8th July 2024 Homework Saturday

Chapter no 02:
"Quadratic Equations"

• Exercise 2.2:

Q1:(i) $(x+7)(x-3)=-7$

Sol: $(x+7)(x-3)=-7$

$[x(x-3)+7(x-3)]=-7$

$x^2-3x+7x-21=-7$

$x^2+4x-21+7=0$

$x^2+4x-14=0$ ... (Standard form)

(ii) $\frac{x^2+4}{3}-\frac{x}{7}=1$

Sol: By cross multiplication:

$\Rightarrow \frac{7(x^2+4)-x(3)}{21}=1\times 1$

$\frac{7x^2+28-3x}{21}=1$

$7x^2-3x+7=0$ ... (Standard form)

(iii) $\frac{x}{x+1} + \frac{x+1}{x} = 6$

Sol:N

* Multiplying $[x(x+1)]$ on both sides :N

$\Rightarrow x(x+1) \times \frac{x}{x+1} + \frac{x+1}{x} \times [x(x+1)] = 6\,[x(x+1)]$

$x^2 + (x+1)^2 = 6x^2 + 6x$

$x^2 + [(x)^2 + 2(x)(1) + (1)^2] = 6x^2 + 6x$

$x^2 + x^2 + 2x + 1 - 6x^2 - 6x = 0$

$2x^2 - 6x^2 + 2x - 6x + 1 = 0$

$-4x^2 - 4x + 1 = 0$ ... (Standard form)

(iv) $\frac{x+4}{x-2} - \frac{x-2}{x} + 4 = 0$

Sol:N Cross Multiplication :N

$\Rightarrow x(x+4) - (x-2)^2 + 4[x(x-2)] = 0$

$x^2 + 4x - [(x)^2 - 2(x)(2) + (2)^2] + 4(x^2 - 2x) = 0$

$x^2 + 4x - x^2 + 4x - 4 + 4x^2 - 8x = 0$

$8x - 8x + 4x^2 - 4 = 0$

$4x^2 - 4 = 0$

Taking "4" common :N

$4(x^2 - 1) = 0$

$x^2 - 1 = 0$ ... Pure Quadratic Equation

v) $\frac{x+3}{x+4} - \frac{x-5}{x} = 1$

Sol: By cross multiplication:

$\Rightarrow x(x+3) - (x+4)(x-5) = 1 \cdot [x(x+4)]$

$x^2+3x - [x(x-5)+4(x-5)] = x^2+4x$

$x^2+3x - [x^2-5x+4x-20] = x^2+4x$

$x^2+3x - x^2+5x-4x+20 = x^2+4x$

$3x+x - x^2-4x+20 = 0$

$4x - x^2 - 4x + 20 = 0$

$-x^2+20 = 0$ ... Pure Quadratic Equation

or $x^2-20 = 0$

vi) $\frac{x+1}{x+2} + \frac{x+2}{x+3} = \frac{25}{12}$

Sol: $\frac{(x+1)(x+3)+(x+2)(x+2)}{(x+2)(x+3)} = 25/12$

$\Rightarrow \frac{(x^2+3x+x+3)(x^2+2x+2x+4)}{x^2+3x+2x+6} = 25/12$

$\frac{(x^2+4x+3)(x^2+4x+4)}{x^2+5x+6} = 25/12$

$\frac{2x^2+8x+7}{x^2+5x+6} = 25/12$

By cross multiplication:

$12(2x^2+8x+7) = 25(x^2+5x+6)$

$24x^2+96x+84 = 25x^2+125x+150$

$24x^2-25x^2+96x-125x+84-150=0$

$x^2-29x-66=0$ ... (Standard form)

