28th June 2024

HomeWork

Chapter 01: "Quadratic Equations"

Exercise 1.3:-

Q1 (ii):- $2x^4 - 11x^2 + 5 = 0$

* Let,

$x^2 = y$ & $x^4 = y^2$

$2x^4 - 11x^2 + 5 = 0$ ... (i)

Putting $x^4 = y^2$ and $x^2 = y$ in eq (i):-

$2y^2 - 11y + 5 = 0$

* By factorization,

$2y^2 - 10y - 1y + 5 = 0$

$2y(y-5) - 1(y-5) = 0$

$(2y-1)(y-5) = 0$

$2y = 1$, $y - 5 = 0$

$y = \frac{1}{2}$, $y = 5$

* Put $y = x^2$

$x^2 = 1/2$, $x^2 = 5$

* Taking $\sqrt{}$ on both sides

$\sqrt{x^2} = \pm\sqrt{1/2}$, $\sqrt{x^2} = \pm\sqrt{5}$

$x = \pm\frac{1}{\sqrt{2}}$, $x = \pm\sqrt{5}$

$\boxed{\text{Sol set} = \left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}}$

(iii) $2x^4 = 9x^2 - 4$

Re-arrange:-

$2x^4 - 9x^2 + 4 = 0$ .. (i)

let,

$x^2 = y$ & $x^4 = y^2$ in eq (i):-

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - 1y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(2y-1)(y-4) = 0$

$2y = 1$, $y - 4 = 0$

$y = 1/2$, $y = 4$

* Put $y = x^2$

$x^2 = 1/2$, $x^2 = 4$

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* Taking $\sqrt{\ }$ on both sides:-

$\sqrt{x^2} = \pm\sqrt{1/2}$, $\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm\sqrt{1/2}$, $x = \pm 2$

$\boxed{\text{Sol set} = \left\{\pm\sqrt{1/2}, \pm 2\right\}}$

(iii) $5x^{1/2} = 7x^{1/4} - 2$

Let,

$x^{1/4} = y$

Taking square on both sides:-

$(x^{1/4})^{2} = y^2$

$x^{1/2} = y^2$

* Re-arrange:-

$5x^{1/2} - 7x^{1/4} + 2 = 0$

Now, put $x^{1/4} = y$ & $x^{1/2} = y^2$:-

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(5y-2)(y-1) = 0$

$5y = 2$, $y - 1 = 0$

$y = 2/5$, $y = 1$

* Put $y = x^{1/4}$

$x^{1/4} = 2/5$, $x^{1/4} = 1$

* Taking "4" on both sides:-

$(x^{1/4})^4 = (2/5)^4$, $(x^{1/4})^4 = (1)^4$

$x = \frac{16}{625}$, $x = 1$

$$\boxed{\text{Sol set} = \left\{\frac{16}{625}, 1\right\}}$$

(iv) $x^{2/3} + 54 = 15x^{1/3}$ ... eq ①

Sol:- let $x^{1/3} = y$

Taking square on both sides:-

$(x^{1/3})^2 = y^2$

$x^{2/3} = y^2$

Putting $x^{1/3} = y$ & $x^{2/3} = y^2$ in eq ①

$y^2 + 54 = 15y$

$y^2 - 15y + 54 = 0$

$y^2 - 9y - 6y + 54 = 0$

$y(y-9) - 6(y-9) = 0$

$(y-6)(y-9) = 0$

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$y - 9 = 0$, $y - 6 = 0$

$y = 9$, $y = 6$

Now $x^{1/3} = 9$ & $x^{1/3} = 6$

* Taking cube on both sides,

$(x^{1/3})^3 = (9)^3 = 729$, $(x^{1/3})^3 = (6)^3 = 216$

$\boxed{\text{Sol set} = \{729, 216\}}$

(v) $3x^{-2} + 5 = 8x^{-1}$

Sol:n $3x^{-2} + 5 = 8x^{-1}$ ... eq ①

let,

$x^{-1} = y$ then $x^{-2} = y^2$

- Equation ① becomes,

$3y^2 + 5 = 8y$

Re-arrange

$3y^2 - 8y + 5 = 0$

$3y^2 - 3y - 5y + 5 = 0$

$3y(y-1) - 5(y-1) = 0$

$(3y-5)(y-1) = 0$

$3y - 5 = 0$, $y - 1 = 0$

$3y = 5$, $y = 1$

$y = 5/3$,

* Put $y = x^{-1}$

$x^{-1} = 1$, $x^{-1} = 5/3$

$x = 1$, $x = 3/5$

$x^{-1} = 1$

$\frac{1}{x} \Rightarrow \frac{1}{1}$

$x = 1$

cross multiplication

Sol set $= \{1, 3/5\}$

(vii) $(2x^2+1) + \frac{3}{2x^2+1} = 4$ .. eq (i)

