TRANSLATORY MOTION :- move without rotation

1. Rectilinear Motion → Straight like car
2. Circular Motion → in a circular path like toy
3. Random Motion → like bees

• Equation of Motion :-

a = Uniform

1- $v_f = v_i + at$

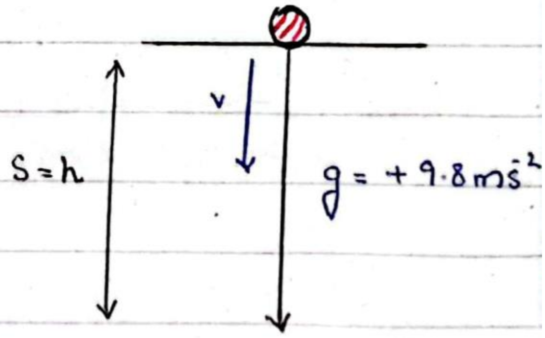
2- $S = v_i t + \frac{1}{2} a t^2$

3- $2aS = (v_f)^2 - (v_i)^2$

if acceleration is uniform than equation of motion works !

• Free Fall Motion :-

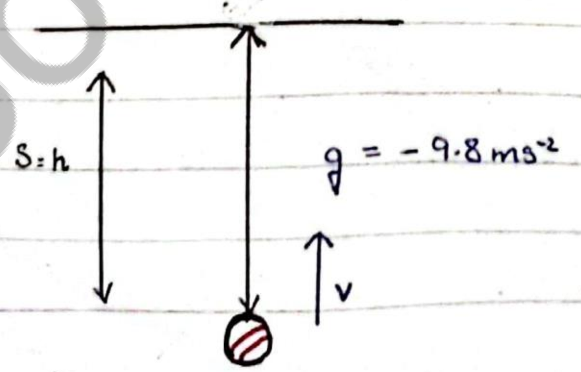
Tip no :- 01



$v_f = v_i + at$

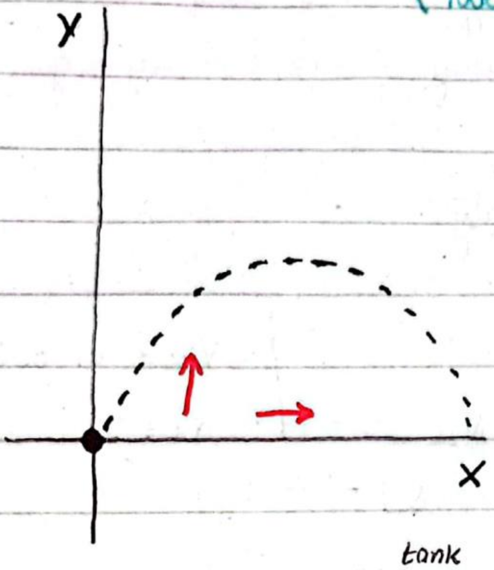
$S = v_i t + \frac{1}{2} a t^2$

$2aS = v_f^2 - v_i^2$



PROJECTILE MOTION :-

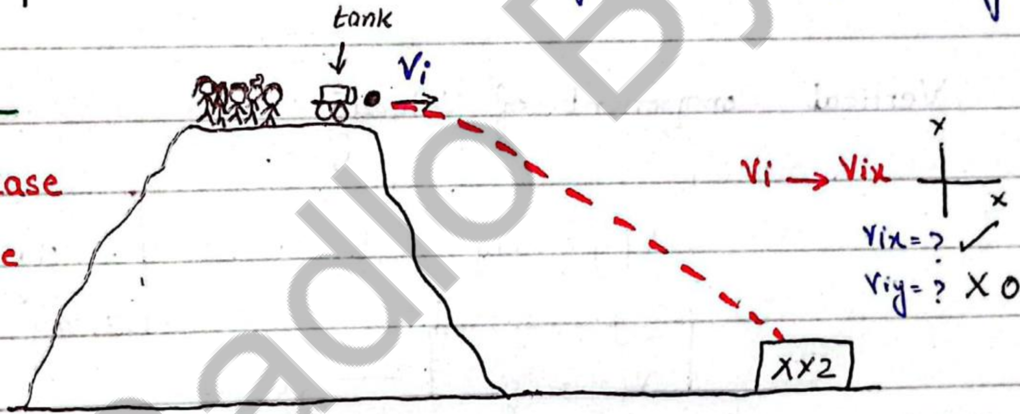
(Two Dimensional Motion under the action of gravity)



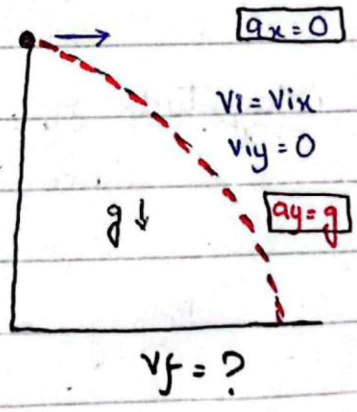
"Projectile motion is a two dimensional motion of an object thrown in the air under the action of gravitational force only."