Q2(iii) $x^2-x-20=0$

Sol:N $x^2-5x+4x-20=0$

$x(x-5)+4(x-5)=0$

$(x-5)(x+4)=0$

$x-5=0$, $x+4=0$

$x=5$, $x=-4$

Sol set = $\{5, -4\}$

(iii) $3y^2 = y(y-5)$

Sol:N $3y^2 = y^2-5y$

Re-arranging*:N

$3y^2-y^2+5y=0$

$2y^2+5y=0$

Taking* "y" common:N

$y(2y+5)=0$

$y=0$, $2y+5=0$

, $2y=-5$

, $y=-5/2$

Sol set = $\{0, -5/2\}$

(iii) $4 - 32x = 16x^2$

Sol:- Re-arranging* the values :-

$-16x^2 - 32x + 4 = 0$

$-16x^2 - 24x + 8x + 4 = 0$

$-16x(x+2) + 2(x+2) = 0$

$(-16x + 2)(x+2) = 0$

$x + 2 = 0$, $-16x + 2 = 0$

$x = -2$, $-16x = -2$

, $x = +2/+16$

, $x = 2/16$

Sol set = $\{-2, 2/16\}$

(iv) $x^2 - 11x = 152$

Sol:- By Re-arranging* :-

$x^2 - 11x - 152 = 0$

$x^2 - 19x + 8x - 152 = 0$

$x(x-19) + 8(x-19) = 0$

$(x+8)(x-19) = 0$

$x + 8 = 0$, $x - 19 = 0$

$x = -8$, $x = 19$

Sol set = $\{-8, 19\}$

(VI) $\frac{x+1}{x} + \frac{x}{x+1} = \frac{25}{12}$

Sol: By cross multiplication:

$\Rightarrow \frac{(x+1)^2 + x^2}{x(x+1)} = \frac{25}{12}$

Formula:
$(a+b)^2 = a^2 + 2ab + b^2$

$\frac{[(x)^2 + 2(x)(1) + (1)^2] + x^2}{x^2 + x} = \frac{25}{12}$

$\frac{x^2 + 2x + 1 + x^2}{x^2 + x} = 25/12$

$\frac{2x^2 + 2x + 1}{x^2 + x} = \frac{25}{12}$

$12(2x^2 + 2x + 1) = 25(x^2 + x)$ ... Cross multiplication

$24x^2 + 24x + 12 = 25x^2 + 25x$

$24x^2 - 25x^2 + 24x - 25x + 12 = 0$

$-1x^2 - 1x + 12 = 0$

Taking "-" common:

$-(x^2 + x - 12) = 0$

$x^2 + x - 12 = 0$

By Factorization:

$x^2+4x-3x-12=0$

$x(x+4)-3(x+4)=0$

$(x-3)(x+4)=0$

$x-3=0$, $x+4=0$

$x=3$, $x=-4$

Sol set $=\{3, -4\}$

(vii) $\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$

Sol:- $\frac{2}{x-9}=\frac{(x-4)-(x-3)}{(x-3)(x-4)}$ ... Cross multiplication

$\frac{2}{x-9}=\frac{x-4-x+3}{x(x-4)-3(x-4)}$

$\frac{2}{x-9}=\frac{-4+3}{x^2-4x-3x+12}$

$\frac{2}{x-9}=\frac{-1}{x^2-7x+12}$

$2(x^2-7x+12)=-1(x-9)$

$2x^2-14x+24=-x+9$

1 $2x^2 - 14x + x + 24 - 9 = 0$

$2x^2 - 13x + 15 = 0$

Sol $2x^2 - 10x - 3x + 15 = 0$ ... Factorization*

$2x(x - 5) - 3(x - 5) = 0$

$2x - 3 = 0$, $x - 5 = 0$

$2x = 3$, $x = 5$

$x = 3/2$,

Sol set $= \{3/2, 5\}$

⊙ Completing square

Q3:(i) $7x^2 + 2x - 1 = 0$

Sol:(i) Dividing each term by "7":

$$\frac{7}{7}x^2 + \frac{2}{7}x - \frac{1}{7} = 0$$

$$x^2 + \frac{2}{2} \cdot \frac{2}{7}x - \frac{1}{7} = 0$$

$$x^2 + 2x\left(\frac{1}{7}\right) - \frac{1}{7} = 0 + \frac{1}{7}$$

* Adding $\left(\frac{1}{7}\right)^2$ on both sides:

$$x^2 + 2x\left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2 = \frac{1}{7} + \left(\frac{1}{7}\right)^2$$

