Chapter 02:

Theory of Quadratic Equation

Exercise 2.1:

Q1: Find the discriminant of following quadratic equation:

(i) $2x^2+3x-1=0$

Sol: $2x^2+3x-1=0$

$ax^2+bx+c=0$

$a=2$, $b=3$, $c=-1$

* $Disc = b^2-4ac$

$= (3)^2 - 4(2)(-1)$

$= 9+8$

$= 17$

Discriminant are unequal, irrational & real.

(ii) $6x^2-8x+3=0$

Sol: $6x^2-8x+3=0$

$ax^2+bx+c=0$

$a=6$, $b=-8$, $c=3$

* $Disc = b^2-4ac$

$= (-8)^2 - 4(6)(3)$

$= 64 - 72$

$= -8$

(iii) $9x^2 - 30x + 25 = 0$

Sol: $9x^2 - 30x + 25 = 0$

$ax^2 + bx + c = 0$

$a = 9, \quad b = -30, \quad c = 25$

★ $Disc = b^2 - 4ac$

$= (-30)^2 - 4(9)(25)$

$= 900 - 900$

$= 0$

(iv) $4x^2 - 7x - 2 = 0$

Sol: $4x^2 - 7x - 2 = 0$

$ax^2 + bx + c = 0$

$a = 4, \quad b = -7, \quad c = -2$

★ $Disc = b^2 - 4ac$

$= (-7)^2 - 4(4)(-2)$

$= 49 + 32$

$= 81$

Q2: Find the nature of roots and verify...

(ii) $x^2 - 23x + 120 = 0$

Sol: $a = 1$, $b = -23$, $c = 120$

* Disc $= b^2 - 4ac$

$= (-23)^2 - 4(1)(120)$

$= 529 - 480$

$= 49$

* As the disc is positive & perfect square, therefore the roots are real, rational & unequal.

Verification:

• By Quadratic Formula:

$$= \frac{-(-23) \pm \sqrt{(-23)^2 - 4(1)(120)}}{2(1)}$$

$$= \frac{23 \pm \sqrt{529 - 480}}{2}$$

$$= \frac{23 \pm 7}{2}$$

$$= \frac{23+7}{2} , \frac{23-7}{2}$$

$= 15$, 8 《So roots are real, rational & unequal》

(ii) $2x^2+3x+7=0$

Sol:- $a=2$, $b=3$, $c=7$

* $Disc = b^2-4ac$

$= (3)^2-4(2)(7)$

$= 9-56$

$= -47$

* The roots are imagenary and unequal as $-47 < 0$.

* Verification :-

* By Q.F :-

$$= \frac{-(3) \pm \sqrt{(3)^2-4(2)(7)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{9-56}}{4}$$

$$= \frac{-3 \pm 47}{4}$$

$$= \frac{-3+47}{4} \, , \, \frac{-3-47}{4}$$

$= 44/4$, $-50/4$

$= 11$, $-25/2$

So the roots are imagenary and unequal.

(iii) $16x^2 - 24x + 9 = 0$

Sol: $a = 16$, $b = -24$, $c = 9$

* Disc $= b^2 - 4ac$

$= (-24)^2 - 4(16)(9)$

$= 576 - 576$

$= 0$

* The roots are rational and equal.

* Verification :-

* By Q.F ,

$$= \frac{-(-24) \pm \sqrt{(-24)^2 - 4(16)(9)}}{2(16)}$$

$$= \frac{24 \pm \sqrt{576 - 576}}{32}$$

$$= \frac{24 \pm 0}{32}$$

$$= \frac{24}{32} , \frac{24}{32}$$

$$= 3/4 , 3/4$$

Hence the roots are rational & equal.

(iv) $3x^2+7x-13=0$

Sol:- $a=3$, $b=7$, $c=-13$

★ Disc $= b^2-4ac$

$= (7)^2-4(3)(-13)$

$= 49+156$

$= 205$

★ Roots are irrational and unequal.

