"VECTORS"

Vector:- →(statement): A quantity that has both magnitude and direction is called vector quantity. →(Representation): It is typically represented by a line with an arrow head.

→ It doesnot have position.

→ It indicates How much (the magnitude) and the direction of the quantity is.

→(example): → Displacement, force etc are the examples of vector quantities.

→(use in daily life): A sign board provides info. about distance and directions to other locations of signboards.

Q.1. Why do we resolve Vectors into components?

Ans. We resolve vector into its components as we want to see the effect of vectors along x-axis and y-axis as A_x and A_y.

RESOLUTION OF VECTORS...

→(Also known As:) → Also known as Rectangular components of a vector.

→(statement:) → The components of a vector which are perpendicular (⊥) to each other are called as rectangular components.

Component:- ↳ Each part of the two-dimensional vector is called a component. ↳ It is an effective part of a vector in different directions.

↳ x-component:- component along x-axis (horizontal)

↳ y-component:- component along y-axis. (verticle)

NOTE: x and y component are perpendicular to each other.

Explination :-

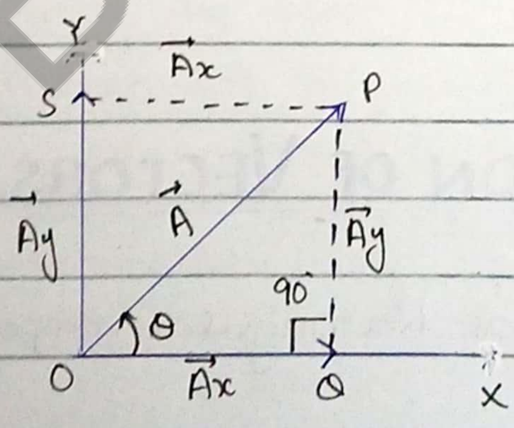
I↳ Consider a vector $\vec{A}$ making an angle θ with horizontal in x-y plane.

I↳ Draw a perpendicular from head of A on x and y-axis.

I↳ OQ of vector A along x-axis is x-component (A_x)

I↳ OS of vector A along y-axis is y-component. (A_y)

Diagram :-



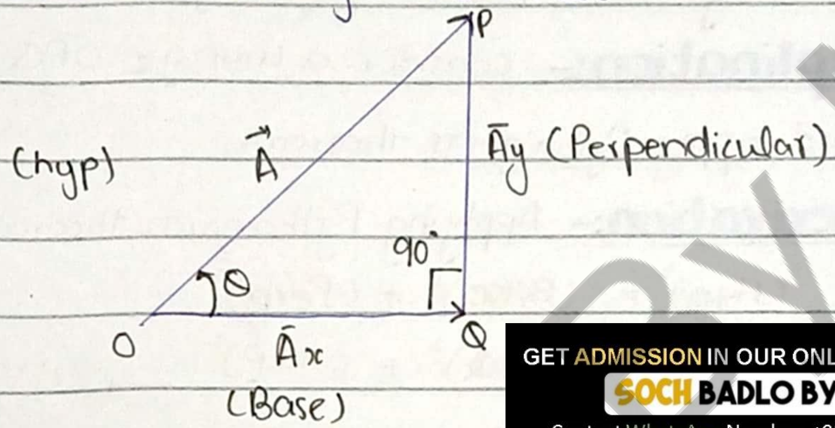
Hence, The components of vector A is:-

$$\boxed{\therefore A = A_x + A_y}$$

Rectangular Components of a Vector:-

NOTE:- To find A_x and A_y we use trigonometric ratios.

→ Consider a triangle as OPQ.



→ For finding A_y-component we will use following trignometry ratio:-

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin\theta = \frac{\vec{A}_y}{\vec{A}}$$

$$\therefore \boxed{\vec{A}_y = \vec{A}\sin\theta}$$

Some = People / Have

Curly = Brown / Hairs

→ For finding A_x-component we will use following trigonometry ratio:-

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\vec{A}_x}{\vec{A}}$$

$$\therefore \vec{A}_x = \vec{A}\cos\theta$$

Finding a Vector from its component:-

- Finding magnitude of A and θ(theta) only when Ax and Ay are given.

