

**Angular Position :-** (1) Position vector is used to show the position of an object. Angle contribute the angular concept in the given condition.

(3) Therefore, the position vector and angle combinally show the angular position of an object.

→ Angle that is formed when a position vector of a moving object displaces with reference to some chosen point.

### Angular Displacement :-

Difference b/w two angular position is called displacement.

$$\text{Initial} - \text{final} = \text{Total}$$

$$\Delta\theta = \theta_f - \theta_i$$

$$\begin{array}{ccc} \text{Radian} & \xrightarrow{\times \frac{1}{180}} & \text{Degree} \\ & \xleftarrow{\times \frac{180}{\pi}} & \end{array}$$

### Angular velocity :-

Instantaneous Angular velocity ( $\omega$ ) Angular velocity at any short interval of time is called instantaneous angular velocity.

$$\omega_{\text{inst}} = \frac{\Delta\theta}{\Delta t}$$

### Direction :-

As, angular velocity depends on angular displacement.

therefore, the direction of angular velocity should be in the

Special tip :-

If we want to convert linear quantities to angular quantities, then we have to multiply moment arm ( $r$ ).

$$s = r\theta$$

$$\theta = \frac{s}{r}$$

How much is one radian?  
→ One radian is angle b/w two radii which covers an arc on

One radian :- If the arc AB is equal to the radius OA then, the angle subtended in the center will be one radian.

$$\Rightarrow \text{Arc AB} = \text{OA (radius)}$$

SLO Based Question :- How can

we find the Direction of Angular Displacement?

Ans: If a body is moving in anti-clockwise direction the angular displacement is positive and if the body is moving in clockwise direction then angular displacement will be negative. (By right hand rule)

of angular displacement



## Angular Acceleration:-

The time rate of change of angular velocity.

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

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units of angular acceleration:-

- (i)  $\text{Rad/s}^2$ , (ii)  $\text{revolution/s}^2$ ,
- (3)  $\text{degree/s}^2$

Direction of Angular ( $\alpha$ ):- -, +?

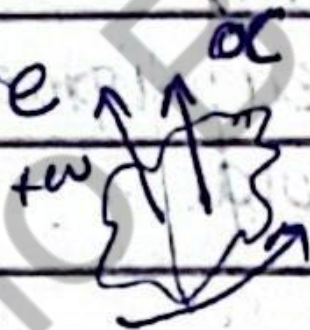
→ The direction of angular acceleration depends on whether the object is speeding up or slowing down in its rotation.

If an object is speeding up, the ( $\alpha$ ) points in the same direction as the rotation.

If the object is slowing down the angular  $\alpha$  points in opposite direction of the rotation.

→ When the angular velocity ( $\omega$ ) of an object is increased then, it has +ive ( $\alpha$ ) acceleration.

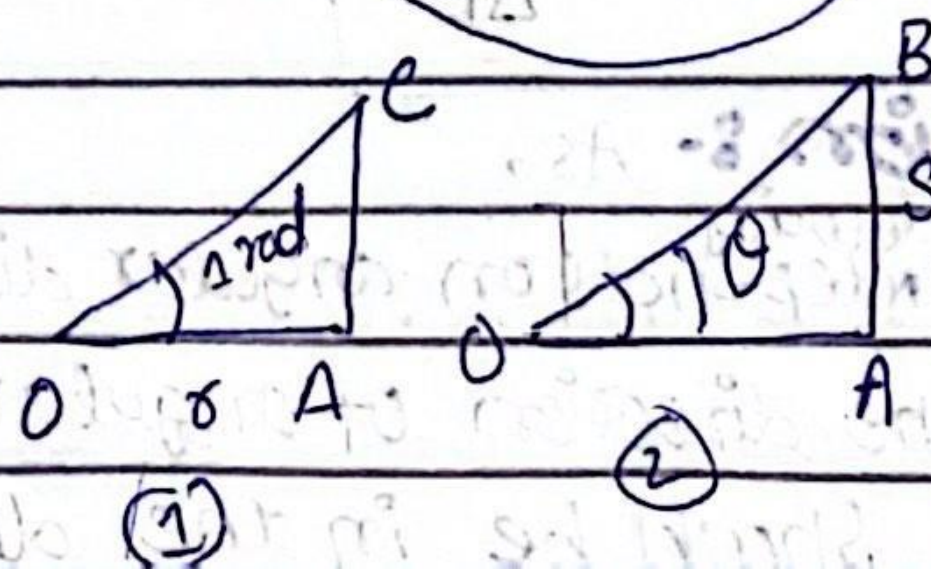
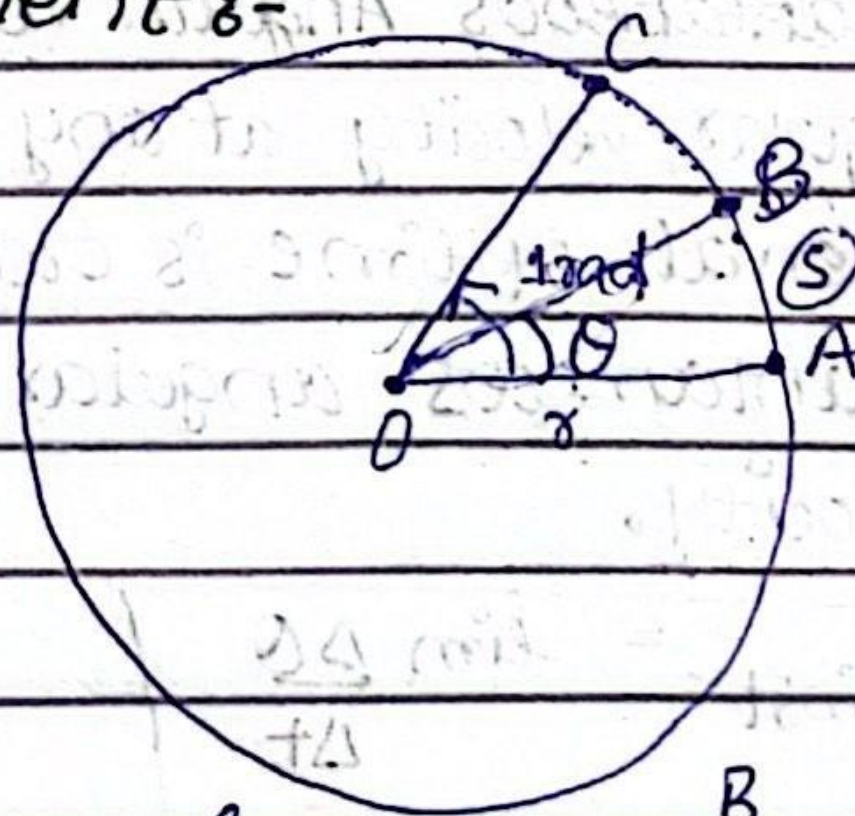
→ When the angular velocity ( $\omega$ ) is decreasing then, it has (-ive) angular acceleration.