Example:-

Special case of Projectile motion



"Vi will exist and its component vix would be maximum for some time and viy would be 0. After some time, due to gravity both vix and viy will be present"



Derivation:-

Velocity => In Horizontal Component of initial velocity = $V_{ix} = V_i$

=> Vertical Component of initial velocity = $V_{iy} = 0$

Acceleration:-

=> Acceleration along x-axis = $a_x = 0$

=> Acceleration along y-axis = $a_y = g$

Horizontal Component of Velocity

Since,

=> $V_{fx} = V_{ix} + at$

$\because V_{ix} = V_i\cos\theta$

$\because a_x = 0$

=> $V_{fx} = V_i\cos\theta + 0 \times t$

=> $\boxed{V_{fx} = V_i\cos\theta}$ —— (eq i)

Vertical Component of Velocity

Since,

=> $V_{fy} = V_{iy} + a_y t$

$\because V_{iy} = 0$

$\because a_y = g$

=> $V_{fy} = 0 + gt$

=> $\boxed{V_{fy} = gt}$ —— (eq (ii))

Magnitude of V_f:-

Since,

$\boxed{V_f = \sqrt{V_{fx}^2 + V_{fy}^2}}$ → (eq iii)

Putting eq (i) and eq (ii) in eq (iii) then

$V_f = \sqrt{(V_i\cos\theta)^2 + (gt)^2}$

$\boxed{V_f = \sqrt{V_i^2\cos\theta^2 + g^2t^2}}$

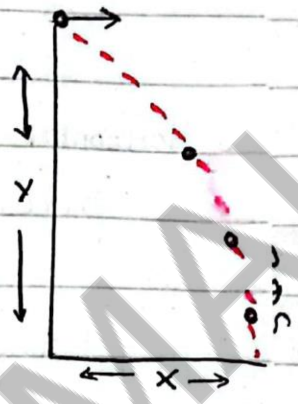
~~Angle~~:- Direction:-

=> Since $\theta = \tan^{-1}\left(\frac{V_{fy}}{V_{fx}}\right)$ => $\tan^{-1}\left(\frac{gt}{V_i\cos\theta}\right)$

Horizontal Displacment:-

Since,

$$\boxed{X = V_{fx} \cancel{\times} t}$$



Vertical Displacment:-

Since

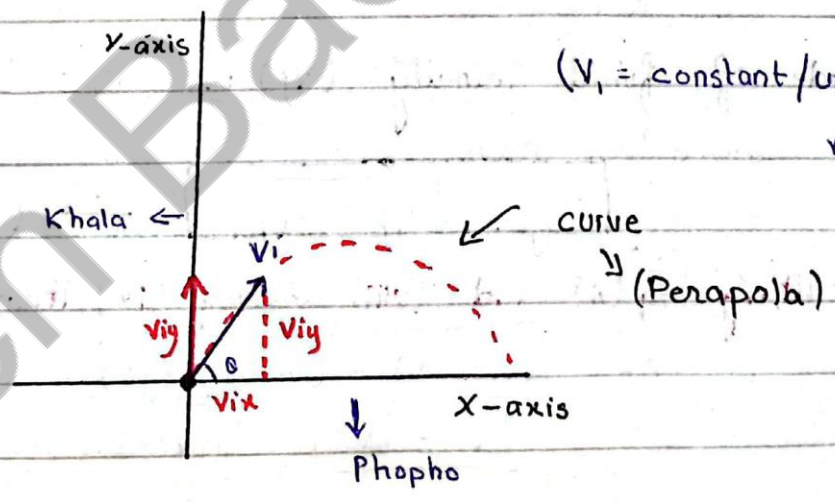
$$S = v_i t + \frac{1}{2} a t^2 \qquad S = Y, \quad v_i = 0, \quad a = g$$

