2nd August 2024 Classwork Friday

Chapter 02:-

"Theory of Quadratic Equations"

Exercise 2.4:-

Q1:- If α, β are roots of equation $x^2+px+q=0$ then evaluate:

(i) $\alpha^2+\beta^2$

Sol:- If α, β are roots of equation $x^2+px+q=0$, therefore

$\alpha+\beta = -b/a = -p$

$\alpha\beta = c/a = q$

Now,

$\alpha^2+\beta^2$

$= \alpha^2+\beta^2+2\alpha\beta-2\alpha\beta$

$= (\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow$ Putting values of $\alpha+\beta$ & $\alpha\beta$

$= (-p)^2-2(q)$

$= p^2-2q$

(ii) $\alpha^3\beta + \alpha\beta^3$

Sol:n

* $\alpha+\beta = -b/a = -p$
* $\alpha\beta = c/a = q$

Now,

$$\alpha^3\beta + \alpha\beta^3$$

$= \alpha\beta(\alpha^2+\beta^2)$

$= \alpha\beta[\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta]$

$= \alpha\beta[(\alpha+\beta)^2-2\alpha\beta]$

* Putting values;

$= q[(-p)^2-2(q)]$

$= q(p^2-2q)$

(iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Sol:n

* $\alpha+\beta = -b/a = -p$
* $\alpha\beta = c/a = q$

Now,

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$= \frac{\alpha^2+\beta^2}{\alpha\beta}$

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$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

* Putting values;

$$= \frac{(-p)^2 - 2(q)}{q}$$

$$= \frac{p^2 - 2q}{q}$$

$$= \frac{1}{q}(p^2 - 2q)$$

Q2:- If α, β are roots of equation $4x^2 - 5x + 6 = 0$, then find values of

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$

Sol:- Since α, β are roots of equation $4x^2 - 5x + 6 = 0$ therefore

$\alpha + \beta = -b/a = -\frac{(-5)}{4} \Rightarrow 5/4$

$\alpha\beta = c/a = 6/4 \Rightarrow 3/2$

Now,

$$\frac{1}{\alpha}+\frac{1}{\beta}$$

$$= \frac{\beta+\alpha}{\alpha\beta}$$

$$\Rightarrow \frac{5/4}{3/2}$$

$$= 5/4 \times 2/3$$

$$= 10/12$$

$$= \boxed{5/6}$$

(ii) $\alpha^2\beta^2$

Sol:n As,

$\alpha+\beta = 5/4$ and $\alpha\beta = 3/2$

So,

$$\alpha^2\beta^2$$

$$= (\alpha\beta)^2$$

$$= (3/2)^2$$

$$= \boxed{9/4}$$

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(iii) $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$

Soln: ∵ $\alpha+\beta = 5/4$ & $\alpha\beta = 3/2$

So,

$$\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$$

$$= \frac{\beta+\alpha}{\alpha^2\beta^2}$$

$$= \frac{5/4}{(\alpha\beta)^2}$$

$$= \frac{5/4}{(3/2)^2}$$

$$= \frac{5}{4} \times \left(2/3\right)^2$$

$$= \frac{5}{4} \times \frac{4}{9}$$

$$= 5/9$$

(iv) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Soln: ∵ $\alpha+\beta = 5/4$ & $\alpha\beta = 3/2$

Now,

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$$

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Formula

$a^3 + b^3 \Rightarrow (a+b)(a^2 - ab + b^2)$

$$= \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)[(\alpha^2 + \beta^2 - \alpha\beta)]}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)[\alpha^2 + \beta^2 - \alpha\beta + 2\alpha\beta - 2\alpha\beta]}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)[(\alpha+\beta) - 3\alpha\beta]}{\alpha\beta}$$

* putting values;

$$= \frac{(5/4)[(5/4)^2 - 3(3/2)]}{3/2}$$

$$= \frac{(5/4)[\frac{25}{16} - 9/2]}{3/2}$$

$$= \frac{5/4\left(\frac{25-72}{16}\right)}{3/2}$$

$$= \frac{5/4\left(\frac{-47}{16}\right)}{3/2}$$

$= \dfrac{\dfrac{-235}{64}}{3/2}$

$= \dfrac{-235}{64_{32}} \times \dfrac{2^{1}}{3}$

$= \boxed{\dfrac{-235}{96}}$

Q31. If α, β are roots of equation $lx^2+mx+n=0$ $(l \neq 0)$ then find value of

(iii) $\alpha^3\beta^2 + \alpha^2\beta^3$

Sol: Since α, β are roots of equation $lx^2+mx+n=0$, therefore

- $\alpha+\beta = -b/a = \dfrac{-m}{l}$

- $\alpha\beta = c/a = \dfrac{n}{l}$

Now,

$\alpha^3\beta^2 + \alpha^2\beta^3$

$= \alpha^2\beta^2 (\alpha+\beta)$

$= (\alpha\beta)^2 (\alpha+\beta)$

* Putting values:-

$= \left(\frac{n}{l}\right)^2 \left(\frac{-m}{l}\right)$

$= \frac{n^2}{l^2} \times \frac{-m}{l}$

$= \frac{-mn^2}{l^3}$

(iii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

Sol:- $\because \alpha+\beta = -m/l$ & $\alpha\beta = n/l$

Now,

$\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$= \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$

$= \frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{(\alpha\beta)^2}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$$= \frac{\left(\frac{-m}{l}\right)^2 - 2\left(\frac{n}{l}\right)}{\left(\frac{n}{l}\right)^2}$$

$$= \frac{-m^2/l^2 - 2n/l}{\frac{n^2}{l^2}}$$

$$= \frac{-m^2}{l^2} - \frac{2n}{l} \div \frac{n^2}{l^2}$$

$$= \frac{-m^2 - 2nl}{l^2} \div \frac{n^2}{l^2}$$

$$= \frac{-m^2 - 2nl}{l^2} \times \frac{l^2}{n^2}$$

$$= \frac{m^2 - 2nl}{n^2}$$

$$= \frac{1}{n^2}(m^2 - 2nl)$$