## Relationship blw Linear angular kinematic Quantity:-

(A) Relation blw linear and angular displacement:-

Consider a particle moving on circular path from A and B i.e; distance blw A and B is equal to radius of a circle, Then the angle covered by particle will be one radian ( $\angle AOC$ ). Consider an other arc AC must be equal to radius "r" and the angle will 1 radian.





By using geometry:

$$\frac{\text{Arc AB}}{\text{Arc AC}} = \frac{\angle AOB}{\angle AOC}$$

So, know putting values

$$\frac{s}{r} = \frac{\theta}{1 \text{ rad}} \Rightarrow (1)$$

$$\Rightarrow \theta = \frac{s}{r} \Rightarrow \boxed{s = r\theta}$$

### (b) Relation for angular velocity:-

Since we have,

$$s = r\theta$$

Dividing  $\Delta t$  on b/s and applying

$\lim_{\Delta t \rightarrow 0}$ :-

$$\Rightarrow \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = r \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t}$$

$$\therefore \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = v, \quad \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \omega$$

$$\boxed{v = r\omega}$$

### (c) Relation b/w Angular acceleration:-

Q: Briefly explain the practical application where the angular and linear velocity are involved?  $\boxed{v = r\omega}$

Derivation of angular acceleration:-

As we have,  $\Rightarrow v = r\omega$

Taking and dividing to sides

by  $\lim_{\Delta t \rightarrow 0} \frac{\Delta}{\Delta t}$ :-

$$\Rightarrow \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = r \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t}$$