So,

$$\cancel{S} = 0 \times t + \frac{1}{2} g t^2$$

$$\boxed{Y = \frac{1}{2} g t^2}$$

Projectile Motion :-



(V_1 = constant / uniform initial velocity)

$$V_{ix} = V_i \cos\theta \quad \text{—— (i)}$$

$$V_{iy} = V_i \sin\theta \quad \text{—— (ii)}$$

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$a_x = 0$ — (iii) ∵ Velocity is uniform

$a_y = -9.8ms^{-2}$ (iv)

Horizontal Component of Final Velocity (V_{fx}) :-

Since, from first equation of motion :-

$\Rightarrow \quad V_{fx} = V_{ix} + a_x t \longrightarrow (v)$

Putting eq (i) and eq (iii) in eq (v) then,

$\Rightarrow \quad V_{fx} = V_{i\cos\theta} + 0(t)$

$$\boxed{V_{fx} = V_i\cos\theta} \longrightarrow (vi)$$

Vertical Component of final Velocity (V_{fy}) :-

Since, from first equation of motion

$\Rightarrow \quad V_{fy} = V_{ix} + at \longrightarrow (vii)$

Putting eq (ii) and eq (iv) in eq (vii) then,

$V_{fy} = V_i\sin\theta + (-g)t$

$$\boxed{V_{fy} = V_i\sin\theta - gt} \longrightarrow (viii)$$

Magnitude of final Velocity (V_f) :-

Since we have

$$\boxed{V_f = \sqrt{V_{fx}^2 + V_{fy}^2}} \longrightarrow (ix)$$

Putting eq (vi) and (viii) in eq (ix) then,

$$V_f = \sqrt{(V_i\cos\theta)^2 + (V_i\sin\theta - gt)^2}$$

$$V_f = \sqrt{V_i^2\cos^2\theta + V_i^2\sin^2\theta - 2V_i\sin\theta gt + g^2t^2}$$

$$V_f = \sqrt{V_i^2(\cos^2\theta + \sin^2\theta) - 2V_i\sin\theta gt + g^2t^2}$$

$$V_f = \sqrt{V_i^2(1) - 2V_i\sin\theta gt + g^2t^2}$$

$$\boxed{V_f = \sqrt{V_i^2 - 2V_i\sin\theta gt + g^2t^2}} \longrightarrow (x)$$

Direction:-

Since

$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)$$

Putting eq (vi) and (viii)

$$\boxed{\theta = \tan^{-1}\left(\frac{V_i\sin\theta - gt}{V_i\cos\theta}\right)} \rightarrow (\#)$$

Horizontal Displacement:-

$$S = v \times t$$

$$X = V_{fx} \times t$$

$$X = V_i\cos\theta \times t$$

Vertical Displacement:-

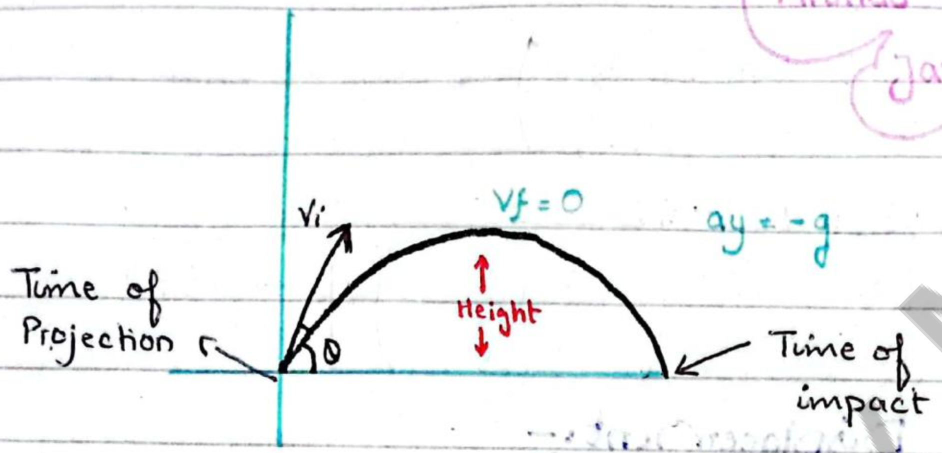
$$S = v_i \times t + \frac{1}{2}at^2$$

$$\Rightarrow \quad Y = V_{iy}t + \frac{1}{2}(-g)t^2$$

$$\boxed{Y = V_i\sin\theta\, t - \frac{1}{2}gt^2}$$

This equation tell that ~~hom~~ how much vertical displacement is covered by a projectile