$$\left(x+\frac{1}{7}\right)^2 = \frac{1}{7} + \frac{1}{49}$$

$$\left(x+\frac{1}{7}\right)^2 = \frac{7+1}{49}$$

$$\left(x+\frac{1}{7}\right)^2 = \frac{8}{49}$$

Formula :-
$(a^2 + 2ab + b^2) = (a+b)^2$

* Taking $\sqrt{}$ on both sides;

$$\sqrt{\left(x+\frac{1}{7}\right)^2} = \sqrt{\frac{8}{49}}$$

$$x+\frac{1}{7} = \pm\frac{\sqrt{4 \times 2}}{7}$$

$$x+\frac{1}{7} = \pm\frac{2\sqrt{2}}{7}$$

$$x = \frac{-1}{7} \pm \frac{2\sqrt{2}}{7}$$

$$x = \frac{-1 \pm 2\sqrt{2}}{7}$$

$$\Rightarrow \frac{-1+2\sqrt{2}}{7}, \frac{-1-2\sqrt{2}}{7}$$

Sol set $= \left\{\frac{-1+2\sqrt{2}}{7}, \frac{-1-2\sqrt{2}}{7}\right\}$

(iii) $ax^2 + bx - a = 0 \quad , a \neq 0$

Sol:- Dividing each term by "a";

$$\Rightarrow \frac{a}{a}x^2 + \frac{b}{a}x - \frac{a}{a} = 0$$

$$x^2 + \frac{b}{a}x - 1 = 0$$

$$x^2 + \frac{b}{a}x = 1$$

* Adding $\left(\frac{b}{a}\right)^2$ on both sides;

$$x^2 + \frac{b}{a}x + \left(\frac{b}{a}\right)^2 = 1 + \left(\frac{b}{a}\right)^2$$

$$\therefore \left(x + \frac{b}{a}\right)^2 = \frac{a^2 + b}{a^2}$$

* Taking $\sqrt{}$ on both sides;

$$\sqrt{\left(x + \frac{b}{a}\right)^2} = \sqrt{\frac{a^2 + b}{a^2}}$$

$$x + \frac{b}{a} = \pm \frac{\sqrt{a^2 + b}}{a}$$

$$x = \frac{-b \pm \sqrt{a^2 + b}}{a}$$

Sol set $= \left\{ \frac{-b + \sqrt{a^2 + b}}{a}, \frac{-b - \sqrt{a^2 + b}}{a} \right\}$

$11x^2 - 34x + 3 = 0$

Solⁿ Dividing each term by "11";

$$\Rightarrow \frac{11x^2}{11} - \frac{34}{11}x + \frac{3}{11} = 0$$

$$x^2 - \frac{2}{2_1} \cdot \frac{34^{17}}{11}x + \frac{3}{11} = 0$$

$$x^2 - 2 \cdot \frac{17}{11}x = 0 - \frac{3}{11}$$

* Adding $\left(\frac{17}{11}\right)^2$ on both sides :-

$$x^2 - 2(x)\left(\frac{17}{11}\right) + \left(\frac{17}{11}\right)^2 = \frac{-3}{11} + \left(\frac{17}{11}\right)^2$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{-33 + 289}{121}$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{256}{121}$$

* Taking $\sqrt{}$ on both sides :-

$$\sqrt{\left(x - \frac{17}{11}\right)^2} = \pm\sqrt{\frac{256}{121}}$$

$$x - \frac{17}{11} = \frac{\pm 16}{11} + \frac{17}{11}$$

$$x = \frac{\pm 16 + 17}{11}$$

$$\Rightarrow \frac{16+17}{11} \quad , \quad \frac{16-17}{11}$$

$$\frac{33}{11} \quad , \quad \frac{-1}{11}$$

$$3 \quad , \quad -1/11$$

Sol set $= \{3, -1/11\}$

(iv) $ax^2 + mx + n = 0 \quad ; a \neq 0$

Sol:- Dividing 'a' with each term;

$$\frac{a}{a}x^2 + \frac{m}{a}x + \frac{n}{a} = 0$$

$$x^2 + \frac{2}{2}(x)\frac{m}{a} = \frac{-n}{a}$$

$$x^2 + 2(x)\frac{m}{2a} = \frac{-n}{a}$$

* Adding $\left(\frac{m}{2a}\right)^2$ on both sides;