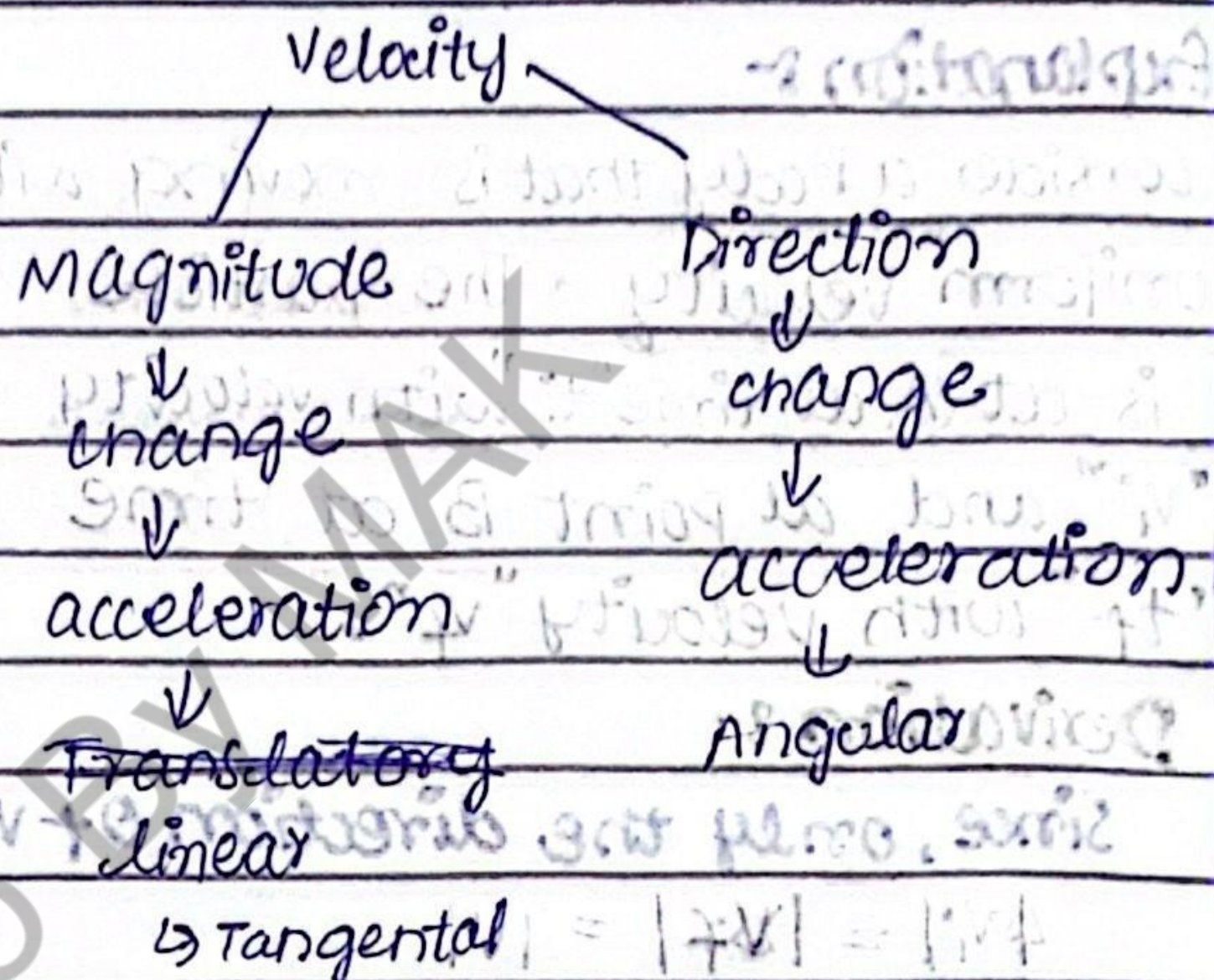
$$\boxed{a_T = r\alpha}$$

### Tangential acceleration ( $a_T$ ):-

Tangential acceleration is produce to change the magnitude of velocity

### Angular $\alpha$ :- ( $\alpha_T$ ):-

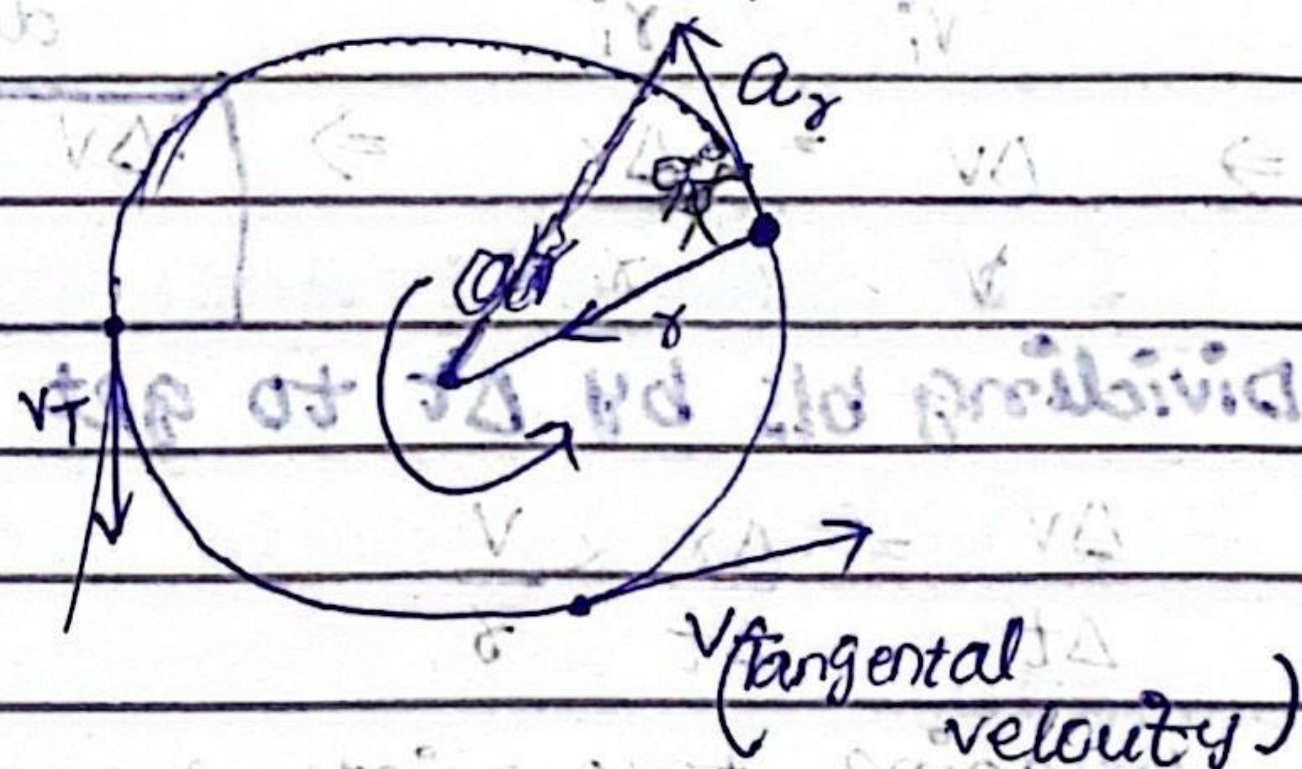
Angular acceleration is produce to change in direction of velocity.



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Diagrams :

etinition :-

Centripetal acceleration is how

videly the direction of velocity

→ the object changes as it moves  
on that circle.

Explanation :-

Consider a body that is moving with uniform velocity. The particle

is at A at time " $t_i$ " with velocity

$t_i$  and at point B at time  $t_f$  with velocity " $v_f$ ".

### Derivation 1-

Since, only the direction of  $v$  changes:

$$4V_i = |x| = |V|$$

• since radius of a circle remains same:

$$|r_i^0| = |r_f| = |r|$$

Now,  $\triangle AOB \leftrightarrow \triangle QPR$

Since these triangles are right angled.  $\tan \theta = \frac{P}{Q}$

$$\frac{\Delta V}{v_i^*} = \frac{\Delta \gamma}{\gamma_i^*}$$

$$v_i \quad y_i$$

$\Delta V$  : is showing the change in direction of  $V$ , By comparing the value

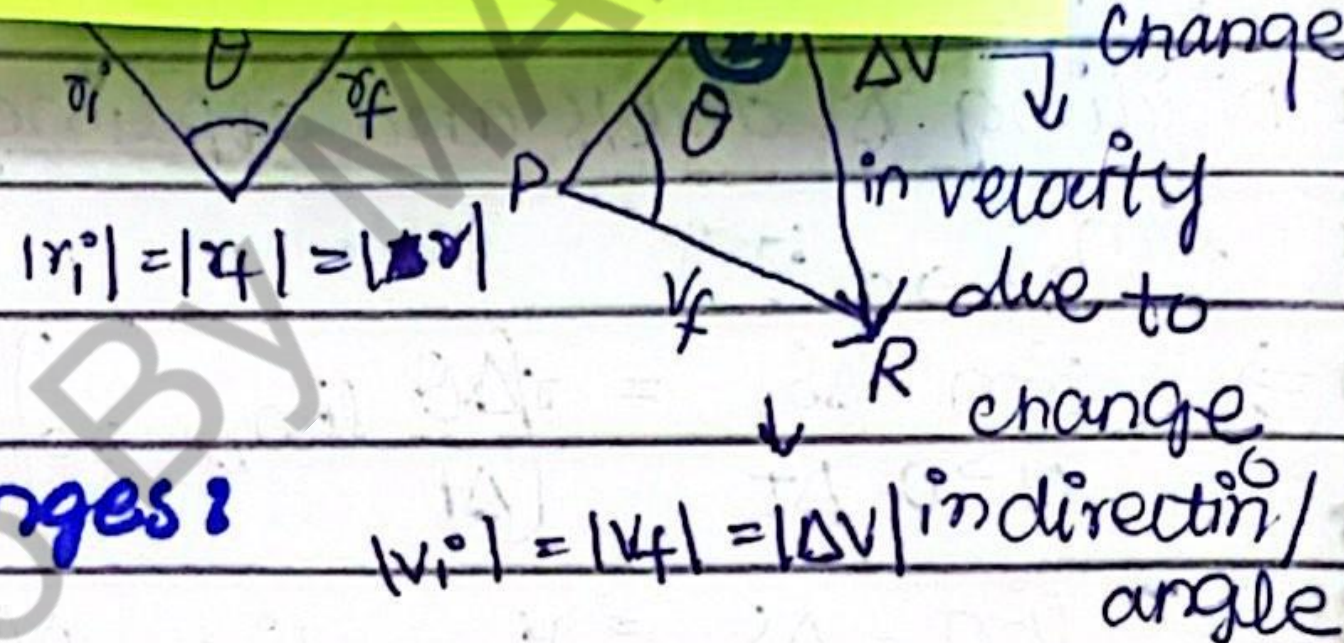
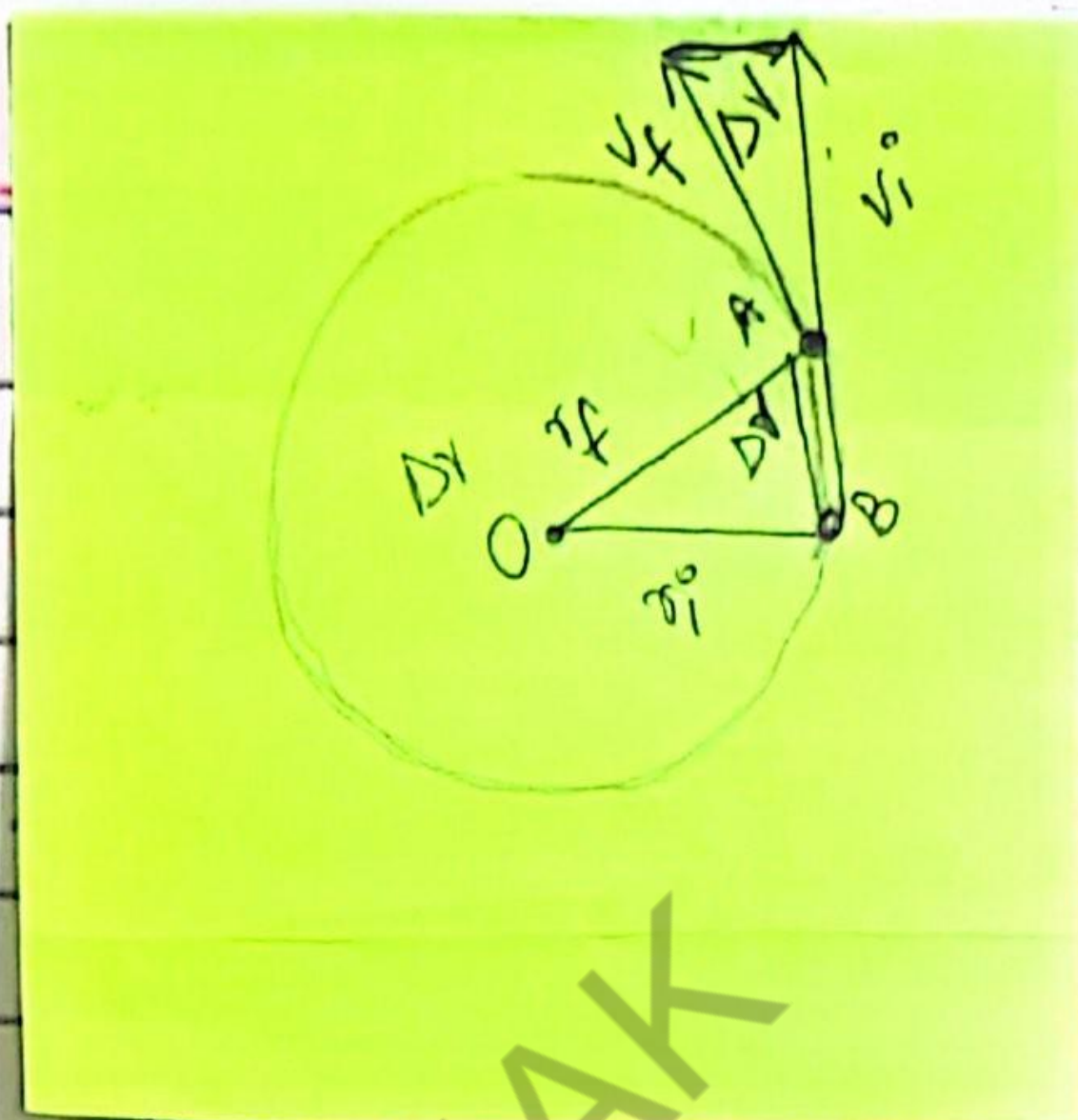
$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta \gamma}{\gamma} \Rightarrow \boxed{\Delta V = V \frac{\Delta \gamma}{\gamma}} \quad \text{of } V \text{ at two different points.}$$

