aug-24

Properties of vector product:-

(1) Commutative property :-

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n} \quad , \quad \vec{B} \times \vec{A} = AB\sin(-\hat{n})$$

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n} \quad , \quad \vec{B} \times \vec{A} = -AB\sin\,\hat{n}$$

Thus $\boxed{\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}}$ —— (i)

$$\Rightarrow \vec{A} \times \vec{B} \neq -\vec{B} \times \vec{A}$$

This property is also known as "anticommuntative property".

(2) Parallel and antiparallel :-

(a) Parallel $\theta = 0°$

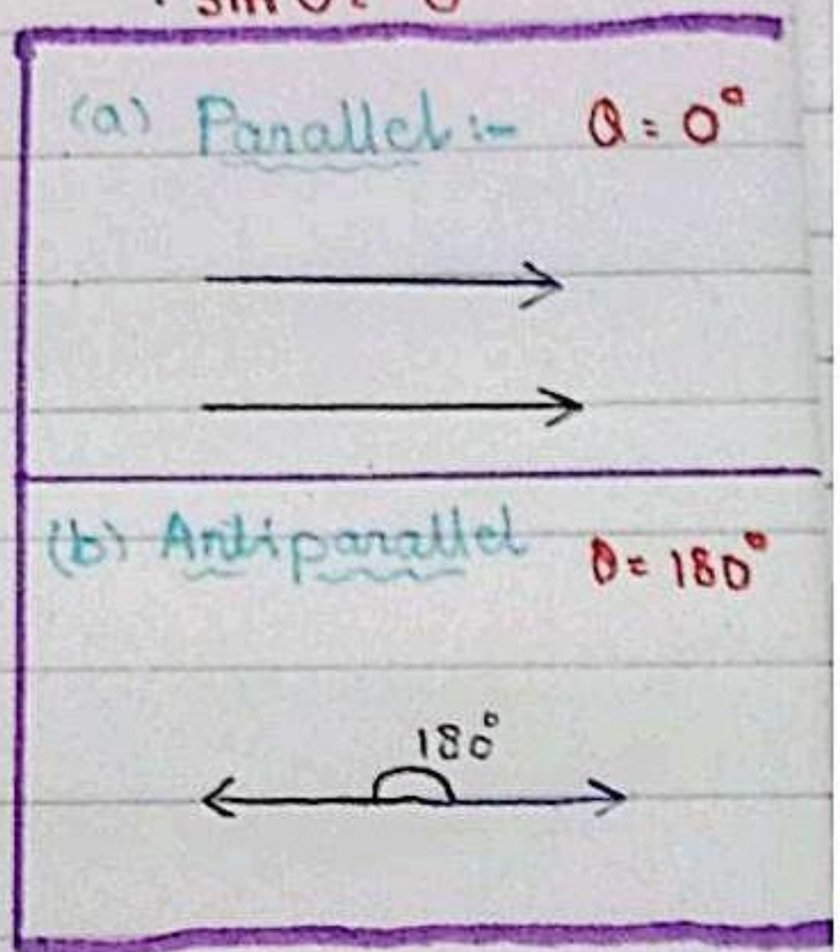
$$\Rightarrow \vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$

$$\Rightarrow \vec{A} \times \vec{B} = AB\sin 0°\,\hat{n}$$

$$\Rightarrow \vec{A} \times \vec{B} = AB(0)\,\hat{n}$$

$$\boxed{\Rightarrow \vec{A} \times \vec{B} = 0} \quad —— (i)$$

$\because \sin 0 = 0°$



(b) Antiparallel $\theta = 180°$

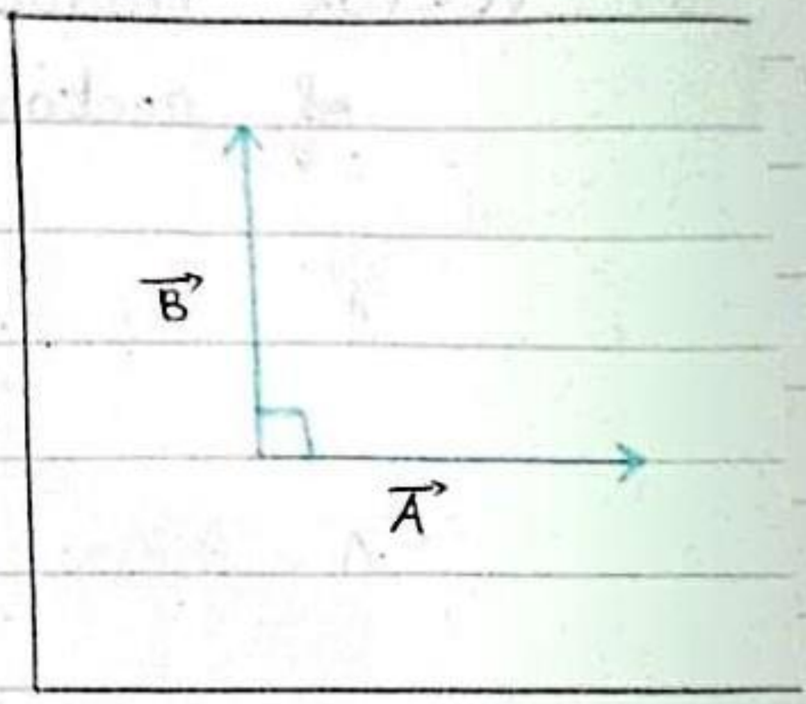
$$\Rightarrow \vec{A} \times \vec{B} = AB\sin\,\hat{n} \quad \Rightarrow \vec{A} \times \vec{B} = AB(\sin 180)\,\hat{n}$$