Height of projectile (H) :-



Since, we have third equation of motion :-

$$\Rightarrow \quad 2as = v_f^2 - v_i^2$$

$$2a_yH = V_{fy}^2 - V_{iy}^2$$

$$2(-g)H = 0 - V_{iy}^2$$

$$-2gH = 0 - V_i^2\sin^2\theta$$

$$H = \frac{\cancel{-}V_i^2\sin^2\theta}{\cancel{-}2g}$$

$$\boxed{H = \frac{V_i^2\sin^2\theta}{2g}}$$

S(H) = ?

a : ✓

Vi : ✓

Vf : ✓

Time of Flight T :

The time taken by a projectile to go from point of projection to point of impact is called "Time of flight T"

T = ?

Vi = ✓

ay = ✓

S(H) = ✓

H = 0

$V_{iy} = V_i\sin\theta$

$a_y = -g$

$$\Rightarrow \quad H = V_{iy}T + \frac{1}{2}a_yT^2$$

$$0 = V_i\sin\theta\, T + \frac{1}{2}(-g)T^2$$

$$\frac{1}{2}gT^{\cancel{2}} = V_i\sin\theta\,\cancel{T}$$

$$\frac{1}{2}gT = V_i\sin\theta$$

$$\boxed{T = \frac{2V_i\sin\theta}{g}}$$

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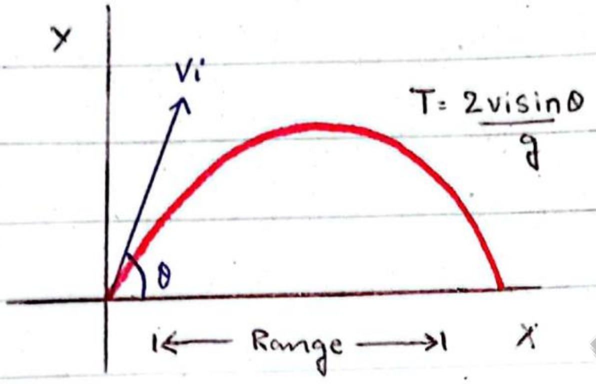
Range of Projectile :-

For finding range we have to use x component



Vertical Distance (y-axis) = Height

Horizontal Distance (x-axis) = Range / distance / displacement

S = V x t

$\Rightarrow$ $\boxed{X = V_x T}$

$R = V_x T$ $\quad \because V_x = V_i \cos\theta$

$R = \frac{V_i \cos\theta \times 2 v_i \sin\theta}{g}$ $\quad \because T = \frac{2 v_i \sin\theta}{g}$

$R = \frac{2 v_i^2 (\cos \times 2 \sin\theta)}{g}$ — (ii)

Extra

From Trigonometry R :-

Since, $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

let $\alpha = \theta$ and $\beta = \theta$

$\sin(\theta + \theta) = \sin\theta . \cos\theta + \cos\theta . \sin\theta$

$\sin(2\theta) = 1 \sin\theta . \cos\theta + 1 \sin\theta . \cos\theta$

$\boxed{\sin(2\theta) = 2 \sin\theta \cos\theta}$ — (iii)

Putting eq (iii) in eq (ii) then,

$$\boxed{R = \frac{v_i^2 \times \sin 2\theta}{g}}$$

Slo Based Question:-

Q- On what factors the value of Range depends?

Ans:- As we know from Range formula:-

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

It depend on three factor:-

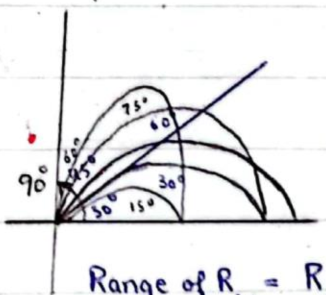
(1) Initial velocity (v_i) (2) Angle (θ)

(3) Gravity (g)

Q Which value of Angle of projection cause the maximum range?

Maximum Range:-

let $\theta = 45°$



Range of $R_{30} = R_{60°}$

$$\Rightarrow R = \frac{v_i^2 \sin(2 \times 45°)}{g} \Rightarrow \frac{v_i^2 \sin(90)°}{g}$$

$\because \sin 90° = 1$

$$R = \frac{v_i^2 \sin(1)}{g} \Rightarrow R = \frac{v_i^2}{g}$$

SAME RANGE:-

For Complementary angle the value range are equal.