of  $V$  at two different points.

Dividing b1s by  $\Delta t$  to get acceleration:

$$\frac{\Delta V}{\Delta t} = \frac{\Delta r}{\Delta t} \times \frac{V}{r}$$

→ Imagine two points A and B extremely close to each other such that  $\Delta t$  approaches to zero and then acceleration at this stage now be instantaneous acceleration.



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So that;

$$\Rightarrow a = \frac{v}{r} \lim_{\Delta t \rightarrow 0} \frac{\Delta r}{\Delta t} \quad \therefore \frac{\Delta v}{\Delta t} = a$$

$$a_c = \frac{v}{r} \times v \quad \therefore \frac{\Delta r}{\Delta t} = v$$

$$\boxed{a_c = \frac{v^2}{r}} \rightarrow \text{Magnitude of centripetal acceleration} \rightarrow \text{eq. (x)}$$



Vectorially, it is given as;

$$\Rightarrow \vec{a}_c = \left( -\frac{v^2}{r} \right) \hat{r} \rightarrow \text{Now, here } (\hat{r}) \text{ is radial unit vector and it has}$$

$$\text{Since, } \boxed{v = r\omega}$$

outward direction and (-)ive sign due to being inward,

Putting eq. in eq. (x)

then,

$$\vec{a}_c = -\frac{(r\omega)^2}{r} \hat{r}$$

$$\Rightarrow \vec{a}_c = -r\omega^2 \hat{r}$$

$$\text{So, } \boxed{\vec{a}_c = -r\omega^2 \hat{r}} \rightarrow \text{centripetal acceleration in terms of angular form.}$$

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**Important points;**

$\rightarrow a_c$  is produced through change in direction and constant velocity.

**Q; In Linear motion;**

can acceleration can be produced when the velocity is constant?

Ans: Since the acceleration is the rate of change of velocity of a body with time, if the velocity is constant, the  $a$  of the body will be zero.

**For Centripetal (Circle) :-** Same Q#

**Q; Ans:** Since, velocity is a vector with both magnitude and direction, a change in either the magnitude or the direction constitutes



compare the two in terms of forces and the other one is in terms of velocities.

a change in the velocity. For this reason, it can be safely concluded that an object moving in a circle at constant speed/velocity is accelerating.

**Centripetal force**  $\rightarrow$  without this force object will move in a straight line due to inertia.

**Definition** - Centripetal force is the push or pull that keeps something moving in a circle.

OR

Centripetal force is the force that pulls an object toward the center of a circle, causing it to move in a circular path even along the curve path.

**Derivation**  $F_c = ma_c$

$$F_c = m(r\omega^2)/r$$

$$F_c = -mr\omega^2$$

$$F_c = ma_c$$

$$F_c = \frac{mv^2}{r}$$

$\therefore$  (-)ive sign indicates the direction toward the center.

$\rightarrow$  It is not a separate force. It is the resultant force of  $g$ , tension, friction, or any other force depending on situation.

**Questions**

**Why  $a_c$  is always directed towards centre?**

toward the center of the circular path bcz it is the result of continuous change in the direction of an object moving in a circle. Even though; the speed of the object remains constant, its direction constantly changing.

$\rightarrow$  And this change in direction requires a force that act perp to the object's velocity.

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⇒ The magnitude of centripetal force depends on the speed of the object and radius of the circular path.

Q# Find the maximum speed of car in  $\text{kmh}^{-1}$  when it is travelling in curved road having 500m radius of curvature's bank at an angle of  $15^\circ$ ?