★ Verification:-

★ By Q.F,

$$= \frac{-(7) \pm \sqrt{(7)^2-4(3)(-13)}}{2(3)}$$

$$= \frac{-7 \pm \sqrt{205}}{6}$$

$$= \frac{-7+\sqrt{205}}{6}, \frac{-7-\sqrt{205}}{6}$$

Hence the roots are irrational and unequal.

Q3. For what value of k, the expression

$k^2x^2+2(k+1)x+4$ is a perfect square.

Sol: $a=k^2$, $b=2(k+1)$, $c=4$

* $Disc = b^2-4ac$

$= [2(k+1)]^2-4(k^2)(4)$

$= (2k+2)^2-16k^2$

$= [(2k)^2+2(2k)(2)+(2)^2]-16k^2$

$= 4k^2+8k+4-16k^2$

$= -12k^2+8k+4=0$

* Taking "-4" common;

$= -4(3k^2+2k+1)=0$

$3k^2+2k+1=0$

$3k^2+3k-1k+1=0$

$3k(k+1)-1(k+1)=0$

$(k+1)(3k-1)=0$

$k+1=0$, $3k-1=0$

$k=-1$, $3k=1$

, $k=1/3$

So the values of k are -1 & $1/3$.

Q4. Find value of K, if roots of following equations are equal.

(ii) $(2k-1)x^2+3kx+3=0$

Sol:

$a=(2k-1)$, $b=3k$, $c=3$

* Disc $= b^2-4ac$

$= (3k)^2-4(2k-1)(3)$

$= 9k^2-12(2k-1)$

$= 9k^2-24k+12$

→ As the roots are equal, therefore disc will be equal to zero.

* Disc $= 0$

$9k^2-24k+12=0$

$3(3k^2-8k+4)=0$

$3k^2-8k+4=0$

$3k^2-6k-2k+4=0$

$3k(k-2)-2(k-2)=0$

$3k-2=0$, $k-2=0$

$3k=2$,

$k=2/3$, $k=2$

Thus, the values of k are 2/3 & 2.

(iii) $x^2 + 2(k+2)x + (3k+4) = 0$

Sol:-

$a = 1$, $b = 2(k+2)$, $c = 3k+4$

* Disc : $b^2 - 4ac$

$= [2(k+2)]^2 - 4(1)(3k+4)$

$= 4(k+2)^2 - 4(3k+4)$

$= 4[(k)^2 + 2(k)(2) + (2)^2] - 12k - 16$

$= 4(k^2 + 4k + 4) - 12k - 16$

$= 4k^2 + 16k + \cancel{16} - 12k - \cancel{16}$

$= 4k^2 + 4k$

* As the roots are equal, therefore disc will be equal to zero.

* Disc $= 0$

$4k^2 + 4k = 0$

* Taking "$4k$" common:-

$4k(k+1) = 0$

$4k = 0$, $k + 1 = 0$

$k = 0/4$,

$k = 0$, $k = -1$

So the values of k are 0 & -1.

(iii) $(3k+2)x^2 - 5(k+1)x + (2k+3) = 0$

Sol:n

$a = 3k+2$, $b = -5(k+1)$, $c = 2k+3$

* Disc $= b^2 - 4ac$

$= [-5(k+1)]^2 - 4(3k+2)(2k+3)$

$= 25(k+1)^2 - 4[3k(2k+3) + 2(2k+3)]$

$= 25[(k)^2 + 2(k)(1) + (1)^2] - 4[6k^2 + 9k + 4k + 6]$

$= 25(k^2 + 2k + 1) - 4(6k^2 + 13k + 6)$

$= 25k^2 + 50k + 25 - 24k^2 - 52k - 24$

$= k^2 - 2k + 1$

* As roots are equal so disc is equal to zero.

• Disc $= 0$

$k^2 - 2k + 1 = 0$

$k^2 - k - k + 1 = 0$

$k(k-1) - 1(k-1) = 0$

$(k-1)(k-1) = 0$

$k - 1 = 0$, $k - 1 = 0$

$k = 1$, $k = 1$

So the values of k are 1 & 1.