$$\Rightarrow \vec{A} \times \vec{B} = AB(0)\,\hat{n}$$

$$\boxed{\Rightarrow \vec{A} \times \vec{B} = 0} \quad —— (ii)$$

3) Perpendicular $\theta = 90^\circ$:-

$$\Rightarrow \vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$
$$\Rightarrow \vec{A} \times \vec{B} = AB\sin 90^\circ$$
$$\Rightarrow \vec{A} \times \vec{B} = AB(1)$$
$$\boxed{\Rightarrow \vec{A} \times \vec{B} = AB}$$



$\because$ $\hat{n}$ is removed because we are interested in magnitude not in direction

4- Vector Product in Unit Vector :-

(a) Same unit Vector :-

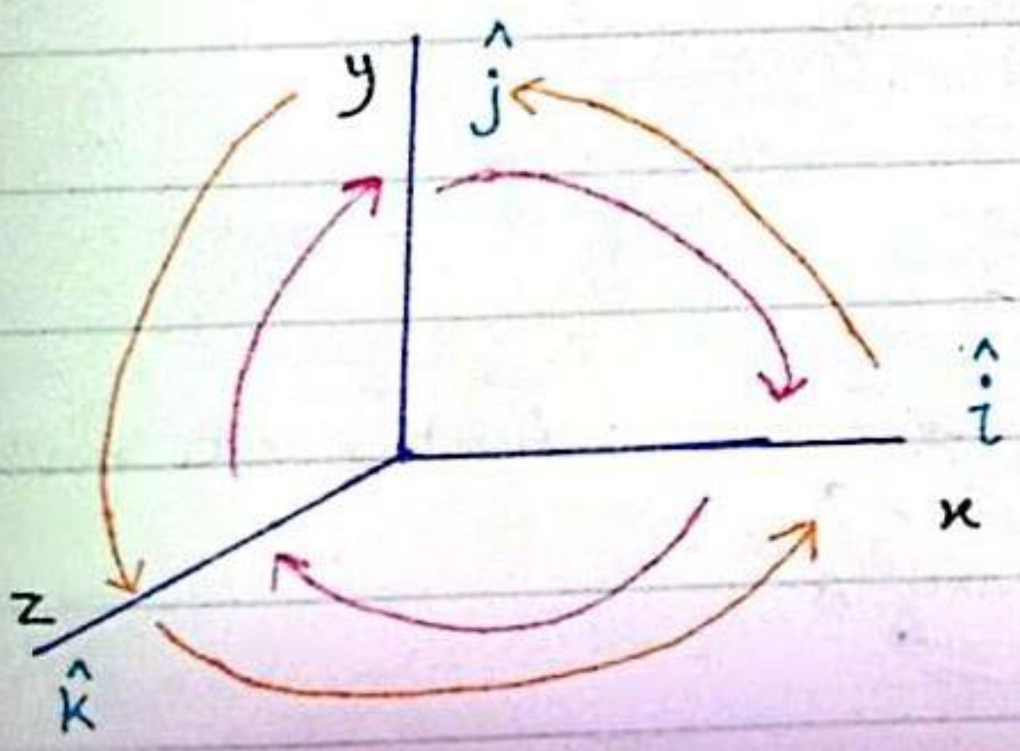
let,

$$\hat{i} \times \hat{i} = \hat{i}\,\hat{i}\,\sin\theta$$
$$\hat{i} \times \hat{i} = \hat{i}\,\hat{i}\,\sin(0)$$
$$\hat{i} \times \hat{i} = 0$$

Thus ;

$$\hat{i}\cdot\hat{i} = 0 \;,\quad \hat{j}\cdot\hat{j} = 0 \;,\quad \hat{k}\cdot\hat{k} = 0$$