(i) 30° and 60°

$$\Rightarrow R_{30°} = \frac{v_i^2 \sin(2 \times 30)}{g} = \frac{v_i^2 \sin(60)}{g} = \boxed{0.866\left(\frac{v_i^2}{g}\right)}$$

$$R_{60^\circ} = \frac{v_i^2 \sin(2\times 60^\circ)}{g} \Rightarrow R_{60^\circ} = \frac{v_i^2 \sin(120^\circ)}{g}$$

$$\boxed{R_{60^\circ} = \frac{v_i^2 (0.866)}{g}}$$

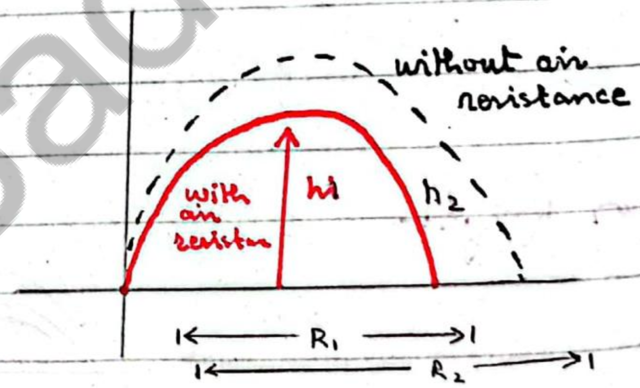
$$R_{30^\circ} = R_{60^\circ} \; ; \; R_{45^\circ} = R_{45^\circ}$$

$$R_{20^\circ} = R_{70^\circ} \; ; \; R_{10^\circ} = R_{80^\circ}$$

SLO Based Question:-

Q Show that the value of Ranges are equal for two complementary angle?

Q How the air resistance affect the projectile motion?



$h_2 > h_1$, $R_2 > R_1$

Air Resistance affect Height and Range of Projectile motion.

It also increase and decrease Time of flight.

LAW OF CONSERVATION OF MOMENTUM :-

no external force

The total momentum of an isolated System of interacting particles is conserved.

constant

$\vec{F}_{external} = 0$

$\vec{F}_{net} = 0$

$\because \vec{F} = \dfrac{\Delta \vec{P}}{\Delta t} \Rightarrow 0 = \dfrac{\Delta \vec{P}}{\Delta t}$

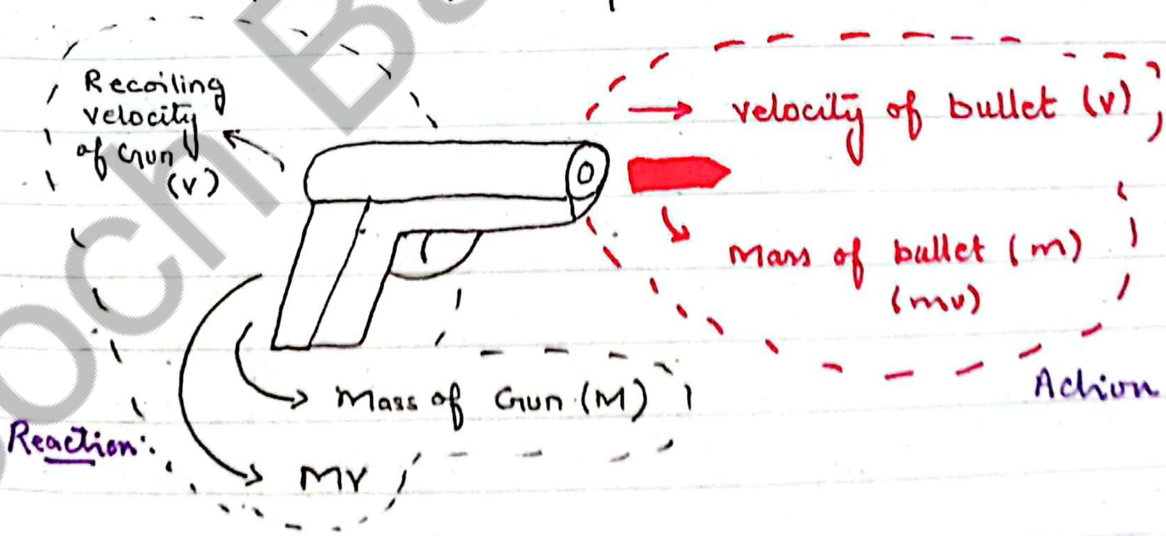
$\Delta \vec{P} = 0 \qquad \therefore \Delta \vec{P} = \vec{P}_f - \vec{P}_i$

$\Rightarrow 0 = \vec{P}_f - \vec{P}_i$

$\boxed{\Rightarrow \vec{P}_f = \vec{P}_i}$

external

∴ if force of any body is zero then momentum remain same that initial momentum is equal to final Momentum.



