3rd July 2024 ClassWork Wednesday

Chapter 02 :N "Quadratic Equations"

Exercise 2.4 :N

* **Introduction :N**

* **Types of radical equation :N**

① $\sqrt{ax+b} = cx+d$ checking.

② $\sqrt{x+a} + \sqrt{x+b} = \sqrt{x+c}$ checking

③ $\sqrt{x^2+px+m} + \sqrt{x^2+px+n} = q$ no checking

Q1 :N (i) $2x+5 = \sqrt{7x+16}$

Sol :N Squaring both sides :N

$$(2x+5)^2 = \left(\sqrt{7x+16}\right)^2$$

$$[(2x)^2 + 2(2x)(5) + (5)^2] = 7x+16$$

$$4x^2 + 20x + 25 = 7x+16$$

$$4x^2 + 20x - 7x + 25 - 16 = 0$$

$$4x^2 + 13x + 9 = 0$$

$$4x^2 + 4x + 9x + 9 = 0$$

$$4x(x+1) + 9(x+1) = 0$$

$(4x+9)(x+1)=0$

$4x+9=0$, $x+1=0$

$4x=-9$,

$x=-9/4$, $x=-1$

* CHECK:

* Put $x=-9/4$ in original equation:

$$2\left(\frac{-9}{4}\right)+5=\sqrt{7\left(\frac{-9}{4}\right)+16}$$

$$=\frac{-9}{2}+5=\sqrt{\frac{-63}{4}+16}$$

$$=\frac{-9+10}{2}=\sqrt{\frac{-63+64}{4}}$$

$$=\frac{1}{2}=\sqrt{\frac{1}{4}}$$

$$=1/2=1/2 \qquad \text{L.H.S} = \text{R.H.S}$$

* Put $x=-1$ in original equation:

$$2(-1)+5=\sqrt{7(-1)+16}$$

$$-2+5=\sqrt{-7+16}$$

$$3=3 \qquad \text{L.H.S} = \text{R.H.S}$$

Sol set $=\{-9/4, -1\}$

(iii) $\sqrt{x+3} = 3x-1$

Sol:N Squaring both sides:N

$(\sqrt{x+3})^2 = (3x-1)^2$

$x+3 = [(3x)^2 - 2(3x)(1) + (1)^2]$

$x+3 = 9x^2 - 6x + 1$

$x + 6x + 3 - 1 = 9x^2$

$7x + 2 = 9x^2$

$+9x^2 + 7x - 2 = 0$

$+9x^2 + 9x + 2x + 2 = 0$

$9x(x+1) - 2(x+1) = 0$

$(9x-2)(x+1) = 0$

$9x+2=0 \quad , \quad x+1=0$

$9x = 2$

$x = -2/9 \quad , \quad x = -1$

* CHECK:N

Put $x = -2/9$

$\sqrt{-2/9 + 3} = 3(2/9) - 1$

$\sqrt{\frac{-2+27}{9}} = \frac{-6}{9_3} - 1$

$\sqrt{\frac{25}{9}} = \frac{-2-3}{3}$

$\frac{5}{3} \neq -\frac{5}{3}$

Not true

* Put $x=1$,

$\sqrt{1+3} = 3(1)-1$

$\sqrt{4} = 3-1$

$2=2$ which is true

Sol set = $\{1\}, \{-2/9\}$ extraneous

(iii) $4x = \sqrt{13x+14} - 3$

Sol: Re-arrange

$4x+3 = \sqrt{13x+14}$

Squaring both sides,

$(4x+3)^2 = (\sqrt{13x+14})^2$

$[(4x)^2 + 2(4x)(3) + (3)^2] = 13x+14$

$16x^2 + 24x + 9 = 13x+14$

$16x^2 + 24x - 13x + 9 - 14 = 0$

$16x^2 + 11x - 5 = 0$

$16x^2 + 16x - 5x - 5 = 0$

$16x(x+1) - 5(x+1) = 0$

$(16x-5)(x+1) = 0$

$16x - 5 = 0$, $x + 1 = 0$

$16x = 5$,

$x = 5/16$, $x = -1$

• CHECK:-

* Put $x = -1$,

$4(-1) = \sqrt{13(-1) + 14} - 3$

$-4 = \sqrt{-13 + 14} - 3$

$-4 = \sqrt{1} - 3$

$-4 \neq -2$ Not true because it is extraneous root

* Put $x = 5/16$,

$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right) + 14} - 3$

$\frac{20}{16} = \sqrt{\frac{65}{16} + 14} - 3$

$\frac{5}{4} = \sqrt{\frac{65 + 224}{16}} - 3$

$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3$

$\frac{5}{4} = \frac{17}{4} - 3$

$\frac{5}{4} = \frac{17-12}{4}$

$\frac{5}{4} = \frac{5}{4}$ true

Sol set = {5/16} , {-1 extraneous}

$\sqrt{3x+100} - x = 4$

$\sqrt{3x+100} = x+4$

Squaring both sides,

$(\sqrt{3x+100})^2 = (x+4)^2$

$3x+100 = [(x)^2 + 2(x)(4) + (4)^2]$

$3x+100 = x^2 + 8x + 16$

$x^2 + 8x + 16 - 3x - 100 = 0$

$x^2 + 5x - 84 = 0$

$x^2 + 12x - 7x - 84 = 0$

$x(x+12) - 7(x+12) = 0$

$(x+12)(x-7) = 0$

$x+12 = 0$, $x-7 = 0$

$x = -12$, $x = 7$

CHECK :-

* Put $x = -12$,

$\sqrt{3(-12)+100} - (-12) = 4$

$\sqrt{-36+100} + 12 = 4$

$\sqrt{64} + 12 = 4$

$8 + 12 = 4$

$20 \neq 4$ not true ((extraneous root))

* Put $x = 7$,

$\sqrt{3(7)+100} - (7) = 4$

$\sqrt{21+100} - 7 = 4$

$\sqrt{121} - 7 = 4$

$11 - 7 = 4$

$4 = 4$ True

Sol set = $\{7\}$, $\{-12$ extraneous$\}$

((V)) $\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$

Sol:n Squaring both sides,

$\left(\sqrt{x+5} + \sqrt{x+21}\right)^2 = \left(\sqrt{x+60}\right)^2$

$\left(\sqrt{x+5}\right)^2 + \left(\sqrt{x+21}\right)^2 + 2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right) = x+60$

$x+5+x+21 + 2\left(\sqrt{x^2+21x+5x+105}\right) = x+60$

$2x+26 + 2\sqrt{x^2+26x+105} = x+60$

$$\sqrt{x^2+26x+105} = x-2x+60-26$$

$$\sqrt{x^2+26x+105} = -x+34$$

$$\sqrt{x^2+26x+105} = 34-x$$

$$4(x^2+26x+105) = (34)^2+(x)^2-2(34)(x)$$

$$4x^2+104x+420 = 156+x^2-68x$$

$$4x^2-x^2+104x+68x+420-156=0$$

$$3x^2+172x-736=0$$

$$a=3 \quad , b=172 \quad , c=-736$$

* By Quadratic formula:-

$$= \frac{-(172)\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$$

$$= \frac{-172\pm\sqrt{29584+8832}}{6}$$

$$= \frac{-172\pm\sqrt{38416}}{6}$$

$$= \frac{-172\pm196}{6}$$

$$= \frac{-172+196}{6} \quad , \frac{-172-196}{6}$$

$= \frac{24}{6}, \frac{-368}{6}$

$= 4, \frac{-184}{3}$

* CHECK:-

Put $x = 4$,

$\sqrt{4+5} + \sqrt{4+21} = \sqrt{4+60}$

$\sqrt{9} + \sqrt{25} = \sqrt{64}$

$3 + 5 = 8$

$8 = 8$ True

Put $x = \frac{-184}{3}$,

$\sqrt{\left(\frac{-184}{3}\right)+5} + \sqrt{\left(\frac{-184}{3}\right)+21} = \sqrt{\left(\frac{-184}{3}\right)+60}$

$\sqrt{\frac{-184+15}{3}} + \sqrt{\frac{-184+63}{3}} = \sqrt{\frac{-184+180}{3}}$

$\sqrt{\frac{-169}{3}} + \sqrt{\frac{-121}{3}} = \sqrt{-4/3}$

$\frac{\sqrt{-1 \times 169}}{\sqrt{3}} + \frac{\sqrt{-1 \times 121}}{\sqrt{3}} = \frac{\sqrt{-1 \times 4}}{3}$