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Q8:- Show that the equation $x^2+(mx+c)^2=a^2$ has equal roots if $c^2=a^2(1+m^2)$.

Sol:- $x^2+(mx+c)^2=a^2$

$x^2+[(mx)^2+2(mx)(c)+(c)^2]=a^2$

$x^2+m^2x^2+2mxc+c^2=a^2$

$(1+m^2)x^2+2mcx+c^2-a^2=0$

$a=1+m^2 \quad , \quad b=2mc \quad , \quad c=c^2-a^2$

* Disc $=b^2-4ac$

$=(2mc)^2-4(1+m^2)(c^2-a^2)$

$=4m^2c^2-4[1(c^2-a^2)+m^2(c^2-a^2)]$

$=4m^2c^2-4(c^2-a^2+m^2c^2-m^2a^2)$

$=\cancel{4m^2c^2}-4c^2-4a^2-\cancel{4m^2c^2}-4m^2a^2$

$=-4c^2-4a^2-4m^2a^2$

* Taking "-4" common:-

$-4(c^2+a^2+m^2a^2)=0$

$c^2+a^2+m^2a^2=0$

$c^2+a^2(1+m^2)=0$

* According to given condition:-

$c^2=a^2(1+m^2)$

Hence proved that

L.H.S = R.H.S

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Q6:- Find the condition that the roots of equation $(mx+c)^2-4ax=0$ are equal.

Sol:- $(mx+c)^2-4ax=0$

$[(mx)^2+2(mx)(c)+(c)^2]-4ax=0$

$m^2x^2+2mxc+c^2-4ax=0$

* Re-arrange:

$m^2x^2+2mxc-4ax+c^2=0$

$m^2x^2+(2mc-4a)x+c^2=0$

$a=m^2$, $b=2mc-4a$, $c=c^2$

* Disc $=b^2-4ac$

$=(2mc-4a)^2-4(m^2)(c^2)$

$=[(2mc)^2-2(2mc)(4a)+(4a)^2]-4m^2c^2$

$=\cancel{4m^2c^2}-16mca+16a^2-\cancel{4m^2c^2}$

$=-16mca+16a^2=0$

$-16a(mc-a)=0$

$-16a=0$, $mc-a=0$

$a=0$, $mc=0$

$a=mc$ is the only condition for equal roots.

Q7. If the roots of equation $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$ are equal then $a=0$ or $a^3+b^3+c^3=3abc$.

Solution: $a=c^2-ab$, $b=-2(a^2-bc)$, $c=b^2-ac$

* Disc: b^2-4ac

$=[-2(a^2-bc)]^2-4(c^2-ab)(b^2-ac)$

$=4(a^2-bc)^2-4[c^2(b^2-ac)-ab(b^2-ac)]$

$=4[(a^2)^2-2(a^2)(bc)+(bc)^2]-4(c^2b^2-ac^3-ab^3+a^2bc)$

$=4(a^4-2abc+b^2c^2)-4c^2b^2+4ac^3+4ab^3-4a^2bc$

$=4a^4-8a^2bc+\cancel{4b^2c^2}-\cancel{4c^2b^2}+4ac^3+4ab^3-4a^2bc$

$=4a^4-12a^2bc+4ac^3+4ab^3$

* Taking "4a" common

$=4a(a^3-3abc+c^3+b^3)$

If roots are equal then

Disc $=0$

$=(4a)(a^3-3abc+c^3+b^3)=0$

$4a=0$, $a^3-3abc+c^3+b^3=0$

$\boxed{a=0}$, $a^3+b^3+c^3=3abc$

Proved that $a=0$ & $a^3+b^3+c^3=3abc$.

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Q8:- Show that roots of following equations are rational.