Given:-

$$\text{Radius} = r = 500\text{m}$$

$$\text{Angle} = \theta = 15^\circ$$

$$\text{Acceleration} = g = 9.8\text{m/s}^2$$

To find:-  $v = ?$

Formula:-

Since, according to the given formula;

$$\tan \theta = \frac{v^2}{rg}$$

$$v^2 = \tan \theta \cdot rg$$

Taking square root on b/s

$$\sqrt{v^2} = \sqrt{\tan \theta \cdot R \cdot g}$$

$$v = \sqrt{\tan \theta \cdot R \cdot g}$$

**Solution:**

In order to find  $\theta = ?$

$$\tan(15^\circ) = 0.2679$$

Now, By putting values in given formula;

$$v = \sqrt{\tan \theta \cdot R \cdot g}$$

$$v = \sqrt{(0.2679) \times (9.8) \times (500)}$$

$$v = \sqrt{1314.45}$$

$$v = 36.2\text{m/s}$$

Now, converting  $v$  into  $\text{kmh}^{-1}$  since,

$$1\text{m/s} = 3.6\text{kmh}^{-1}$$

$$v = 36.2 \times 3.6$$

$$\boxed{v = 130.4\text{kmh}^{-1}}$$

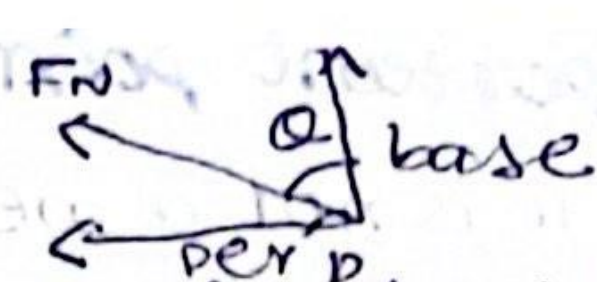
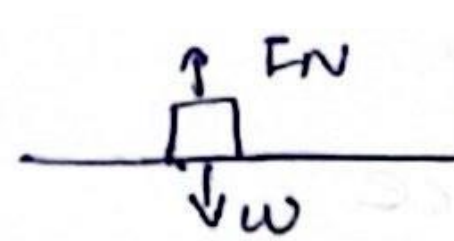
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## Application of centripetal force :-



**Banking of Roads :-** Banking of roads refers to the angle at which road is inclined in order to help vehicles negotiate curves more safely and efficiently.

**Centripetal force :-** when a ~~force~~ <sup>vehicle</sup> moves in a circular path, it requires centripetal force directed toward the center of the circle.

**Banking angle :-** The angle at which the road is inclined with respect to the horizontal.

**Normal force:** The perpendicular force exerted by the surface of the road on the vehicles.

## Forces acting on a vehicle on a Banked curve :-

1) Weight ( $W$ ) ( $mg$ ) :- This force acts downward vertically, due to gravity. It has magnitude of  $mg$ , where ' $m$ ' is mass and ' $g$ ' is gravity.

2) This force acts perpendicular to the surface of the road.

**Derivation :-** vertical component of the normal balance the  $w$ .

$$F_N \cos \theta = mg \rightarrow (i)$$

The hor

Dividing eq (ii) by eq (i)

$$\frac{F_N \sin \theta}{F_N \cos \theta} = \frac{mv^2/r}{mg}$$

$$\rightarrow \tan \theta = \frac{v^2}{rg}$$

$$\rightarrow \theta = \tan^{-1} \frac{v^2}{rg}$$

$$v^2 = rg \tan \theta$$

$$\sqrt{v^2} = \sqrt{rg \tan \theta}$$

$$\rightarrow v = \sqrt{rg \tan \theta}$$

The horizontal component of the normal force provide necessary  $F_c$  :-  $F_N \sin \theta = F_c$

$$F_N \sin \theta = \frac{mv^2}{r} \rightarrow (ii)$$

## Centrifuge :- A

Centrifuge is a device that uses  $F_c$  to separate substances based on density.

When device spins, it generates a centripetal force that acts on the substances inside. This force keeps objects moving in a circular path.

**Applications :-**

- cream separator
- washing machine dryer.

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## Moment of inertia:- ( $\text{kgm}^2$ )

→ The property of body which resists change in angular motion

or rest.

→ The tendency of a body to resist any change due to rotational motion of a body for increase in angular velocity.

**For point of mass** It is a property of an object that measures how difficult it is to make it rotate around a certain axis. → The larger moment of I harder to change its rotation.

$$I = mr^2$$

**Rigid body:-**

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2$$
$$\sum_{i=1}^n m_i r_i^2$$

more radius = harder to rotate  
less radius = easy to rotate

## Angular Momentum ( $\text{kgm}^2\text{s}^{-1}$ ), ( $\text{ML}^2\text{T}^{-1}$ )

### Definitions

1) The momentum possessed by a body during its angular motion is called angular momentum.

2) The cross product of moment arm " $\vec{r}$ " with linear momentum " $\vec{p}$ " is called angular momentum.

$$\vec{L} = \vec{r} \times \vec{p}$$

Derive a relation b/w angular momentum with momentum of

**For point mass:-** AS, we have from

Angular momentum:-

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = r p \sin \theta \hat{n}$$

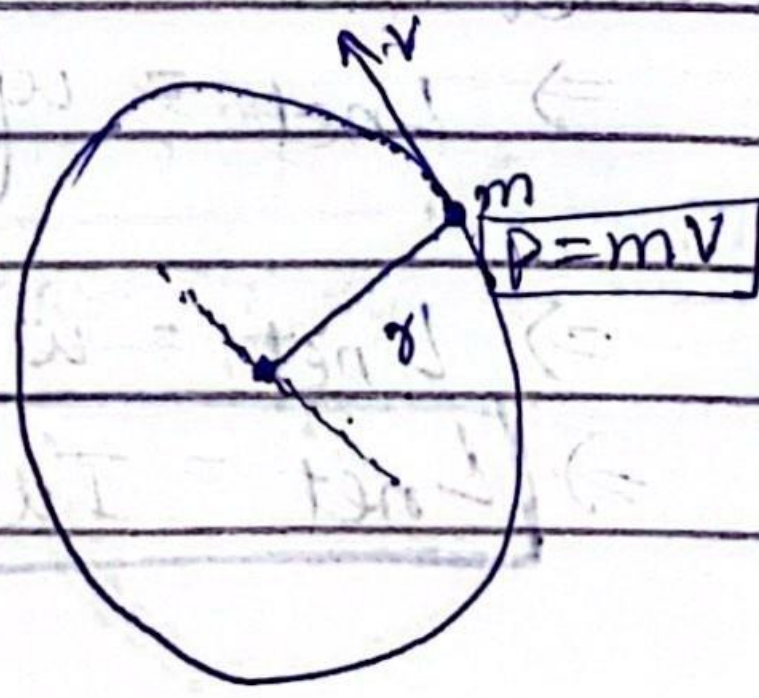
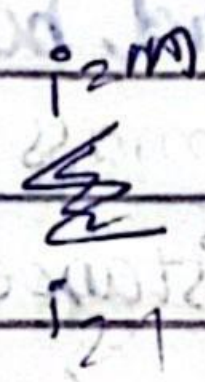
→ If you rotate a rigid body about different axis how does moment of inertia change?

Ans: The moment of inertia depends on how far the mass is distributed from the axis. The farther the mass is from the axis, the greater the moment of inertia.