(B) Different unit Vector :-



Anticlockwise = +   clockwise = −

$\hat{i} \times \hat{j} = \hat{k}$ $\hat{i} \times \hat{k} = -\hat{j}$

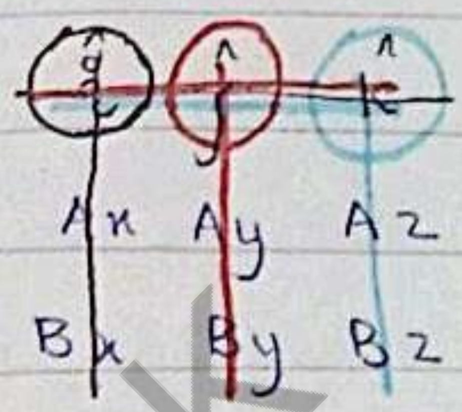
$\hat{j} \times \hat{k} = \hat{i}$ $\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\hat{j} \times \hat{i} = -\hat{k}$

5) Vector Product of two vectors in term of rectangular Component :-

$\vec{A} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$, $\vec{B} = (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



Expanding from 1st Row

$$= +\hat{i}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

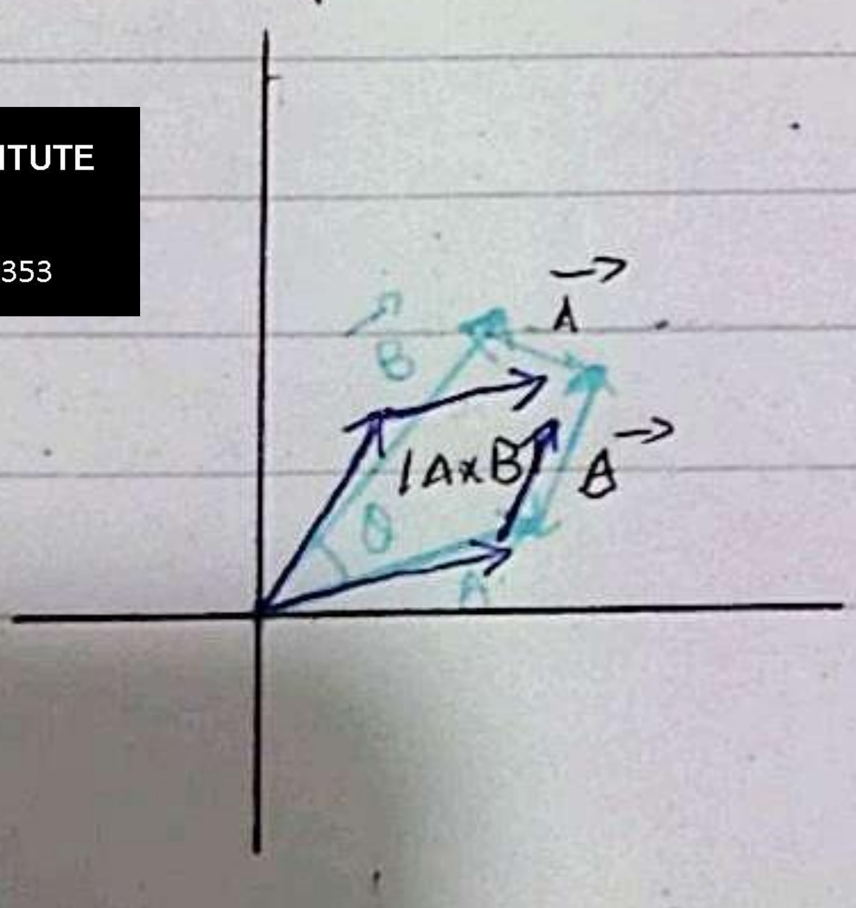
$$= +\hat{i}\,[A_yB_z - A_zB_y] - \hat{j}\,[A_xB_z - A_zB_x] + \hat{k}\,[A_xB_y - A_yB_x]$$

$$\Rightarrow \vec{A}\times\vec{B} = \hat{i}\,[A_yB_z - A_zB_y] + \hat{j}\,[-A_xB_z + A_zB_x] + \hat{k}\,[A_xB_y - A_yB_x]$$

—— × —— × —— × —— × ——

Physical Significance of vector product :-

Q Show that the magnitude of vector product gives the areas of parallelogram ?



$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

Magnitude of two vector $|AB|$

" The vector or Cross product of two Vectors A and B give the area of parallelogram through its magnitude "

$$\text{Area of Parallelogram} = |A \times B|$$

Derivation :-

$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$$

$$|A \times B| = \sqrt{(A_x) + (A_y)}$$

Area of Pallo(gm)