Chapter 02:

"Theory of Quadratic Equations"

Exercise 2.3:

Q1: Without solving, find the sum and product of roots of following quadratic equations.

(i) $x^2-5x+3=0$

Sol: $a=1$, $b=-5$, $c=3$

* Sum $= -b/a$

$= \frac{-(-5)}{1} = 5$

* Product $= c/a$

$= \frac{3}{1} = 3$

(ii) $3x^2+7x-11=0$

Sol: $a=3$, $b=7$, $c=-11$

* Sum $= -b/a$

$= -7/3$

* Product $= c/a$

$= -11/3$

(iii) $px^2 - qx + r = 0$

Sol:n $a = p$, $b = -q$, $c = r$

* Sum $= -b/a$

$= \frac{-(-q)}{p} \Rightarrow q/p$

* product $= c/a$

$= r/p$

(iv) $(a+b)x^2 - ax + b = 0$

Sol:n $a = a+b$, $b = -a$, $c = b$

* Sum $= -b/a$

$= \frac{-(-a)}{a+b} = a/a+b$

* product $= c/a$

$= b/a+b$

(v) $(l+m)x^2 + (m+n)x + n-l = 0$

Sol:n $a = l+m$, $b = m+n$, $c = n-l$

* Sum $= -b/a$

$= \frac{-(m+n)}{l+m} = \frac{-m-n}{l+m} = -n/l$

* Product $= c/a$

$= \frac{n-l}{l+m} = n/m$

(vii) $7x^2 - 5mx + 9n = 0$

Sol:- $a = 7$, $b = -5m$, $c = 9n$

* Sum $= -b/a$

$= \frac{-(-5m)}{7} = 5m/7$

* Product $= c/a$

$= \frac{9n}{7}$

Q2:- Find value of k, if

(ii) Sum of the roots of equation $2kx^2 - 3x + 4k = 0$ is twice the product of roots.

Sol:- $2kx^2 - 3x + 4k = 0$

$a = 2$, $b = -3$, $c = 4$

$S = -b/a$

$= \frac{-(-3)}{2k} \Rightarrow S = \frac{3}{2k}$

$P = c/a$

$= {}^{2}\cancel{4}/\cancel{2} \rightarrow P = 2$

* According to given condition

$S = 2P$

$\frac{3}{2k} = 2(2)$

$3 = 4 \times 2k$

$3 = 8k$

$\frac{3}{8} = k$ or $k = 3/8$

ii) Sum of roots of equation $x^2 + (3k-7)x + 5k = 0$ is $\frac{3}{2}$ times the product of the roots.

Given $x^2 + (3k-7)x + 5k = 0$

$a = 1$, $b = 3k-7$, $c = 5k$

$S = -b/a$

$= \frac{-(3k-7)}{1}$

$S = -3k+7$

$P = c/a$

$= \frac{5k}{1}$

$P = 5k$

* According to given condition

$S = \frac{3}{2} P$

$-3k+7 = \frac{3}{2}(5k)$

$-3k+7 = \frac{15k}{2}$

$2(-3k+7) = 15k$

$-6k+14 = 15k$

$14 = 21k$

$\frac{14}{21} = k$

$k = 2/3$

Q3:- Find k, if

(ii) Sum of the square of roots of the equation $4kx^2+3kx-8=0$ is 2.

Sol:- $a=4k$, $b=3k$, $c=-8$

$S = -b/a$

$$= \frac{-3k}{4k}$$

$$S = -3/4$$

$$P = c/a$$

$$= \frac{-8}{4k}$$

$$P = \frac{-2}{k}$$

* According to given condition

$$\alpha^2 + \beta^2 = 2$$

$$\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta = 2$$

$$\left(\frac{-3}{4}\right)^2 - 2\left(\frac{-2}{k}\right) = 2$$

$$\frac{9}{16} + \frac{4}{k} = 2$$

$$\frac{4}{k} = \frac{2}{1} - \frac{9}{16}$$

$$\frac{4}{k} = \frac{32-9}{16}$$

$$\frac{4}{k} = \frac{23}{16}$$

$4 \times 16 = 23k$

$64 = 23k$

$\frac{64}{23} = k$

$k = \frac{64}{23}$

(iii) Sum of squares of the roots of equation $x^2 - 2kx + 2k + 1 = 0$ is 6.