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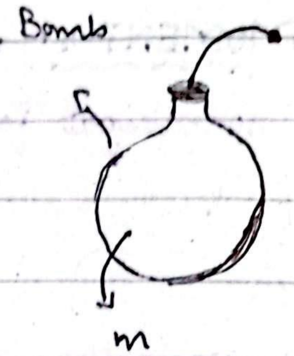
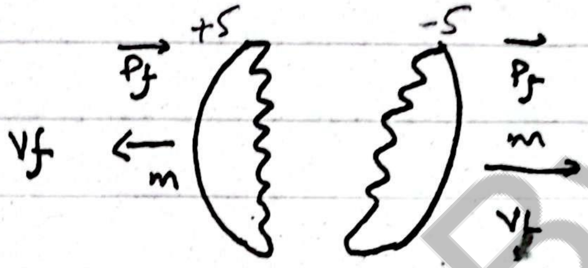


$\vec{V_i} = 0$

$\boxed{P_i = 0}$

2



$\boxed{P_f = 0}$

So $\vec{P_f} = \vec{P_i}$

$0 = 0$ So this example is also an example of law of Conservation of moment

Interaction

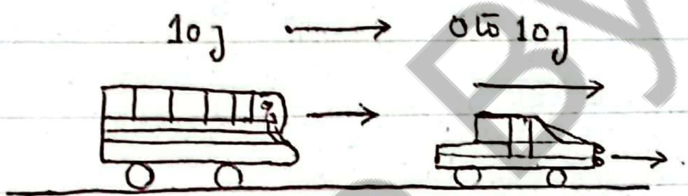
COLLISION :-

Collision is the interacting between two elements. OR

~~when 2 or more bodies~~

"The interaction between two bodies causing change in the motion of bodies."

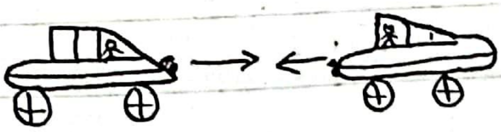
Elastic Collision:-



"Elastic collision is such a collision in which both the momentum and the kinetic energy are conserved."

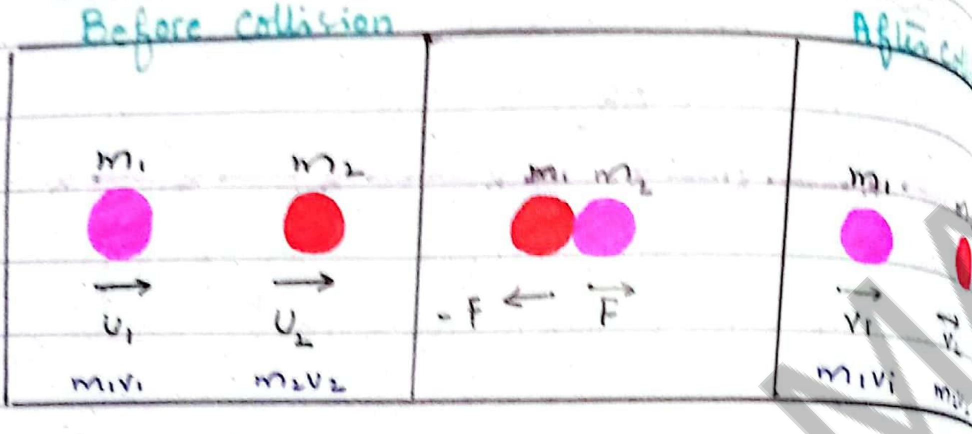
Inelastic Collision:-

"InElastic collision is such a collision in which the momentum is conserved but kinetic energy is not conserved"



Kinetic energy transfer into sound energy, Heat energy.