$\frac{13i}{\sqrt{3}} + \frac{11i}{\sqrt{3}} = \frac{2i}{\sqrt{3}}$

$$\frac{13i + 11i}{\sqrt{3}} = \frac{2i}{\sqrt{3}}$$

$$\frac{24i}{\sqrt{3}} \neq \frac{2i}{\sqrt{3}} \quad \text{not true}$$

Sol set = $\{4\}$, $\{\frac{-184}{3}$ extraneous$\}$

ii) $\sqrt{x+1} + \sqrt{x-2} = \sqrt{x+6}$

Sol^n

Squaring* both sides*,

$$\left(\sqrt{x+1} + \sqrt{x-2}\right)^2 = \left(\sqrt{x+6}\right)^2$$

$$\left(\sqrt{x+1}\right)^2 + \left(\sqrt{x-2}\right)^2 + 2\left(\sqrt{x+1}\right)\left(\sqrt{x-2}\right) = x+6$$

$$x+1 + x-2 + 2\left(\sqrt{x^2-2x+x-2}\right) = x+6$$

$$2x-1+2\sqrt{x^2-x-2} = x+6$$

$$2\sqrt{x^2-x-2} = x-2x+6+1$$

$$2\sqrt{x^2-x-2} = -x+7$$

$$4(x^2-x-2) = (7)^2+(x)^2-2(7)(x)$$

$$4x^2-4x-49-8-x^2+14x = 0$$

$$3x^2+10x-57 = 0$$

$$3x^2-9x+19x-57 = 0$$

$$3x(x-3)+19(x-3) = 0$$

$$(3x+19)(x-3) = 0$$

$3x+19=0$, $x-3=0$

$3x=-19$,

$x=\frac{-19}{3}$, $x=3$

* CHECKING

Put $x=3$,

$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$

$\sqrt{4}+\sqrt{1}=\sqrt{9}$

$2+1=3$

$3=3$ true

Put $x=\frac{-19}{3}$,

$\sqrt{\left(\frac{-19}{3}\right)+1}+\sqrt{\left(\frac{-19}{3}\right)-2}=\sqrt{\left(\frac{-19}{3}\right)+6}$

$\sqrt{\frac{-19+3}{3}}+\sqrt{\frac{-19-6}{3}}=\sqrt{\frac{-19+18}{3}}$

$\sqrt{\frac{-16}{3}}+\sqrt{\frac{-25}{3}}=\sqrt{\frac{-1}{3}}$

$\frac{\sqrt{-1\times16}}{3}+\frac{\sqrt{-1\times25}}{3}=\frac{\sqrt{-1\times1}}{3}$

$\Rightarrow \frac{4i}{\sqrt{3}} + \frac{5i}{\sqrt{3}} = \frac{1i}{\sqrt{3}}$

$\frac{9i}{\sqrt{3}} = \frac{1i}{\sqrt{3}}$ not true

Sol set = $\{3\}$, $\{\frac{-19}{3}$ extraneous$\}$

(vii) $\sqrt{11-x} - \sqrt{6-x} = \sqrt{27-x}$

Sol:- Squaring both sides :-

$(\sqrt{11-x} - \sqrt{6-x})^2 = (\sqrt{27-x})^2$

$(\sqrt{11-x})^2 - (\sqrt{6-x})^2 - 2(\sqrt{11-x})(\sqrt{6-x}) = 27-x$

$11-x+6-x-2\sqrt{66-11x-6x+x^2} = 27-x$

$17-2x-2\sqrt{66-17x+x^2} = 27-x$

$-2\sqrt{66-17x+x^2} = 27-17-x+2x$

$-2\sqrt{66-17x+x^2} = 10+x$

* Again Squaring both sides :-

$(-2\sqrt{66-17x+x^2})^2 = (10+x)^2$

$4(66-17x+x^2) = 100+x^2+20x$

$264-68x+4x^2 = 100+x^2+20x$

$264-100-68x-20x+4x^2-x^2=0$

$3x^2-88x+164=0$

$a=3$, $b=-88$, $c=164$

* By Quadratic formula:-

$$= \frac{-(-88) \pm \sqrt{(-88)^2 - 4(3)(164)}}{2(3)}$$

$$= \frac{88 \pm \sqrt{7744 - 1968}}{6}$$

$$= \frac{88 \pm \sqrt{5776}}{6}$$

$$= \frac{88 \pm 76}{6}$$

$$= \frac{88+76}{6} \quad , \frac{88-76}{6}$$

$$= \frac{164}{6} \quad , \frac{12}{6}$$

$$= \frac{82}{3} \quad , 2$$

* CHECK:-

• Put $x = 2$,

$$\sqrt{11-2} - \sqrt{6-2} = \sqrt{2^2-2}$$

$$\sqrt{9} - \sqrt{4} = \sqrt{25}$$

$3-2=5$

$1 \neq 5$ not true

- put $x = 8\pi/3$

$$\sqrt{11-\frac{8\pi}{3}} - \sqrt{6-\frac{8\pi}{3}} = \sqrt{2\pi-\frac{8\pi}{3}}$$