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$$L = r p \sin 90^\circ$$

$$L = r p \rightarrow (i)$$

$$\text{Since, } p = mv \rightarrow (ii)$$

Putting eq (ii) in eq (i) then,

$$L = r(mv) = mvr \text{ so, } L = mvr \rightarrow (3)$$

$$\text{Since, } v = r\omega \rightarrow (iv)$$

Putting eq (iv) in eq (iii) then,

$$\Rightarrow L = m(r\omega)r \Rightarrow L = mr^2\omega \rightarrow (v)$$

the product of moment of inertia and its angular velocity.

Since from moment of inertia,

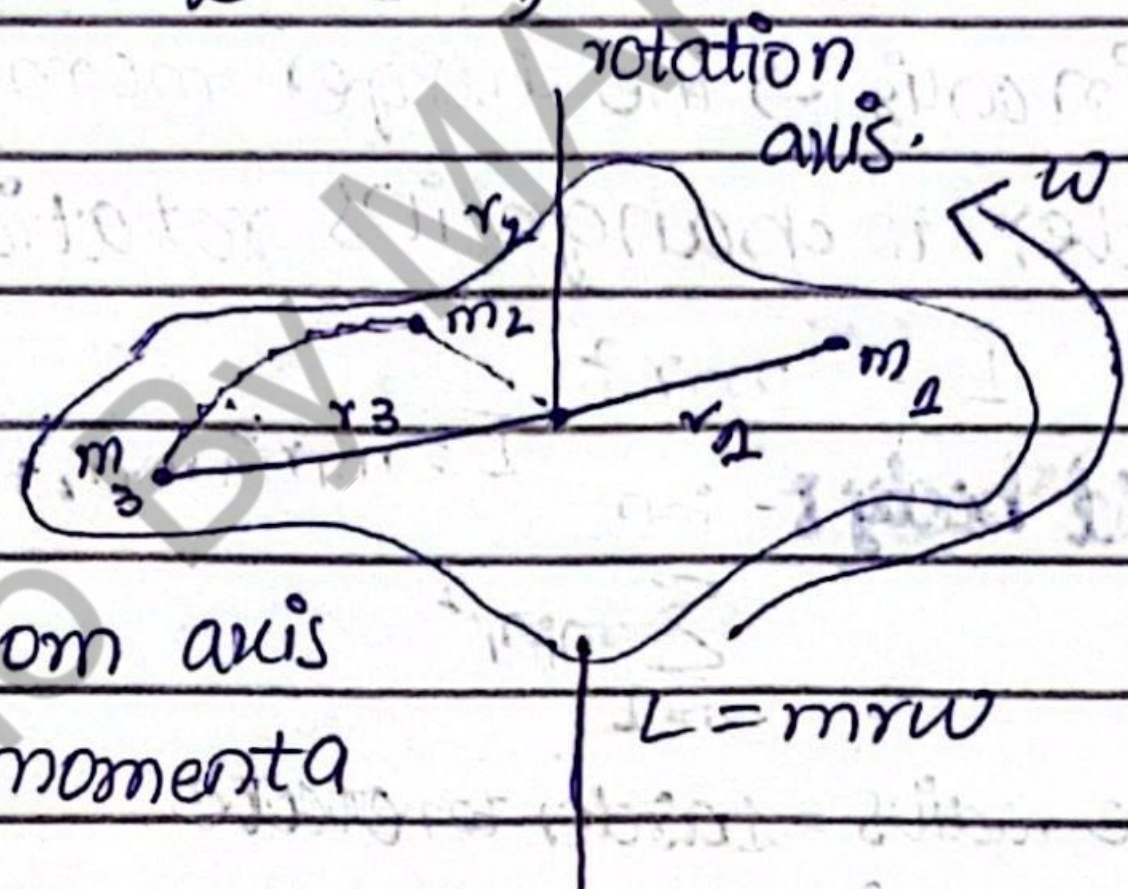
$$I = mr^2 \rightarrow (vi) \text{ Putting eq (vi) in eq (v) then;}$$

$$\Rightarrow L = I\omega$$

### For a Rigid body :-

consider mass  $m_1, m_2, \dots, m_n$

having distance of  $r_1, r_2, \dots, r_n$  from axis of rotation. So, the net angular momenta of this system is given as:-



$$L_{\text{net}} = L_1 + L_2 + L_3 + \dots + L_n$$

Since, we know that the angular momentum of single particles is given by:-  $L = mrv$

$$\Rightarrow L_{\text{net}} = m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega + \dots + m_n r_n^2 \omega$$

Taking common " $\omega$ " from the above.

$$\Rightarrow L_{\text{net}} = \omega(m_1 r_1^2) + \omega(m_2 r_2^2) + \omega(m_3 r_3^2) + \dots + \omega(m_n r_n^2)$$

$$L_{\text{net}} = \omega(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2)$$

$$\Rightarrow L_{\text{net}} = \omega \left( \sum_{i=1}^n m_i r_i^2 \right) \quad \therefore I = \sum_{i=1}^n m_i r_i^2$$

$$\Rightarrow L_{\text{net}} = \omega(I)$$

$$\Rightarrow L_{\text{net}} = I\omega$$

Hence, this shows that the total angular momenta of rigid body.

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# Relation b/w Torque and Angular Momentum:-

Since, we have the angular momentum,

$$\Rightarrow \vec{L} = \vec{r} \times \vec{p}$$

Multiplying both sides by " $\frac{\Delta}{\Delta t}$ "  $\Rightarrow$  dono taraf time ka except sa change liya

$$\frac{\Delta \vec{L}}{\Delta t} = \frac{\Delta}{\Delta t} (\vec{r} \times \vec{p})$$

$$\Rightarrow \frac{\Delta \vec{L}}{\Delta t} = \vec{r} \times \frac{\Delta \vec{p}}{\Delta t} \quad \because \frac{\Delta \vec{p}}{\Delta t} = \vec{F}$$

$$\Rightarrow \frac{\Delta \vec{L}}{\Delta t} = \vec{r} \times \vec{F} \quad \therefore \boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

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$$\Rightarrow \boxed{\frac{\Delta \vec{L}}{\Delta t} = \vec{\tau}}$$

**Conclusion:-** Thus we conclude that the rate of change in angular momentum produce torque.

## SLO Based Questions:-

1) Derive the relation of angular momentum with moment of inertia and angular velocity for a rigid body.

2) Drive  $\boxed{L = I\omega}$  for a rigid body. or  $\frac{\Delta L}{\Delta t} = \vec{\tau}$

### Special tip:-

(i) linear momentum me change angular momentum ki value ko change hoga. (ii) Jab hi angular momentum change hoga to torque produce hoga.