→ **explination:-** consider a triangle OPQ and apply Pythagoras theorem.

→ **Derivation:-** Applying Pythagoras theorem.

$$(Hyp)^2 = (Base)^2 + (Perp)^2$$

$$(OP)^2 = (OQ)^2 + (QP)^2 \longrightarrow (i)$$

As $OS = QP = \vec{A}_y$ so,

$$(OP)^2 = (OQ)^2 + (OS)^2 \longrightarrow (ii)$$

As, $OP = A$, $OQ = A_x$, $OS = A_y$, then

$$(A)^2 = (A_x)^2 + (A_y)^2$$

Taking square root on b.sides.

$$\therefore A = \sqrt{(A_x)^2 + (A_y)^2}$$

↳ Finding the direction angel θ by using following trigonometry ratio:-

$$\tan\theta = \frac{Perpendicular}{Base}$$

Till = Painted / Black

$$\tan\theta = \frac{A_y}{A_x} \longrightarrow (a)$$

Rearranging the above equ(a),

$$\therefore \theta = \tan^{-1}\frac{A_y}{A_x}$$

UNIT VECTORS.

(statement):

→ A vector which has magnitude of one (1) and only indentify direction is called a unit vector.

OR

→ A unit vector is a vector that has the magnitude equal to 1 (one).

Representated As:- It is generally represented by the "cap" (^) symbol over it.

e.g., $\vec{A} = \hat{A}$.

Obtained By:- It is formed by dividing vector by its own magnitude.

Formula:- It's formula is given as:-

$$\therefore \text{Unit Vector} = \frac{\text{Vector}}{\text{Vector's Magnitude}}$$

e.g., A vector $\vec{A}$ as:-

$$\vec{A} = |A|\,\hat{A}$$

$$\therefore \hat{A} = \frac{\vec{A}}{|A|}$$

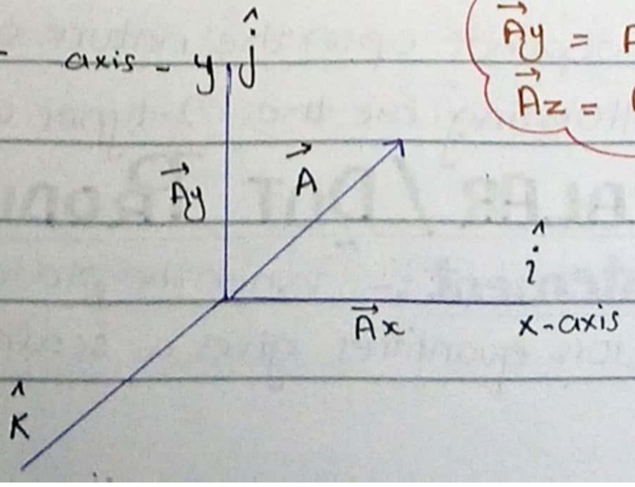
Unit Vectors.

$\vec{A}_x = A_x\hat{i}$

$\vec{A}_y = A_y\hat{j}$

$\vec{A}_z = A_z\hat{k}$

Diagram:-



Types:- unit vectors have 3-forms

→ **$\hat{i}$ unit vector:** It is the unit vector along x-axis.

→ **$\hat{j}$ unit vector;** It is the unit vector along y-axis.

→ **$\hat{K}$ unit vector:** It is the unit vector along z-axis.

They donot change magnitude or dimension of any thing.

→ **Normal unit vector:** It is represnted by $\hat{n}$.

PRODUCT OF VECTORS.

(statements)

→ When two or more vector quantities are multiplyied the required product can be scalar or vector.

→ It depends upon the nature of a vector.

→ Following are the 2-types of Product.

• SCALAR / DOT PRODUCTS.

Statement:- "When the product of two vectors quantities gives a scalar quantity then the product is called scalar quantity."