Elastic Collision in One dimension :-



Momentum :-

According to the law of Conservation of Momentum

$$\Rightarrow \quad P_i = P_f$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{—(i)}$$

$$\Rightarrow m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{—(ii)}$$

Kinetic Energy:-

According to the law of conservation of kinetic energy.

$$\Rightarrow K.E_i = K.E_f$$

$$\Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

After cancelling 1/2 factor rewrite above equation

$$\Rightarrow m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1+v_1)(u_1-v_1) = m_2(v_2+u_2)(v_2-u_2) \quad \text{—(iii)}$$

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Dividing equation (iii) by (ii), we get

$$\frac{\cancel{m_1}(u_1^2 - v_1^2)}{\cancel{m_1}(u_1 - v_1)} = \frac{\cancel{m_2}(v_2^2 - u_2^2)}{\cancel{m_2}(v_2 - u_2)}$$

$$\cancel{m_1}(u_1+v_1)\cancel{(u_1-v_1)} = \cancel{m_2}(u_2+v_2)\cancel{(u_1-v_1)}$$

$$u_1 + v_1 = u_2 + v_2$$

$$u_1 - u_2 = v_2 - v_1$$

$$u_1 - u_2 = -(-v_2 + v_1)$$

$$u_1 - u_2 = -(v_1 - v_2)$$

Before		After	
$+u_1$	$-u_2$	$-v_1$	$+v_2$
↑	↓	↓	↑
net change is equal in both			

velocity of approch. ← $u_1 - u_2$

velocity of separation ← $-(v_1 - v_2)$
Recession.

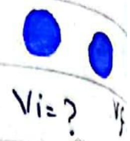
~~Thus~~ This equation states that before the collision u_1 has higher energy than u_2 and v_1 After Collision v_1 has less energy than v_2 but in net change the values remain same but there is a change of + and − value.

Example

Before collision	⟹	After collision
+10	⟶	−10
$(u_1 - u_2)$	⟶	$-(v_1 - v_2)$
$(15-5)$	⟶	$-(15-5)$
10	⟶	−10

ELASTIC COLLISION IN ONE DIMENSION:

Since,



$$u_1 - u_2 = v_2 - v_1$$

$$\Rightarrow u_1 - u_2 + v_1 = v_2$$

$$\Rightarrow \boxed{v_2 = u_1 - u_2 + v_1} \rightarrow \text{(iv)}$$

Velocity first object after collision (v_1):-

Putting eq(iv) in eq(i)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 - u_2 + v_1)$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1$$

$$m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2 = m_1v_1 + m_2v_1$$

$$m_1u_1 - m_2u_1 + 2m_2u_2 = v_1(m_1 + m_2)$$

$$u_1(m_1 - m_2) + 2m_2u_2 = v_1(m_1 + m_2)$$

$$v_1 = \frac{u_1(m_1 - m_2) + 2m_2u_2}{m_1 + m_2}$$

$$\boxed{v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2} \text{ —(v)}$$

Since,

$$\boxed{v_1 = v_2 - u_1 + u_2} \text{ — (vi)}$$

Velocity Second object after collision

Putting eq(vi) in eq(i)

$$m_1u_1 + m_2u_2 = m_1(v_2 - u_1 + u_2) + m_2v_2$$

$$m_1u_1 + m_2u_2 = m_1v_2 - m_1u_1 + m_1u_2 + m_2v_2$$

$$m_1u_1 + m_1u_1 - m_1u_2 = m_1v_2 + m_2v_2$$

$$m_2u_2 + 2m_1u_1 - m_1u_2 = v_2(m_1 + m_2)$$

$$2m_1u_1 - (m_1 - m_2)u_2 = v_2(m_1 + m_2)$$

$$\boxed{v_2 = \frac{2m_1u_1}{(m_1+m_2)} - \frac{m_1 - m_2}{m_1+m_2}u_2} \quad \text{— (vii)}$$

Cases - I :-

When colliding bodies have the same mass. i.e $\boxed{m_1 = m_2 = m}$ — (viii)

Thus from eq(vi) we get

$$v_1 = \left(\frac{\cancel{m_1} - \cancel{m_2}}{m_1 + m_2}\right)u_1 + \frac{2m}{m+m}u_2$$

$$v_1 = \left(\frac{0}{2m}\right)u_1 + \frac{\cancel{2m}}{\cancel{2m}}u_2$$

$$v_1 = 0 + u_2$$

$$\boxed{v_1 = u_2}$$

Putting given equation in eq(vii)

$$v_2 = \frac{2mu_1}{(m+m)} - \frac{\cancel{m} - \cancel{m}}{m+m}u_2$$

$$V_2 = \frac{2\cancel{m}}{2\cancel{m}} u_1 - \frac{0}{2m} u_2$$

$$V_2 = 1(u_1) - 0$$

$$\boxed{V_2 = u_1}$$

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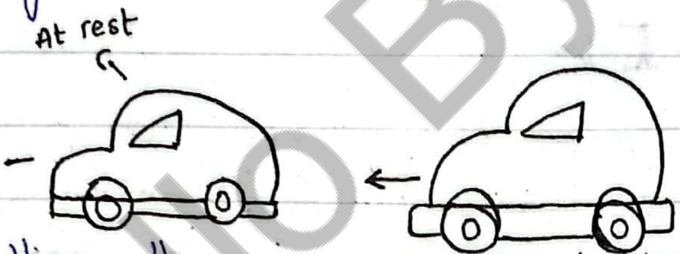
CASE-II