## Conservation of Angular Momentum:-

As, we know:-

$$\Rightarrow \frac{\Delta \vec{L}}{\Delta t} = \vec{\tau}$$

If we consider an isolated system then the torque (external)  $\vec{\tau} = 0$ ,

$$\Rightarrow \frac{\Delta \vec{L}}{\Delta t} = 0 \Rightarrow \Delta \vec{L} = 0 \quad \therefore$$

$$\Rightarrow \vec{L}_f - \vec{L}_i = 0$$

$$\Rightarrow \boxed{\vec{L}_f = \vec{L}_i} \rightarrow (i)$$

The final momentum should be equal to initial angular momentum.

Since  $\boxed{L = I\omega}$  then,

$$L_i = I_i \omega_i, \quad L_f = I_f \omega_f$$

$$\boxed{I_i \omega_i = I_f \omega_f} \rightarrow (ii)$$

Since,  $I = m r^2$

$$I_i = m_i r_i^2, \quad I_f = m_f r_f^2 \rightarrow (iii)$$

$$\Rightarrow m_i r_i^2 \omega_i = m_f r_f^2 \omega_f \rightarrow$$

By putting eq. 3 in eq. 2

(2)

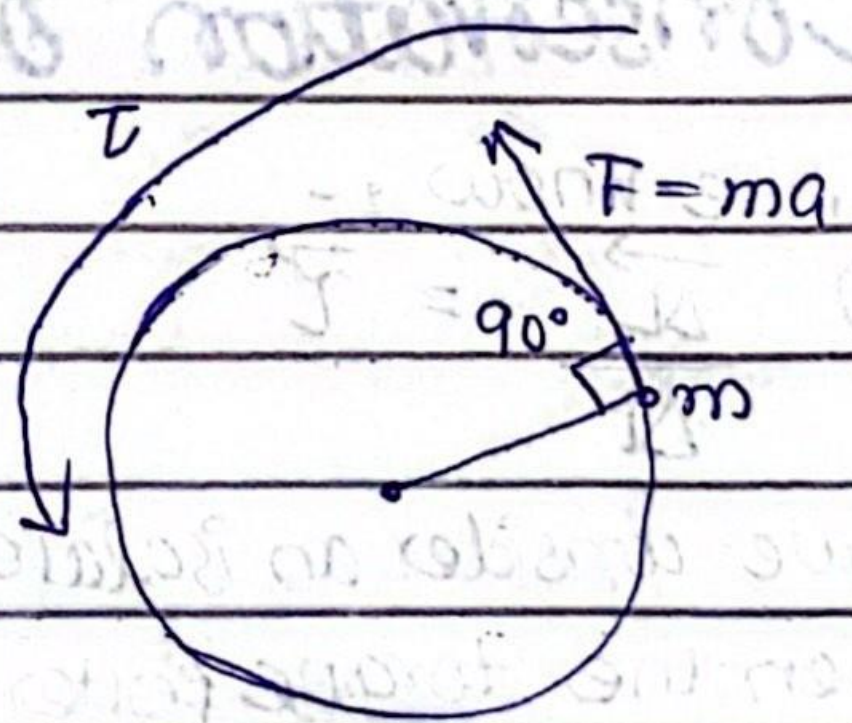


## Applications Conservation of Angular Momentum :-

- Ice skater:** while spinning, ice skaters when pull in their arms, they reduce their moment of inertia and causes increase in angular velocity. but when they contract their arms this increase their moment of inertia and causes decrease in angular velocity.
- Board divers:-** when a diver begin their dive, they pull their limbs close to their body, which reduces their moment of inertia, But According to the conservation of  $\vec{L}$  the angular velocity increases.
- Gyroscope:** maintain its orientation due to the momentum. when an external torque is applied, such as when the gyroscope is tilted, it experience a change  $\Delta L$ .
  - This change doesn't result in a simple shift in direction. instead, it causes precession, where the gyroscope rotates around a vertical axis rather than tipping over.
  - when spinning, they resist changes in direction,
  - It

$$\begin{aligned} r \times F &= r m a \\ r m \alpha &= I \alpha \end{aligned}$$

$$\tau = I \alpha$$



### Torque and Angular acceleration:

$$\text{Since, } \tau = r F \sin 90^\circ \rightarrow \tau = r F \sin 90^\circ$$

$$\therefore F = ma \rightarrow (ii)$$

$$\tau = r F \rightarrow (i)$$

$$\Rightarrow a = r \alpha \rightarrow (iii)$$

Putting eq (3) in eq (ii) then,

$$\Rightarrow F = m (r \alpha)$$

$$F = m r \alpha \rightarrow (iv)$$

Putting eq (iv) in (i)

$$\Rightarrow \tau = r (m r \alpha) \Rightarrow \tau = m r^2 \alpha$$

$$\therefore \tau = I \alpha$$

$$\therefore I = m r^2$$

### SLO Q:-

- How Torque is related with  $\alpha$ ?

- Derive  $\tau = I \alpha$

- Derive the relation b/w Torque and angular acceleration.



# Artificial Gravity :-

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## linear velocity :-

Since, we have :-

$$\Rightarrow a_c = \frac{V^2}{R} \Rightarrow V^2 = a_c R$$

$$\Rightarrow V = \sqrt{a_c R} \Rightarrow V = \sqrt{gR} \rightarrow (i)$$

Angular velocity of satellite :-

Since, we have  $V = R\omega \rightarrow (ii)$

By comparing eq (i) and eq (ii) then,

$$\Rightarrow \sqrt{gR} = R\omega$$

$$\Rightarrow R\omega = \sqrt{gR}$$

$$\Rightarrow \omega = \frac{\sqrt{gR}}{R}$$

$$\Rightarrow \omega = \frac{\sqrt{gR}}{R}$$

$$\Rightarrow \omega = \frac{\sqrt{g}}{R} \rightarrow (iii)$$

Frequency of satellite :-

Since, frequency is required

$$\therefore f = \frac{1}{T}$$

$$f = \frac{1}{2\pi \sqrt{\frac{R}{g}}} \rightarrow (iv)$$

Exercise Short Questions :-

(2) Is centripetal force a fundamental force ... ?

Centripetal force is not a fundamental force itself, rather it is a

Time period of satellites :-

The time required by a rise of satellite to complete one rotation is called Time period of Satellite.

Since,

$$\Rightarrow S = VT \Rightarrow T = \frac{S}{V}$$

$$T = \frac{2\pi R}{V}$$

S = circumference of circle

$$\Rightarrow T = \frac{2\pi R}{R\omega} \because V = R\omega$$

$$\Rightarrow T = \frac{2\pi}{\omega} \rightarrow (v)$$

Putting eq (iii) in eq (v) then,

$$T = 2\pi \times \sqrt{\frac{R}{g}}$$

$$T = 2\pi \sqrt{\frac{R}{g}} \rightarrow (vi)$$

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It is the net force required to keep an object moving in a circular path. It arises from other fundamental forces such as; gravitational, electromagnetic or tension forces therefore by combination of these  $\vec{F}$ ,  $F_c$  can be provided.

5) double ties on one side of axis + - - - ?