* **Also known As:**

It is also known as DOT PRODUCT.

* **Represented by:** It is represented by putting a dot (.) between two vectors.

e.g., $\vec{A}.\vec{B}$

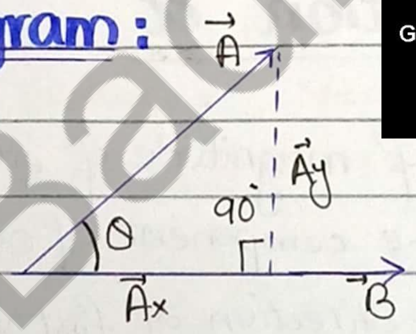
Vector . Vector = Scalar

OR $(\text{vector})^2$ = Scalar

* **Explination:**

↳ consider two vectors A and B making an angel θ.

* **Diagram:**



↳ By the above diagram we can say that components of $\vec{A}$ are parallel to $\vec{B}$ or projection of vector A ($\vec{A}$) on vector B ($\vec{B}$).

↳ The dot product between these two vectors become:-

$\vec{A}\cdot\vec{B} \longrightarrow$ Magnitude of $\vec{B}$

$\downarrow$

component of $\vec{A}$ parallel to $\vec{B}$

Hence,

$$\vec{A}\cdot\vec{B} = \vec{A_x}.\ B$$

$$= (A\cos\theta).B \qquad \because \vec{A_x} = A\cos\theta$$

$$\therefore\ \boxed{\vec{A}\cdot\vec{B} = AB\cos\theta} \longrightarrow (\alpha)$$

The scalar product of two vectors is obtained by multiplying their magnitudes with the cosine of the angel between them.

Statement with respect to equation α

"The product of magnitude of either vector with the component of other vector in the direction of first vector."

Properties of Scalar Quantities

Commutative Property.

commutative property holds in scalar product.

$$\therefore \vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

- ## Orthagonal / Perpendicular ($\theta = 90^\circ$)

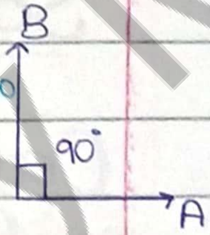
If $\vec{A}$ is perpendicular with $\vec{B}$ then,

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$= AB\cos(90^\circ) \quad \because \cos 90^\circ = 0$$

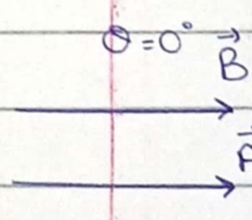
$$= AB(0)$$

$$\therefore \vec{A}\cdot\vec{B} = 0 \quad \text{(zero)}$$



We can say that 2 vectors are orthogonal if they are perpendicular to each other.

- ## Parallel ($\theta = 0^\circ$)



If $\vec{A}$ is parallel to $\vec{B}$ then,

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

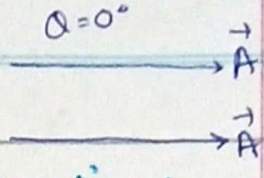
$$\vec{A}\cdot\vec{B} = AB\cos(0^\circ) \quad \therefore \cos(0^\circ) = 1$$

$$= AB(1)$$

$$\therefore \vec{A}\cdot\vec{B} = AB$$

- ## Self Product

When $\vec{A}$ is multiplied with $\vec{A}$ OR when we take square of $\vec{A}$ OR take a self product of $\vec{A}$, then



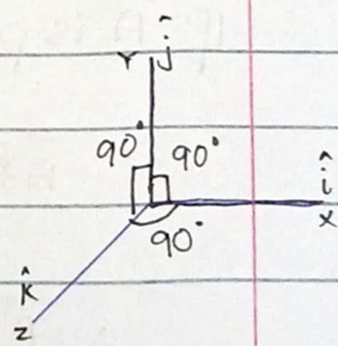
$$\vec{A}\cdot\vec{A} = AA\cos\theta$$
$$= A^2\cos(0^\circ) \quad \therefore \cos 0^\circ = 1$$
$$= A^2(1)$$
$$\therefore \vec{A}\cdot\vec{A} = A^2$$