$$\sqrt{\frac{33-8\pi}{3}} - \sqrt{\frac{18-8\pi}{3}} = \sqrt{\frac{81-8\pi}{3}}$$

$$\sqrt{\frac{-49}{3}} - \sqrt{\frac{-64}{3}} = \sqrt{\frac{-1}{3}}$$

$$\frac{\sqrt{49\times-1}}{\sqrt{3}} - \frac{\sqrt{64\times-1}}{\sqrt{3}} = \frac{\sqrt{-1\times1}}{\sqrt{3}}$$

$$\frac{7i}{\sqrt{3}} - \frac{8i}{\sqrt{3}} = \frac{1i}{\sqrt{3}}$$

$$\frac{7i-8i}{\sqrt{3}} = \frac{i}{\sqrt{3}}$$

$$\frac{-i}{\sqrt{3}} \neq \frac{i}{\sqrt{3}}$$ not true

As $\frac{8\pi}{3}$ & 2π are extraneous roots

So, Sol set = { }

((viii)) $\sqrt{4a+x} - \sqrt{a-x} = \sqrt{a}$

Sol:N Squaring both sides,

$$(\sqrt{4a+x} - \sqrt{a-x})^2 = (\sqrt{a})^2$$

$$(\sqrt{4a+x})^2 + (\sqrt{a-x})^2 - 2(\sqrt{4a+x})(\sqrt{a-x}) = a$$

$$4a+x+a-x-2\sqrt{4a^2-4ax+ax-x^2} = a$$

$$-2\sqrt{4a^2-4ax+ax-x^2} = a-a-4a-x$$

$$-2\sqrt{4a^2-3ax-x^2} = a-5a$$

$$\sqrt{4a^2-3ax-x^2} = \frac{-4a}{-2}$$

$$(\sqrt{4a^2-3ax-x^2})^2 = (2a)^2$$

$$4a^2-3ax-x^2 = 4a^2$$

$$4a^2-4a^2-3ax-x^2 = 0$$

$$0 = 3ax+x^2$$

$$x(3a+x) = 0$$

$x=0$, $3a+x=0$

, $x=-3a$

* CHECK:N

. Put $x=0$,

$$\sqrt{4a+0} - \sqrt{a-0} = \sqrt{a}$$

$$\sqrt{4a} - \sqrt{a} = \sqrt{a}$$

$$2\sqrt{a} - \sqrt{a} = \sqrt{a}$$

$\sqrt{a} = \sqrt{a}$ true

• Put $x = -3a$,

$\sqrt{4a-3a} - \sqrt{a+3a} = \sqrt{a}$

$\sqrt{1a} - \sqrt{4a} = \sqrt{a}$

$\sqrt{a} - 2\sqrt{a} = \sqrt{a}$

$-\sqrt{a} \neq \sqrt{a}$ not true

Sol set = {0}

(xx) $\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$

Soln

let $y = x^2+x$

$\sqrt{y+1} - \sqrt{y-1} = 1$

* Squaring both sides;

$(\sqrt{y+1} - \sqrt{y-1})^2 = (1)^2$

$(\sqrt{y+1})^2 + (\sqrt{y-1})^2 - 2(\sqrt{y+1})(\sqrt{y-1}) = 1$

$y+1 + y-1 - 2\sqrt{y^2-y+y-1} = 1$

$2y - 2\sqrt{y^2-1} = 1$

$2y - 1 = 2\sqrt{y^2-1}$

$(2y-1)^2 = (2\sqrt{y^2-1})^2$

$(2y)^2 + (1)^2 - 2(2y)(1) = 4(y^2-1)$

$4y^2 + 1 - 4y = 4y^2 - 4$

$4y^2 - 4y^2 + 1 - 4y + 4 = 0$

$-4y + 5 = 0$

$-(4y+5) = 0$

$4y - 5 = 0$

* put $y = x^2 + x$

$4(x^2 + x) - 5 = 0$

$4x^2 + 4x - 5 = 0$

* By Quadratic formula:

$a = 4 \quad , b = 4 \quad , c = -5$

$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(4)(-5)}}{2(4)}$$

$$= \frac{-4 \pm \sqrt{16 + 80}}{8}$$

$$= \frac{-4 \pm \sqrt{96}}{8}$$

$$= \frac{-4 \pm 4\sqrt{6}}{8}$$

$$= \frac{4(-1 \pm \sqrt{6})}{8_2}$$

$$= \frac{-1 \pm \sqrt{6}}{2}$$

$$= \frac{-1+\sqrt{6}}{2} \quad , \frac{-1-\sqrt{6}}{2}$$

Sol set $= \left\{ \frac{-1+\sqrt{6}}{2} , \frac{-1-\sqrt{6}}{2} \right\}$

(xii) $\sqrt{x^2+3x+8} + \sqrt{x^2+3x+2} = 3$

Sol:

- let $y = x^2+3x$

$$\sqrt{y+8} + \sqrt{y+2} = 3$$

* Squaring both sides:

$$(\sqrt{y+8} + \sqrt{y+2})^2 = (3)^2$$

$$(\sqrt{y+8})^2 + (\sqrt{y+2})^2 + 2(\sqrt{y+8})(\sqrt{y+2}) = 9$$

$$y+8+y+2+2\sqrt{y^2+2y+8y+16} = 9$$

$$2y+10+2\sqrt{y^2+10y+16} = 9$$

$$2\sqrt{y^2+10y+16} = 9-2y-10$$

$$2\sqrt{y^2+10y+16} = -2y-1$$

$$2\sqrt{y^2+10y+16} = -(2y+1)$$

$$(2\sqrt{y^2+10y+16})^2 = [-(2y+1)]^2$$

$$4(y^2+10y+16) = (2y)^2+(1)^2+2(2y)(1)$$

$$4y^2+40y+64 = 4y^2+1+4y$$

$$4y^2-4y^2+40y-4y+64-1=0$$

$$36y+63=0$$

Put $y = x^2+3x$

$$36(x^2+3x)+63=0$$

$$36x^2+108x+63=0$$

Using Quadratic Formula:

$$a=36,\ b=108,\ c=63$$

$$= \frac{-(108) \pm \sqrt{(108)^2 - 4(36)(63)}}{2(36)}$$

$$= \frac{-108 \pm \sqrt{11664 - 9072}}{72}$$

$$= \frac{-108 \pm \sqrt{2592}}{72}$$

$$= \frac{-108 \pm 36\sqrt{2}}{72}$$

$$= \frac{36(-3 \pm \sqrt{2})}{72}$$

$$= \frac{-3 \pm \sqrt{2}}{2}$$

$$= \frac{-3+\sqrt{2}}{2}, \frac{-3-\sqrt{2}}{2}$$

$$\text{Sol set} = \left\{\frac{-3+\sqrt{2}}{2}, \frac{-3-\sqrt{2}}{2}\right\}$$

((xi)) $\sqrt{x^2+3x+9} + \sqrt{x^2+3x+4} = 5$

Solin

- let $y = x^2+3x$

$$\sqrt{y+9} + \sqrt{y+4} = 5$$

Squaring both sides in

$$(\sqrt{y+9}+\sqrt{y+4})^2=(5)^2$$

$$(\sqrt{y+9})^2+(\sqrt{y+4})^2+2(\sqrt{y+9})(\sqrt{y+4})=25$$

$$y+9+y+4+2\sqrt{y^2+4y+9y+36}=25$$

$$2y+13+2\sqrt{y^2+13y+36}=25$$

$$2\sqrt{y^2+13y+36}=25-2y-13$$

$$2\sqrt{y^2+13y+36}=12-2y$$

$$2\sqrt{y^2+13y+36}=12-2y)$$

$$(2\sqrt{y^2+13y+36})^2=(12-2y)^2$$

$$4(y^2+13y+36)=(12)^2+(2y)^2-2(12)(2y)$$

$$4y^2+52y+144=144+4y^2-48y$$

$$4y^2-4y^2+52y+48y+144-144=0$$

$$100y=0 \Rightarrow y=0$$

Put $y=x^2+3x$

$$x^2+3x=0$$

$$x(x+3)=0.$$

$x=0$, $x=-3$

CHECK in • Put $x=0$:

$$\sqrt{(0)^2+3(0)+9}+\sqrt{(0)^2+3(0)+4}=5$$

$$\sqrt{9}+\sqrt{4}=5$$

$$3+2=5$$

$5=5$ true

- put $x = -3$

$$\sqrt{(-3)^2+3(-3)+9}+\sqrt{(-3)^2+3(-3)+4}=5$$

$$\sqrt{9(-9)+9}+\sqrt{9-9+4}=5$$

$$\sqrt{9}+\sqrt{4}=5$$

$$3+2=5$$

$$5=5 \quad \text{true}$$

Sol set $= \{0, -3\}$