(ii) $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$

Sol:- $a = a(b-c)$, $b = b(c-a)$, $c = c(a-b)$

* Disc $= b^2 - 4ac$

$= [b(c-a)]^2 - 4[a(b-c)][c(a-b)]$

$= b^2(c-a)^2 - 4(ab-ac)(ac-bc)$

$= b^2[(c)^2 - 2(c)(a) + (a)^2] - 4[ab(ac-bc) - ac(ac-bc)]$

$= b^2(c^2 - 2ac + a^2) - 4(a^2bc - ab^2c - a^2c^2 - abc^2)$

$= b^2c^2 - 2ab^2c + a^2b^2 - 4a^2bc + 4ab^2c + 4a^2c^2 + 4abc^2$

$= b^2c^2 + 2ab^2c + a^2b^2 - 4a^2bc + 4a^2c^2 - 4abc^2$

$= (bc)^2 + (ab)^2 + (2ac)^2 + 2(bc)(ab) + 2(ab)(2ac) + 2(2ac)($

$= [bc + ab + (-2ac)]^2$

$(bc + ab - 2ac)^2 = 0$

The roots are rational.

(ii) $(a+2b)x^2 + 2(a+b+c)x + (a+2c) = 0$

Sol: $a = a+2b$, $b = 2(a+b+c)$, $c = a+2c$

* Disc $= b^2 - 4ac$

$= [2(a+b+c)]^2 - 4(a+2b)(a+2c)$

$= (a+b+c)^2 - 4[a(a+2c) + 2b(a+2c)]$

$= 4(a^2+b^2+c^2+2ab+2bc+2ca) - 4(a^2+2ac+2ba+4bc)$

$= \cancel{4a^2} + 4b^2 + 4c^2 + \cancel{8ab} + 8bc + \cancel{8ca} - \cancel{4a^2} - \cancel{8ac} - \cancel{8ba} - 16bc$

$= 4b^2 + 4c^2 - 8bc$

$= 4(b^2 + c^2 - 2bc)$

$= 4(b-c)^2$

$= [2(b-c)]^2$

Hence, the roots are rational.

Q:N For all values of k, prove that the roots of equation $x^2 - 2\left(k + \frac{1}{k}\right)x + 4 = 0$, are real.

Sol:N $a = 1$, $b = -2\left(k + \frac{1}{k}\right)$, $c = 4$

* Disc $= b^2 - 4ac$

$= \left[-2\left(k + \frac{1}{k}\right)\right]^2 - 4(1)(4)$

$= 4\left(k + \frac{1}{k}\right)^2 - 16$

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$= 4\left[k^2 + 2(\cancel{k})\left(\frac{1}{\cancel{k}}\right) + \left(\frac{1}{k}\right)^2\right] - 16$

$= 4\left(k^2 + 2 + \frac{1}{k^2}\right) - 16$

$= 4k^2 + 8 + \frac{4}{k^2} - 16$

$= 4k^2 - 8 + \frac{4}{k^2}$

$= 4\left(k^2 - 2\ {}^{1}/_{k^2}\right)$

$= 4\left(k - \frac{1}{k}\right)^2$

$\Rightarrow \left[2\left(k - \frac{1}{k}\right)\right]^2 > 0$

which is positive for all values of k and it shows roots are real.

Q10:- Show that roots of equation $(b-c)x^2 + (c$

$+ (a-b) = 0$ are real.

Sol:- $a = b-c$, $b = c-a$, $c = a-b$

* Disc $= b^2 - 4ac$

$= (c-a)^2 - 4(b-c)(a-b)$

$= [(c)^2 - 2(c)(a) + (a)^2] - 4[b(a-b) - c(a-b)]$

$= c^2 - 2ac + a^2 - 4(ba - b^2 - ca + bc)$

$= c^2 - 2ac + a^2 - 4ab + 4b^2 + 4ac - 4bc$

$= c^2 + 2ac + a^2 - 4ab + 4b^2 - 4bc$

$= (c)^2 + (a)^2 + (2b)^2 + 2(c)(a) + 2(a)(2b) + 2(2b)(c)$

$= (c)^2 + (a)^2 + (-2b)^2 + 2ac - 4ab - 4bc$

$= (c + a - 2b)^2$

So, The roots of given equation are real.