Scalar Product of Unit Vector:

Case 01:
Similar Unit Vector

When we multiply similar unit vectors:-

$$\hat{i}\cdot\hat{i} = \hat{i}\cdot\hat{i}\cos\theta \quad \therefore \theta = 0^\circ$$
$$= \hat{i}\hat{i}\cos(0^\circ) \quad \therefore \cos(0^\circ) = 1$$
$$= \hat{i}\hat{i}(1) \quad \therefore \hat{i} = 1$$
$$= (1)(1)(1)$$
$$\therefore \hat{i}\cdot\hat{i} = 1$$



Thus we can say that:-

$$\hat{j}\cdot\hat{j} = 1 \; ; \; \hat{k}\cdot\hat{k} = 1$$

$$\therefore \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

Case 02:
Different Unit Vector

when we multiply different unit vectors:-

Let, $\hat{i}\cdot\hat{j} = ij\cos\theta$ $\quad \therefore \theta = 90^\circ$

$\hat{i}\cdot\hat{j} = ij\cos(90^\circ)$ $\quad \therefore \cos 90^\circ = 0$

$\hat{i}\cdot\hat{j} = ij(0)$

$\therefore \hat{i}\cdot\hat{j} = 0$

Thus we can say that,

$$\therefore \boxed{\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0}$$

Scalar Product of two Vectors in terms of Rectangular Components —

let,

$\vec{A} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$

$\vec{B} = (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= (A_x\hat{i}B_x\hat{i} + A_y\hat{j}B_y\hat{j} + A_z\hat{k}B_z\hat{k})$$

$$= (A_xB_x(\hat{i}\cdot\hat{i}) + A_yB_y(\hat{j}\cdot\hat{j}) + A_zB_z(\hat{k}\cdot\hat{k}))$$

$$= (A_xB_x(1) + A_yB_y(1) + A_zB_z(1))$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \longrightarrow (i)$$

As,

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

then,

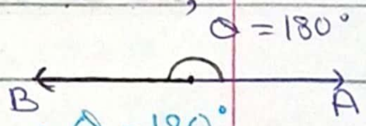
$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

Hence,

$$\therefore \cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

Anti Parallel : $(\theta = 180°)$

If $\vec{A}$ is antiparallel with $\vec{B}$ then,



$$\vec{A}\cdot\vec{B} = AB\cos\theta$$ $\therefore \theta = 180°$

$$= AB\cos(180°)$$ $\therefore \cos(180°) = -1$

$$= AB(-1)$$

$$\therefore \vec{A}\cdot\vec{B} = -AB$$

Points To Remember

→ • Dot product → cos θ → Base with 0°

× cross product → sin θ → Perp with 90°

→ Base has no direction / Prep has direction

→ Base is sometimes known as directionless and zero (0°) angel.

e.g., $Z = xy\cos\theta$ (scalar) ; $M = cd\sin\theta$ (vector)

VECTOR / CROSS PRODUCTS.

Statement:- "When the product of two vectors quantities gives a vector quantity then the product is called vector Product."

Also Known As:-

It is also known as "Cross Product."

Represented by:- It is represented by putting a cross (x) between two vectors. e.g., $\vec{A} \times \vec{B}$

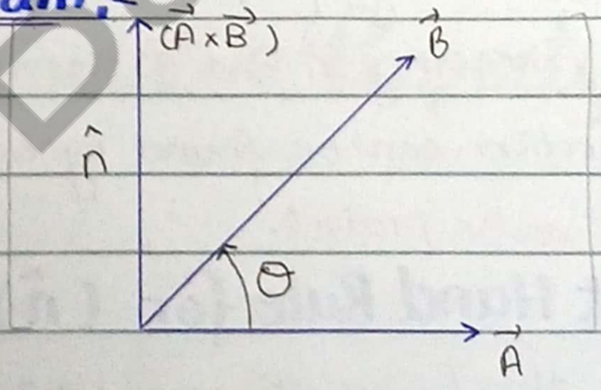
Vector x Vector = Vector

OR (Vector)^2 = Vector

Explination:-

Consider two vectors A and B making an angel θ with each other.