$$x^2 + 2(x)\left(\frac{m}{2a}\right) + \left(\frac{m}{2a}\right)^2 = \frac{-n}{a} + \left(\frac{m}{2a}\right)^2$$

$$\left(x+\frac{m}{2l}\right)^2 = \frac{-n}{l} + \left(\frac{m}{2l}\right)^2$$

$$\left(x+\frac{m}{2l}\right)^2 = \frac{-n}{l} + \frac{m^2}{4l^2}$$

Taking $\sqrt{}$ on both sides:-

$$\sqrt{\left(x+\frac{m}{2l}\right)^2} = \pm\sqrt{\frac{-4ln+m^2}{4l^2}}$$

$$x+\frac{m}{2l} = \pm\frac{\sqrt{-4ln+m^2}}{2l}$$

$$x = \frac{-m}{2l} \pm \frac{\sqrt{-4ln+m^2}}{2l}$$

$$\Rightarrow \frac{-m \pm \sqrt{4ln+m^2}}{2l}$$

Sol set $= \left\{ \frac{-m+\sqrt{4ln+m^2}}{2l}, \frac{-m-\sqrt{-4ln+m^2}}{2l} \right\}$

(V) $3x^2+7x=0$

Sol:- Dividing each term by "3":-

$$\frac{3x^2}{3} + \frac{7}{3}x = 0$$

$$x^2 + \frac{2}{2}\cdot\frac{7}{3}x = 0$$

$$x^2 + 2\left(\frac{\pi}{6}\right)x = 0$$

* Adding "$\left(\frac{\pi}{6}\right)^2$" on both sides:-

$$x^2 + 2\left(\frac{\pi}{6}\right)x + \left(\frac{\pi}{6}\right)^2 = 0 + \left(\frac{\pi}{6}\right)^2$$

$$\left(x + \frac{\pi}{6}\right)^2 = \frac{49}{36}$$

* Taking $\sqrt{}$ on both sides:-

$$\sqrt{\left(x + \frac{\pi}{6}\right)^2} = \pm\sqrt{\frac{49}{36}}$$

$$x + \frac{\pi}{6} = \pm\frac{7}{6}$$

$$x = \frac{-\pi \pm 7}{6}$$

$$\Rightarrow \frac{-\pi + 7}{6}, \frac{-\pi - 7}{6}$$

$$0, -14/6$$

$$, \frac{-\pi}{3}$$

$$\boxed{\text{Sol set} = \left\{0, \frac{-\pi}{3}\right\}}$$

((vi)) $x^2 - 2x - 195 = 0$

Sol:N $x^2 - 2x - 195 = 0$

$x^2 - 2x = 195$

* Adding "$(1)^2$" on both sides;

$(x)^2 - 2(x)(1) + (1)^2 = 195 + 1$

$(x-1)^2 = 196$

* Taking $\sqrt{}$ on both sides:N

$\sqrt{(x-1)^2} = \pm\sqrt{196}$

$x - 1 = \pm 14$

$x = 1 \pm 14$

$1 + 14 \quad , \quad 1 - 14$

$15 \quad ; -13$

Sol set $= \{15, -13\}$

((vii)) $-x^2 + \frac{15}{2} = \frac{7}{2}x$

Sol:N $\frac{15}{2} = x^2 + \frac{7}{2}x$

$x^2 + \frac{7}{2}x = \frac{15}{2}$

$$x^2 + \frac{2}{2}(x)\left(\frac{7}{2}\right) = \frac{15}{2}$$

$$x^2 + 2x\left(\frac{7}{4}\right) = \frac{15}{2}$$

* Adding "$\left(\frac{7}{4}\right)^2$" on both sides :-

$$(x^2) + 2(x)\left(\frac{7}{4}\right) + \left(\frac{7}{4}\right)^2 = \frac{15}{2} + \left(\frac{7}{4}\right)^2$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{15}{2} + \frac{49}{16}$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{120 + 49}{16}$$