Sol:- let $2x^2+1 = y$

* So equation (i) becomes

$y + \frac{3}{y} = 4$

multiplying "y" with each term

$y \times y + \frac{3}{\cancel{y}} \times \cancel{y} = 4 \times y$

$y^2 + 3 = 4y$

$y^2 - 4y + 3 = 0$

$y^2 - 1y - 3y + 3 = 0$

$y(y-1) - 3(y-1) = 0$

$(y-1)(y-3) = 0$

$y-1=0$, $y-3=0$

$y=1$, $y=3$

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* Put $y = 2x^2+1$

$2x^2+1=1$, $2x^2+1=3$

$2x^2=1-1$, $2x^2=3-1$

$\frac{\cancel{2}x^2}{\cancel{2}}=0/2$, $\frac{\cancel{2}x^2}{\cancel{2}}=\frac{\cancel{2}}{\cancel{2}}$

$x^2=0$, $x^2=1$

Taking $\sqrt{}$ on both sides

$\sqrt{x^2}=\pm\sqrt{0}$, $\sqrt{x^2}=\pm\sqrt{1}$

$x=0$, $x=\pm 1$

$\boxed{\text{Sol set} = \{1,-1,0\}}$

(viii) $\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$

Sol: $\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$ ... eq (i)

let, $y=\frac{x}{x-3}$ then $\frac{1}{y}=\frac{x-3}{x}$

* Equation (i) becomes,

$y+4\left(\frac{1}{y}\right)=4$

$y+\frac{4}{y}=4$

* Multiplying each term with "y" :-

$$y \times y + \frac{4}{\cancel{y}} \times \cancel{y} = 4 \times y$$

$$y^2 + 4 = 4y$$

* Re-arrange :-

$$y^2 - 4y + 4 = 0$$

$$y^2 - 2y - 2y + 4 = 0$$

$$y(y-2) - 2(y-2) = 0$$

$$(y-2)(y-2) = 0$$

$$(y-2)^2 = 0$$

* Taking $\sqrt{}$ on both sides :-

$$\sqrt{(y-2)^2} = \pm\sqrt{0}$$

$$y - 2 = 0$$

$$\boxed{y = 2}$$

* Put $y = \frac{x}{x-3}$

$$\frac{x}{x-3} = 2$$

* By Cross multiplication :-

$$x = 2(x-3)$$

$$x = 2x - 6$$

$$x - 2x = -6$$

$$= \frac{\cancel{-1}x}{\cancel{-1}} = \frac{-6}{-1}$$

$$x = 6$$

$$\boxed{\text{Sol set} = \{6\}}$$

(viii) $\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = 2\frac{1}{6}$

Sol:- $\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = 2\frac{1}{6}$ ... eq (i)

let: $\frac{4x+1}{4x-1} = y$

$\frac{4x-1}{4x+1} = \frac{1}{y}$

* Equation (i) becomes,

$y + \frac{1}{y} = \frac{13}{6}$

* Multiplying "6y" with each term,

$6y \times y + \frac{1}{\cancel{y}} \times 6\cancel{y} = \frac{13}{\cancel{6}} \times \cancel{6}y$

$6y^2 + 6 = 13y$

* Re-arrange

$6y^2 - 13y + 6 = 0$

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$6y^2 - 9y - 4y + 6 = 0$

$3y(2y-3) - 2(2y-3) = 0$

$(3y-2)(2y-3) = 0$

$3y-2=0 \quad , \quad 2y-3=0$

$3y=2 \quad , \quad 2y=3$

$y=2/3 \quad , \quad y=3/2$

* Put $y = \frac{4x+1}{4x-1}$

$\frac{4x+1}{4x-1} = 2/3 \quad , \quad \frac{4x+1}{4x-1} = \frac{3}{2}$

$2(4x-1) = 3(4x+1) \quad , \quad 2(4x+1) = 3(4x-1)$

$8x-2 = 12x+3 \quad , \quad 8x+2 = 12x-3$

$8x-12x = 3+2 \quad , \quad 8x-12x = -3-2$

$-4x = 5 \quad , \quad -4x = -5$

$\frac{\cancel{-4}}{\cancel{-4}}x = \frac{5}{-4} \quad , \quad \frac{\cancel{-4}x}{\cancel{-4}} = \frac{+5}{+4}$