Diagram:-



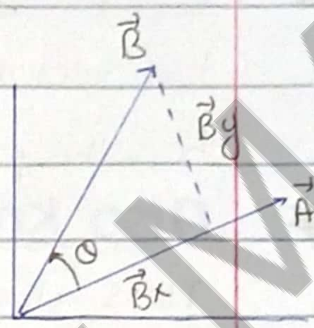
Hence, the ~~do~~ cross product between these two vectors becomes-

$\vec{A} \times \vec{B}$ =

Where, $\vec{A} \times \vec{B}$ → component of $\vec{B}$ perpendicular to $\vec{A}$.

↓

Magnitude of $\vec{A}$



Hence,

$$\vec{A} \times \vec{B} = AB_y$$

$$\vec{A} \times \vec{B} = A(B\sin\theta)(+\hat{n})$$

$$\therefore \vec{A} \times \vec{B} = AB\sin\theta\,\hat{n} \longrightarrow (B)$$

↳ The vector product of two vectors is obtained by multiplying their magnitudes with the sine of the angle between them.

★ Normal ($\hat{n}$) unit Vector :-

↳ It represents the Direction of the vector product.

↳ It will be always perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

↳ Its direction can be found by Right hand rule of vector product.

★ Right Hand Rule for ($\hat{n}$):-

↳ Rotate the fingers of your right hand. It should be in the direction from 1st. vector to the 2nd. vector through smaller angel of the 2 possibel angels with erect thumb.

I, The direction of the product will be along the direction of erect thumb.

Statement with respect to equation (B)

"The product of the projection of the first vector onto the second vector and the magnitude of the second vector."

Properties of Vector Product

• Commutative Property.

→ It is also known as Anti-commutative property.

As we know that,

$$\boxed{\therefore \vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}}$$

changing the property order of vectors the direction of vector product get reversed.

$$\vec{B} \times \vec{A} = AB \sin\theta \, (-\hat{n})$$

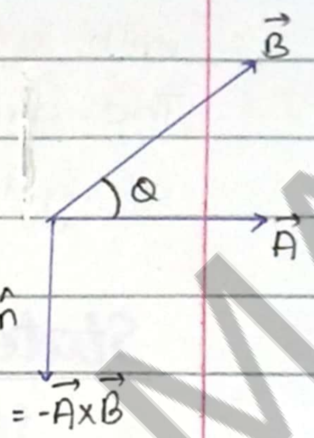
OR $\quad \vec{B} \times \vec{A} = -AB \sin\theta \; \hat{n}$

Thus, we can say that

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

Therefore,

$$\boxed{\therefore \quad \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$$



Parallel and Anti-Parallel.

PARALLEL ($\theta = 0°$).

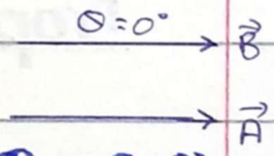
As we know that,

$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n} \qquad \therefore \theta = 0°$$

$$= AB \sin(0°) \hat{n} \qquad \therefore \sin 0° = 0$$

$$= AB\hat{n}(0)$$

$$\boxed{\therefore \vec{A} \times \vec{B} = 0 \text{ (zero)}}$$



ANTI-PARALLEL ($\theta = 180°$)

As we know that,

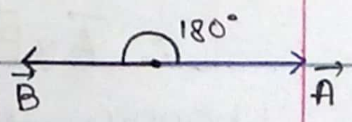
$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n} \qquad \therefore \theta = 180°$$

$$= AB \sin(180°) \hat{n} \qquad \therefore \sin 180° = 0$$

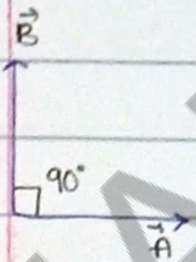
$$= AB(0)\hat{n}$$

$$= AB\hat{n}(0)$$

$$\boxed{\therefore \vec{A} \times \vec{B} = 0 \text{ (zero)}}$$



Perpendicular ($\theta = 90^\circ$)