Yes, the moment of inertia for a system with double ties on one side of axis be different from single tie bcz double has larger mass and increase its distribution of axis of rotation. Therefore moment of inertia will be greater..

6) why it is best to have blades to rotate in opposite...?

Having the blades rotate in opposite direction helps counter torque, preventing the helicopter from spinning and enhancing stability and control.

7) If the diameter of earth become half.....?

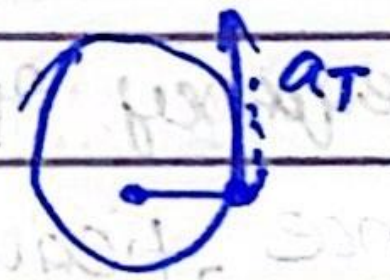
If Earth's diameter become half while its mass remain same, its moment of inertia decreases, To conserve  $\vec{L}$ , the rotational speed must increase, causing Earth to spin faster around its axis.

8)  $a_{at}$  magnitude is changed but not direction of its velocity?

In circular motion, tangential acceleration changes the magnitude of the object but not its direction because it acts along the tangent path.

9) Why artificial gravity is less than  $9.8 \text{ m/s}^2$ ?

Artificial gravity is usually small than  $9.8 \text{ m/s}^2$  bcz it is produced by centripetal acceleration in rotating system, like space station;  $(a_c = \frac{v^2}{r})$  and also prevent discomfort and allow easier movement.



10) How rotation of a flywheel helps to even out the power - ?

The rotation of a flywheel helps even out power delivery from an engine by storing kinetic energy. When the engine produces power, the flywheel absorbs some of that energy, reducing fluctuation.



## Numerical problems 8-

①

Given:  $t = 60s$

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revolution = 3000 per minute

$$= 2\pi \text{ rad}$$

To find:  $\omega = ?$

Solution: As we know that,

$$\omega = \frac{2\pi}{t} = \frac{2(3.14) \times 3000}{60}$$

$$\boxed{\omega = 314 \text{ rad/s}}$$

X: = : X: = : X: = : X: = :

② Given:  $r = 14.5m$

$$g = 9.81 \text{ m/s}^2$$

To find:  $v = ?$

Solution:  $v = \sqrt{rg}$ ,  $F_c = \frac{mv^2}{r}$

In order to find (t and) =

Here, centripetal force is provided by gravitational force. So,

$$F_c = F_g$$

$$\frac{mv^2}{r} = mg \quad [v = \sqrt{rg}]$$

By putting values

=

Now,

$$v = \sqrt{(14.5)(9.81)}$$

$$v = \sqrt{142.245}$$

$$\boxed{v = 11.9 \text{ m/s}}$$

③

$$200/1000$$

Given:  $m = 200g = 0.2kg$

$$r = l = 0.8m$$

Find:  $I = ?$

Solution: Since,

$$I = \frac{1}{2}ML^2$$

$$\text{So, } I = \frac{1}{2}(0.2)(0.8)^2$$

$$= \frac{1}{2}(0.2)(0.64)$$

$$I = \frac{1}{12}(0.2)(0.64)$$

$$I = 0.128/12$$

$$\boxed{I = 0.02 \text{ kgm}^2}$$

X: = : X: = : X: = :

④ Given:  $m = 450g = 0.45kg$

$$r = 11cm = 0.11m$$

revolution = 10 revolution per second

To find:  $L$

Solution: - In order to find

$$L = I\omega$$

But to find  $I = ?$  (sphere)

$$I = \frac{2}{5}mr^2$$

$$I = \frac{2}{5}(0.45)(0.0121)$$

$$I = \frac{2}{5}0.002178$$

Now,  $\omega = 10 \times 2\pi$

$$\omega = 10 \times 2(3.14)$$

$$\omega = 62.8$$



$$\text{So, } L = I\omega$$

$$\vec{L} = (0.002178)(62.8)$$

$$\vec{L} = 0.136 / 0.136 \text{ kg m}^2 \text{ s}^{-1}$$

$$x = x = x = x$$

Given:-

(5) (9)

$$\omega = 10 \text{ rad/s}$$

$$I' = \frac{1}{3} I$$

To find:-

$$\omega' = ?$$

Solution:-

$$L = I\omega \rightarrow (i)$$

$$L' = I'\omega' \rightarrow (ii)$$

Since angular momentum is conserved;

$$L' = L$$

$$I'\omega' = I\omega$$

$$\frac{1}{3} I\omega' = I\omega$$

$$\omega' = 3\omega$$

$$\omega' = 3(10)$$

$$\omega' = 30 \text{ rad/s}$$

When he close his arms the angular velocity will be  $30 \text{ rad/s}$ .

velocity will be  $30 \text{ rad/s}$ .

Data:- (6)

To find:-

$$F = 200 \text{ N} \quad (i) a = ?$$

$$m = 30 \text{ kg} \quad (ii) a = ?$$

$$r = 2 \text{ m}$$

$$m_b = 20 \text{ kg}$$

$$r_b = 1.5 \text{ m}$$

Solution:-

$$(a) \alpha = \frac{\tau}{I_m \text{ (disk shape)}}$$

$$\alpha = \frac{\tau}{\frac{1}{2} m r^2} = \frac{r \times F}{\frac{1}{2} m r^2}$$

By putting values

$$\alpha = \frac{(200)(2)}{\frac{1}{2}(30)(2)^2} = \frac{400}{60}$$

$$\alpha = 6.667 \text{ rad/s}^2$$

Now,

$$(b) \alpha = \frac{F r}{I_m + I_{boy}}$$

$$I_m \text{ for many go round (solid disk)} = \frac{1}{2} m r^2$$

$$I \text{ for boy} = m r_b^2$$

Formula will be:-

$$\alpha = \frac{F r}{\frac{1}{2} m r^2 + m_b r_b^2}$$

By putting value.

$$\alpha = \frac{(200)(2)}{60 + (20)(1.5)^2}$$

$$60 + 45$$

$$\alpha = \frac{400}{105}$$

$$\alpha = 3.81 \text{ rad/s}^2$$

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$$\alpha = \frac{400}{60+45}$$

$$\alpha = \frac{400}{105} = 3.81$$

$$\alpha = 3.81 \text{ rad/s}^2$$

$$\textcircled{7} \quad X \otimes X \equiv \otimes X X \equiv \otimes \otimes X X$$

Data :-

$$g_0 = 5 \text{ ms}^{-2}$$

$$D = 100 \text{ m}$$

$$R = D/2 = 50 \text{ m}$$

To find :-

$$\omega = ?$$

Solution :-

$$\omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{5}{50}}$$

$$\omega = \sqrt{0.1}$$

$$\omega = 0.316 \text{ rad/s}$$

rotation per min

Now, converting in rpm

$$= \frac{0.316}{2\pi/60} = \frac{0.316 \times 60}{2(3.14)}$$

$$\omega = 18.96 / 6.28 = 2.961$$

$$\omega = 3.02$$

$$\omega = 3 \text{ rpm}$$

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