$x = -5/4 \quad , \quad x = 5/4$

$\boxed{\text{Sol set} = \{\pm 5/4\}}$

((ix)) $\dfrac{x-a}{x+a} - \dfrac{x+a}{x-a} = \dfrac{7}{12}$

Sol:- $\dfrac{x-a}{x+a} - \dfrac{x+a}{x-a} = \dfrac{7}{12}$ .. eq ①

let, $\dfrac{x-a}{x+a} = y$ then $\dfrac{x+a}{x-a} = \dfrac{1}{y}$

* Equation ① becomes,

$$y - \frac{1}{y} = \frac{7}{12}$$

* Multiplying each term by "12y":-

$$12y \times y - \frac{1}{\cancel{y}} \times 12\cancel{y} = \frac{7}{\cancel{12}} \times \cancel{12}y$$

$$12y^2 - 12 = 7y$$

$$12y^2 - 7y - 12 = 0$$

$$12y^2 - 9y - 16y - 12 = 0$$

$$3y(4y+3) - 4(4y+3) = 0$$

$$(3y-4)(4y+3) = 0$$

$$3y - 4 = 0 \quad , \quad 4y + 3 = 0$$

$$3y = 4 \quad , \quad 4y = -3$$

$$y = 4/3 \quad , \quad y = -3/4$$

* put $y = \dfrac{x-a}{x+a}$

$$\frac{x-a}{x+a}=\frac{4}{3} \quad , \quad \frac{x-a}{x+a}=-3/4$$

$$3(x-a)=4(x+a) \quad , \quad 4(x-a)=-3(x+a)$$

$$3x-3a=4x+4a \quad , \quad 4x-4a=-3x-3a$$

$$3x-4x=4a+3a \quad , \quad 4x+3x=-3a+4a$$

$$\frac{\cancel{-1}x}{\cancel{-1}}=\frac{7a}{-1} \quad , \quad 7x=1a$$

$$x=\frac{a}{7}$$

$$x=-\frac{7}{1}a \quad ,$$

$$x=-7a$$

$$\boxed{\text{Sol set}=\left\{-7a\,,\ a/7\right\}}$$

(x) $x^4-2x^3-2x^2+2x+1=0$

Sol: Dividing x^2 with each term,

$$\frac{x^4}{x^2}-\frac{2x^3}{x^2}-\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=0$$

$$x^2-2x-2+2x+\frac{1}{x^2}=0$$

$$\left(x^2+\frac{1}{x^2}\right)-2x+\frac{2}{x}-2=0$$

$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-2=0 \quad \ldots \text{eq (i)}$$

let $x-\frac{1}{x}=y$

* Taking square on both sides,

$$\left(x-\frac{1}{x}\right)^2=y^2$$

$$x^2+\frac{1}{x^2}=y^2+2$$

* So equation (i) becomes,

$$y^2+2-2(y)-2=0$$

$$y^2+\cancel{2}-2y-\cancel{2}=0$$

$$y^2-2y=0$$

* Taking "y" common,

$$y(y-2)=0$$

$$y=0 \quad , \quad y=2$$

* Put $y=x-\frac{1}{x}$

$$x-\frac{1}{x}=0 \quad , \quad x-\frac{1}{x}=2$$

$$\frac{x^2-1}{x}=0 \quad , \quad \frac{x^2-1}{x}=2$$

$$x^2-1=0\times x \quad , \quad x^2-1=2(x)$$

$$x^2=1 \quad , \quad x^2-2x-1=0$$

$\sqrt{x^2} = \pm\sqrt{1}$, By Quadratic formula

$x = \pm 1$, $a=1$, $b=-2$, $c=-1$

$$\Rightarrow \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4+4}}{2}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$= \frac{\cancel{2}(1 \pm \sqrt{2})}{\cancel{2}}$$

$$x = 1 \pm \sqrt{2}$$

$\boxed{\text{Sol set} = \{\pm 1 , 1 \pm \sqrt{2}\}}$

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(xi) $2x^4+x^3-6x^2+x+2=0$

Sol: Dividing each term by "x^2",

$$\Rightarrow \frac{2x^4}{x^2}+\frac{x^3}{x^2}-\frac{6x^2}{x^2}+\frac{x}{x^2}+\frac{2}{x^2}=0$$

$$2x^2+x-6+\frac{1}{x}+\frac{2}{x^2}=0$$

$$2\left(x^2+\frac{1}{x^2}\right)+1\left(x+\frac{1}{x}\right)-6=0 \quad \ldots eq\ (i)$$