As we know that,

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n} \quad \therefore \sin 90^\circ = 1$$

$$= AB\sin(90^\circ)\,\hat{n} \quad \therefore \theta = 90^\circ$$

$$= AB(1) \quad \therefore \text{ignoring } \hat{n} \text{ due to its direction.}$$

$$\boxed{\therefore \vec{A} \times \vec{B} = AB}$$

Vector Product in Unit Vectors

Case 01 :

Similar Unit Vector

When we multiply similar unit vector:-

$$\hat{i} \times \hat{i} = ii\sin\theta$$

In similar unit vectors the angel $\theta = 0^\circ$

then, $\hat{i} \times \hat{i} = ii\sin(0)^\circ \quad \therefore \theta = 0^\circ$

$$\boxed{\hat{i} \times \hat{i} = 0 \text{ (zero)}} \quad \therefore \sin 0^\circ = 0$$

Thus,

$$\boxed{\therefore \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0}$$

Case 02:

Different Unit Vector

when we are multiplying different unit vectors:-

$\hat{i} \times \hat{j} = 1$ (Magnitude)

1st cond: $\therefore \hat{i} \times \hat{j} = +\hat{k}$ (for Direction)

2nd cond: $\therefore \hat{i} \times \hat{k} = -\hat{j}$

Tip to Remember

$\hat{j}$

$\hat{k}$ $\hat{i}$

Anticlockwise $\Rightarrow$ + $\Rightarrow$ ↺

clockwise $\Rightarrow$ − $\Rightarrow$ ↻

3rd cond: $\therefore \hat{j} \times \hat{k} = +\hat{i}$

4th cond: $\therefore \hat{j} \times \hat{i} = +\hat{k}$

5th cond: $\therefore \hat{k} \times \hat{i} = -\hat{j}$

6th cond: $\therefore \hat{k} \times \hat{j} = -\hat{i}$

Vector Product of two Vectors in terms of Rectangular Components —

As we know that,

$$\vec{A} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$$

$$\vec{B} = (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & B_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Expanding from 1st Row:-

$$= +\hat{i}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= +\hat{i}\,(A_yB_z - A_zB_y) - \hat{j}\,(A_xB_z - A_zB_x) + \hat{k}\,(A_xB_y - A_yB_x)$$

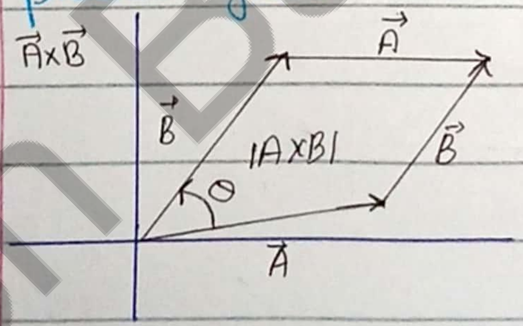
$$\Rightarrow \vec{A}\times\vec{B} = \hat{i}(A_yB_z - A_zB_y) + \hat{j}(-A_xB_z + A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

OR

$\therefore$ $$\vec{A}\times\vec{B} = \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$$

Physical Significance of Vector Product _.

Q. Show that the magnitude of vector product gives the area of parallelogram.



As we know that,

$$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n}$$

The magnitude of two vectors,

$$\therefore\ |\vec{A}\times\vec{B}| = AB$$

explination:

↳ Vector product gives us the area of a plane.

↳ (adjacent sides):
Consider two vectors $\vec{A}$ and $\vec{B}$ which represents adjacent sides.

↳ (Area):
The magnitude of $\vec{A}$ and $\vec{B}$ gives us the area of rectangular or of parallelogram.

↳ unit vector ($\hat{n}$):

It gives the direction of the calculated area which is normal to the plane and can be found by Right hand Rule.