Sol:n $a = 1$, $b = -2k$, $c = 2k+1$

$S = -b/a$

$= \frac{-(-2k)}{1}$

$= S = 2k$

$P = c/a$

$= \frac{2k+1}{1}$

$P = 2k+1$

* According to given condition

$\alpha^2 + \beta^2 = 6$

$\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta = 6$

$(\alpha+\beta)^2 - 2\alpha\beta = 6$

$(2k)^2 - 2(2k+1) = 6$

$4k^2 - 4k - 2 = 6$

$4k^2 - 4k - 2 - 6 = 0$

$4k^2 - 4k - 8 = 0$

$4(k^2 - k - 2) = 0$

$k^2 - k - 2 = 0$

$k^2 - 2k + 1k - 2 = 0$

$k(k-2) + 1(k-2) = 0$

$(k+1)(k-2) = 0$

$k - 2 = 0 \quad , \quad k + 1 = 0$

$k = 2 \quad , \quad k = -1$

Q4: Find p if

(ii) The roots of equation $x^2 - x + p^2 = 0$ differ by unity.

Sol: $x^2 - x + p = 0$

$a = 1 \ , b = -1 \ , c = p$

let $\alpha, \alpha - 1$ are roots

then, $S = \alpha + (\alpha - 1) = -b/a$

$\Rightarrow \alpha + \alpha - 1 = \frac{-(-1)}{1}$

$2\alpha - 1 = 1$

$2\alpha = 1+1$

$\alpha = 2/2$

$\alpha = 1$ ...(i)

and

$P = (\alpha)(\alpha-1) = c/a$

$\alpha^2 - \alpha = P/1$

$\alpha^2 - \alpha = P$ ...(ii)

Putting value of α from eq (i) & (ii)

$\alpha^2 - \alpha = P$

$(1)^2 - 1 = P$

$1 - 1 = P$

$0 = P$

Now,

$P = 0$

(iii) The roots of equation $x^2 + 3x + p - 2 = 0$ differ by 2.

Sol: $a = 1$, $b = 3$, $c = p - 2$

let α, $\alpha - 2$ are roots then

$S = \alpha + (\alpha - 2) = -b/a$

$\Rightarrow \alpha + \alpha - 2 = -3/1$

$2\alpha - 2 = -3$

$2\alpha = -3 + 2$

$\alpha = \frac{-1}{2}$ ... (i)

$$P = (\alpha)(\alpha - 2) = c/a$$

$$\alpha^2 - 2\alpha = \frac{P-2}{1}$$

$$\alpha^2 - 2\alpha = P - 2 \quad \ldots (iii)$$

putting value of α from eq (i) in eq (ii)

$$\alpha^2 - 2\alpha = P - 2$$

$$\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) = P - 2$$

$$\frac{1}{4} + 1 = P - 2$$

$$\frac{1}{4} + 1 + 2 = P$$

$$\frac{1}{4} + 3 = P$$

$$\frac{1+12}{4} = P$$

$$P = \frac{13}{4}$$

Q5:- Find m if,

(ii) the roots of equation $x^2 - 7x + 3m - 5 = 0$ satisfy the relation $3\alpha + 2\beta = 4$.

Sol:- $a = 1$, $b = -7$, $c = 3m - 5$

let α, β are roots then,

$$S = \alpha + \beta = -b/a$$

$$= \frac{-(-7)}{1}$$

$$\alpha + \beta = 7 \quad \ldots (i)$$

$$P = \alpha\beta = c/a$$

$$= \frac{3m-5}{1}$$

$$\alpha\beta = 3m - 5 \quad \ldots (ii)$$

From eq (i)

$$\alpha + \beta = 7$$

$$\alpha = 7 - \beta$$

* Given relation :-

$$3\alpha + 2\beta = 4 \quad \ldots (iii)$$

Putting α from eq (i) in eq (iii)

$$3(7 - \beta) + 2\beta = 4$$

$$21 - 3\beta + 2\beta = 4$$

$$-\beta = 4 - 21$$

$-\beta = -17$

$\beta = 17$

Putting value of β in eq (i)

$\alpha + \beta = 7$

$\alpha + 17 = 7$

$\alpha = 7 - 17$

$\alpha = -10$

Putting values of α & β in eq (ii)

$\alpha\beta = 3m - 5$

$(-10)(17) = 3m - 5$

$-170 = 3m - 5$

$-170 + 5 = 3m$

$\frac{-165}{3} = m$

$m = -55$

(iii) The roots of equation $x^2 + 7x + 3m - 5 = 0$ satisfy the relation $3\alpha - 2\beta = 4$.