Let $x+\frac{1}{x}=y$ then $x^2+\frac{1}{x^2}=y^2-2$

* Equation (i) becomes

$2(y^2-2)+1(y)-6=0$

$2y^2-4+y-6=0$

$2y^2+y-10=0$

$2y^2+5y-4y-10=0$

$y(2y+5)-2(2y+5)=0$

$(2y+5)(y-2)=0$

$2y+5=0$, $y-2=0$

$2y=-5$,

$y=-5/2$, $y=2$

* Put $y=x+\frac{1}{x}$

$x + \frac{1}{x} = -5/2$

$\frac{x^2+1}{x} = -5/2$

$2(x^2+1) = -5(x)$

$2x^2+2 = -5x$

$2x^2+5x+2=0$

$2x^2+1x+4x+2=0$

$x(2x+1)+2(2x+1)=0$

$(x+2)\ (2x+1)=0$

$x+2=0\ ,\ 2x+1=0$

$x=-2\ ,\ x=-1/2$

$x+\frac{1}{x}=2$

$\frac{x^2+1}{x}=2$

$x^2+1=2x$

$x^2-2x+1=0$

$x^2-x-x+1=0$

$x(x-1)-1(x-1)=0$

$(x-1)(x-1)=0$

$(x-1)^2=0$

$\sqrt{(x-1)^2} = \pm\sqrt{0}$

$x-1=0$

$x=1$

$\boxed{\text{Sol set} = \{-2, -1/2, 1\}}$

(xiii) $4.2^{2x+1} - 9.2^{x} + 1 = 0$

Sol:n $4.2^{2x}.2^{1} - 9.2^{x} + 1 = 0$

$8(2^{x})^2 - 9.2^{x} + 1 = 0$ .. eq (i)

let '2^x' $= y$

* Equation (i) becomes,

$8y^2-9y+1=0$

$8y^2-8y-1y+1=0$

$8y(y-1)-1(y-1)=0$

$(8y-1)(y-1)=0$

$8y-1=0 \quad , \quad y-1=0$

$8y=1 \quad ,$

$\boxed{y=1/8} \quad , \quad \boxed{y=1}$

* Put $y = 2^x$

$2^x = 1 \quad , \quad 2^x = 1/8$

$\frac{2^x}{2} = \frac{2}{2} \quad , \quad 2^x = \frac{1}{2^3}$

$\boxed{x=0} \quad , \quad \frac{2^x}{2} = \frac{2^{-3}}{2}$

$, \boxed{x=-3}$

$\boxed{\text{Sol set} = \{0, -3\}}$

(xiii) $3^{2x+2} = 12.3^x - 3$

Sol: Re-arrange

$3^{2x+2} - 12.3^x + 3 = 0$

$3^{2x}.3^2 - 12.3^x + 3 = 0$

$\cancel{3^{2x}.9 - 12.3^x +}$

$9.3^{2x} - 12.3^x + 3 = 0$

let $y = 3^x$ then $y^2 = 3^{2x}$

* Equation (i) becomes,

$9y^2-12y+3=0$

$9y^2-3y-9y+3=0$

$3y(3y-1)-3(3y-1)=0$

$(3y-1)(3y-3)=0$

$3y-1=0 \quad , \quad 3y-3=0$

$3y=1 \quad , \quad \frac{\cancel{3}y}{\cancel{3}}=\frac{\cancel{3}}{\cancel{3}}$

$y=1/3 \quad , \quad y=1$

* Put $y=3^x$

$3^x=1/3 \quad , \quad 3^x=1$

$3^x=3^{-1} \quad , \quad 3^x=3^0$

$x=-1 \quad , \quad x=0$

$\boxed{\text{Sol set}=\{-1,0\}}$

(xiv) $2^x+64.2^{-x}-20=0$

Sol:- $2^x+\frac{64}{2^x}-20=0$ .. eq (i)

let $y=2^x$

* Equation (i) becomes,

$y+\frac{64}{y}-20=0$

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* Multiplying each term with 'y':-