When colliding bodies have the same mass i.e ($m_1 = m_2 = m$) and second body (target) is at rest ($u_2 = 0$)



then putting the given condition (values) in eq (v)

$$v_1 = \left(\frac{\cancel{m}-\cancel{m}}{m+m}\right) u_1 + \frac{2m}{m+m}(0)$$

$$v_1 = 0 + \frac{2m}{2m}(0)$$

$$v_1 = 0 + 1(0)$$

$$\boxed{v_1 = 0}$$

then putting the given condition in eq (vii)

$$v_2 = \left(\frac{2m}{m+m}\right) u_1 - \left(\frac{\cancel{m_1}+\cancel{m_2}}{m+m}\right) 0$$

$$v_2 = \left(\frac{2\cancel{m}}{2\cancel{m}}\right) u_1 - 0$$

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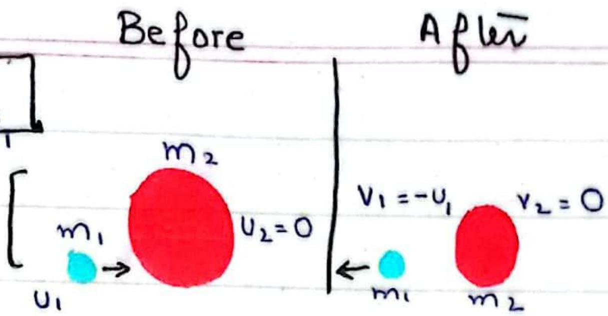
$$\boxed{v_2 = u_1}$$

Case-III

$m_2 >> m_1$



When ligher body (m_1) Collides with a massive body ($m_2 >> m_1$) at rest ($u_2 = 0$) then under such condition m_1 can be neglected

i.e:- $m_1 \approx 0$, $u_2 = 0$

So thus we put this in eq (v)

$$v_1 = \left(\frac{0 - m_2}{0 + m_2}\right) u_1 + \frac{2m_2}{(0+m_2)} 0$$

$$v_1 = \left(\frac{-\cancel{m_2}}{\cancel{m_2}}\right) u_1 + 0$$

$$\boxed{v_1 = -u_1}$$

Now we put this equation in eq (vii)

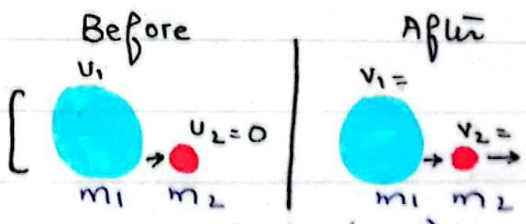
$$v_2 = \frac{2(0)}{(0+m_2)} u_1 - \left(\frac{0 - m_2}{0 + m_2}\right) 0$$

$$v_2 = \frac{0}{m_2}(u_1) - 0$$

$$\boxed{v_2 = 0}$$

Case - IV :-

$m_1 >> m_2$



When a massive body (m_1) Collides with a ligher body ($m_1 >> m_2$) at rest ($u_2 = 0$) then in such condition we neglect m_2

i.e:- $m_2 \approx 0$, $u_2 = 0$

Thus by putting this in eq (v)

$$v_1 = \left(\frac{m_1 - 0}{m_1 + 0}\right) u_1 + \frac{2(0)}{(m_1 + 0)} \times 0$$

$$v_1 = \left(\frac{\cancel{m_1}}{\cancel{m_1}}\right) u_1 + 0$$

$$\boxed{v_1 = u_1}$$

Now putting this in eq (vii)

$$v_2 = \frac{2m_1}{(m_1 + 0)} u_1 - \left(\frac{m_1 - 0}{m_1 + 0}\right) 0,$$

$$v_2 = \frac{2\cancel{m_1}}{\cancel{m_1}} (u_1) - \left(\frac{m_1}{m_1}\right) 0$$

$$v_2 = 2u_1 - 0$$

$$\boxed{v_2 = 2u_1}$$