Solution: $a = 1$, $b = 7$, $c = 3m - 5$

let α, β are roots then

$S = \alpha + \beta = -b/a$

$= -7/1$

$\alpha + \beta = -7$ ... (i)

$P = \alpha\beta = c/a$

$= \frac{3m-5}{1}$

$\alpha\beta = 3m-5 \quad \ldots (ii)$

From eq (i)

$\alpha+\beta = -7$

$\beta = -7-\alpha$

* Given relation:-

$3\alpha - 2\beta = 4 \quad \ldots (iii)$

Putting β from eq (i) in eq (iii)

$3\alpha - 2\beta = 4$

$3\alpha - 2(-7-\alpha) = 4$

$3\alpha + 14 + 2\alpha = 4$

$5\alpha = -14+4$

$\alpha = \frac{-10}{5}$

$\alpha = -2$

Putting value of α in eq (i)

$\alpha + \beta = -7$

$-2 + \beta = -7$

$\beta = -7+2$

$\beta = -5$

Putting values of α, β in eq (iii)

$$\alpha\beta = 3m - 5$$

$$(-2)(-5) = 3m - 5$$

$$5 + 10 = 3m$$

$$\frac{15}{3} = m$$

$$m = 5$$

(ii) the roots of equation $3x^2 - 2x + 7m + 2 = 0$ satisfy the relation $7\alpha - 3\beta = 18$.

Sol: $a = 3$, $b = -2$, $c = 7m + 2$

let α, β are roots then

$$S = \alpha + \beta = -b/a$$

$$= -\frac{(-2)}{3}$$

$$\alpha + \beta = 2/3 \quad \ldots (i)$$

$$P = \alpha\beta = c/a$$

$$\alpha\beta = \frac{7m+2}{3} \quad \ldots (ii)$$

From eq (1)

$$\alpha + \beta = 2/3$$

$$\alpha = 2/3 - \beta$$

Scanned with CamScanner

* Given relation :-

$$7\alpha - 3\beta = 18 \quad \dots \text{(iii)}$$

Putting α value from eq (i) in eq (iii)

$$7\alpha - 3\beta = 18$$

$$7\left(\frac{2}{3} - \beta\right) - 3\beta = 18$$

$$\frac{14}{3} - 7\beta - 3\beta = 18$$

$$-10\beta = 18 - \frac{14}{3}$$

$$-10\beta = \frac{54 - 14}{3}$$

$$-10\beta = \frac{40}{3}$$

$$\beta = \frac{40}{3 \times 10}$$

$$\beta = \frac{-4}{3}$$

Putting β in eq (i)

$$\alpha + \beta = \frac{2}{3}$$

$$\alpha + \left(\frac{-4}{3}\right) = \frac{2}{3}$$

Scanned with CamScanner

$\alpha - \frac{4}{3} = 2/3$

$\alpha = \frac{2+4}{3}$

$\alpha = 6/3$

$\alpha = 2$

put α & β values in eq (ii)

$\alpha\beta = \frac{7m+2}{3}$

$2\left(\frac{-4}{3}\right) = \frac{7m+2}{3}$

$\cancel{3} \times \frac{-8}{\cancel{3}} = \frac{7m+2}{\cancel{3}} \times \cancel{3}$

$-8 = 7m+2$

$-8-2 = 7m$

$-10 = 7m$

$\boxed{m = \frac{-10}{7}}$

Q6:- Find m, if sum and product of roots of following equation is equal to a given number. (λ)

(ii) $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

Sol:- $a = 2m+3$, $b = 7m-5$, $c = 3m-10$

* Let α, β are roots then

$S = \alpha+\beta = -b/a$

$\alpha+\beta = \dfrac{-(7m-5)}{2m+3}$ ... (i)

$P = \alpha\beta = c/a$

$\alpha\beta = \dfrac{3m-10}{2m+3}$ ... (ii)

Since sum and product of root is equal to a number λ, then

$\alpha+\beta = \lambda$ & $\alpha\beta = \lambda$

So, $\alpha+\beta = \alpha\beta$

* Putting values ...

$(2m+3) \times \dfrac{-(7m-5)}{2m+3} = \dfrac{3m-10}{2m+3} \times (2m+3)$

$-(7m-5) = 3m-10$

$-7m+5 = 3m-10$

$-7m-3m = -10-5$

$-10m = -15$

$m = \dfrac{15}{10}$

$m = 3/2$

$4x^2 - (3+5m)x - (9m-17) = 0$

$a = 4$, $b = -(3+5m)$, $c = -(9m-17)$

let α, β are roots then

$S = \alpha + \beta = -b/a$

$= \frac{-[-(3+5m)]}{4}$

$\alpha + \beta = \frac{3+5m}{4}$

$P = \alpha\beta = c/a$

$= \frac{-(9m-17)}{4}$

$\alpha\beta \Rightarrow \frac{-9m-17}{4}$

Since sum & product of root is equal to a number λ, then

$\alpha + \beta = \lambda$ &

$\alpha\beta = \lambda$

So,

$\alpha + \beta = \alpha\beta$

* Putting values ...

$\frac{3+5m}{\cancel{4}} = \frac{-9m+17}{\cancel{4}}$

$$3+5m=-9m+17$$

$$5m+9m=17-3$$

$$14m=14$$

$$m=14/14$$

$$m=1$$