$y \times y + \frac{64}{\cancel{y}} \times \cancel{y} - 20 \times y = 0$

$y^2 + 64 - 20y = 0$

* Re-arrange:-

$y^2 - 20y + 64 = 0$

$y^2 - 16y - 4y + 64 = 0$

$y(y-16) - 4(y-16) = 0$

$(y-16)(y-4) = 0$

$y - 16 = 0 \quad , \quad y - 4 = 0$

$y = 16 \quad , \quad y = 4$

* Put $y = 2^x$

$2^x = 4 \quad , \quad 2^x = 16$

$2^x = (2)^2 \quad , \quad 2^x = (2)^4$

$x = 2 \quad , \quad x = 4$

$\boxed{\text{Sol set} = \{2, 4\}}$

(XVI) $(x+1)(x+3)(x-5)(x-7)=192$

Sol: $[(x+1)(x-5)][(x+3)(x-7)]=192$

$[x(x-5)+1(x-5)][x(x-7)+3(x-7)]=192$

$(x^2-5x+1x-5)(x^2-7x+3x-21)=192$

$(x^2-4x-5)(x^2-4x-21)=192$

let, $y=x^2-4x$

* Equation ① becomes,

$(y-5)(y-21)=192$

$y(y-21)-5(y-21)=192$

$y^2-21y-5y+105=192$

$y^2-26y+105-192=0$

$y^2-26y-87=0$

$y^2+3y-29y-87=0$

$y(y+3)-29(y+3)=0$

$(y-29)(y+3)=0$

* Put $y=x^2-4x$

$(x^2-4x+3)(x^2-4x-29)=0$

$x^2-4x+3=0$

$x^2-1x-3x+3=0$

$x(x-1)\mp3(x+1)=0$

$(x-1)(x\mp3)=0$

$x=1$, $x=+3$

, $x^2-4x-29=0$

, By Quadratic formula

, $a=1$, $b=-4$, $c=-29$

,

, $\dfrac{-(-4)\pm\sqrt{(-4)^2-4(1)(-29)}}{2(1)}$

$$= \frac{4 \pm \sqrt{16+116}}{2}$$

$$= \frac{4 \pm \sqrt{132}}{2}$$

$$= \frac{4 \pm 2\sqrt{33}}{2}$$

$$= \frac{\cancel{2}(2 \pm \sqrt{33})}{\cancel{2}}$$

$$x = 2 \pm \sqrt{33}$$

$$\boxed{\text{Sol set} = \{1, 3, 2 \pm \sqrt{33}\}}$$

((xvi)) $(x-1)(x-2)(x-8)(x+5)+360=0$

Sol: $[(x-1)(x-2)][(x-8)(x+5)]+360=0$

$[x(x-2)-1(x-2)][x(x+5)-8(x+5)]+360=0$

$(x^2-2x-1x+2)(x^2+5x-8x-40)+360=0$

$(x^2-3x+2)(x^2-3x-40)+360=0$

Let $y = x^2-3x$

* Equation (i) becomes,

$(y+2)(y-40)+360=0$

$y^2-40y+2y-80+360=0$

$y^2-38y+280=0$

$y^2-10y-28y+280=0$

$y(y-10)-28(y-10)=0$

$(y-28)(y-10)=0$

$y-10=0 \quad , \quad y-28=0$

$y=10 \quad , \quad y=28$

* Put $y = x^2-3x$

$x^2-3x-10=0 \quad , \quad x^2-3x-28=0$

$x^2-5x+2x-10=0 \quad , \quad x^2-7x+4x-28=0$

$x(x-5)+2(x-5)=0 \quad , \quad x(x-7)+4(x-7)=0$

$(x+2)(x-5)=0 \quad , \quad (x-7)(x+4)=0$

$x+2=0 \quad , \quad x-5=0 \quad , \quad x-7=0 \quad , \quad x+4=0$

$x=-2 \quad , \quad x=5 \quad , \quad x=7 \quad , \quad x=-4$

$\boxed{\text{Sol set} = \{-2, 5, 7, -4\}}$