* Taking $\sqrt{}$ on both sides :-

$$\sqrt{\left(x + \frac{7}{4}\right)^2} = \sqrt{\frac{169}{16}}$$

$$x + \frac{7}{4} = \pm\frac{13}{4}$$

$$x = -\frac{7}{4} \pm \frac{13}{4}$$

$$\Rightarrow \frac{-7+13}{4}, \frac{-7-13}{4}$$

$$6/4, -20/4$$

$\{$Sol set $= \{3/2, -5\}$

(vii) $x^2 + 17x + \frac{33}{4} = 0$

Sol:- $x^2 + 17x = -\frac{33}{4}$

$x^2 + \frac{2}{2} \cdot 17x = -\frac{33}{4}$

$x^2 + 2(x)\left(\frac{17}{2}\right) = -\frac{33}{4}$

* Adding "$\left(\frac{17}{2}\right)^2$" on both sides:-

$(x^2) + 2(x)\left(\frac{17}{2}\right) + \left(\frac{17}{2}\right)^2 = -\frac{33}{4} + \left(\frac{17}{2}\right)^2$

$\left(x + \frac{17}{2}\right)^2 = -\frac{33}{4} + \frac{289}{4}$

$\left(x + \frac{17}{2}\right)^2 = \frac{-33 + 289}{4}$

$\left(x + \frac{17}{2}\right)^2 = \frac{256}{4}$

* Taking $\sqrt{}$ on both sides:-

$\sqrt{\left(x + \frac{17}{2}\right)^2} = \pm\sqrt{\frac{256}{4}}$

$$x + \frac{17}{2} = \pm\frac{16}{2}$$

$$x = \frac{-17 \pm 16}{2}$$

$$\Rightarrow \frac{-17+16}{2}, \frac{-17-16}{2}$$

$$\frac{-1}{2}, \frac{-33}{2}$$

Sol set $= \{-1/2, -33/2\}$

(ix) $$4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$$

Soln $$4 = \frac{3x^2+5}{3x+1} + \frac{8}{3x+1}$$

$$4 = \frac{3x^2+5+8}{3x+1}$$

$$4 = \frac{3x^2+13}{3x+1}$$

$4(3x+1) = 3x^2+13$ ... Cross multiplication

$12x + 4 = 3x^2 + 13$

$4 - 13 = 3x^2 - 12x$

$3x^2 - 12x = -9$

* Dividing each term by "3":-

$$\Rightarrow \frac{3}{3}x^2 - \frac{12}{3_1}x = \frac{-9_3}{3_1}$$

$$x^2 - 4x = -3$$

$$x^2 - \frac{2}{2_1} \cdot 4^2(x) = -3$$

$$x^2 - 2(x)(2) = -3$$

* Adding "$(2)^2$" on both sides:-

$$(x^2) - 2(x)(2) + (2)^2 = -3 + (2)^2$$

$$(x-2)^2 = -3+4$$

$$(x-2)^2 = 1$$

* Taking $\sqrt{}$ on both sides:-

$$\sqrt{(x-2)^2} = \pm\sqrt{1}$$

$$x-2 = \pm 1$$

$$x = 2 \pm 1$$

$$\Rightarrow 2+1 \ , \ 2-1$$

(3) , (1)

{Sol set = [3, 1]}

((X)) $7(x+2a)^2+3a^2=5a(7x+23a)$

Sol:N $7(x^2+4ax+4a^2)+3a^2=35ax+115a^2$

$7x^2+28ax+28a^2+3a^2=35ax+115a^2$

$7x^2+28ax-35ax+31a^2-115a^2=0$

$7x^2-7ax-84a^2=0$

* Dividing each term by "7":N

$\frac{7x^2}{7}-\frac{7}{7}ax-\frac{84a^2}{7}=0$

$x^2-ax-12a^2=0$

$x^2-ax=12a^2$

$x^2-\frac{2}{2}.ax=12a^2$

$x^2-2(x)\left(\frac{a}{2}\right)=12a^2$

* Adding "$\left(\frac{a}{2}\right)^2$" on both sides:N

$(x^2)-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$

$\left(x-\frac{a}{2}\right)^2=\frac{48a^2+a^2}{4}$

* Taking $\sqrt{}$ on both sides:N

$\sqrt{\left(x-\frac{a}{2}\right)^2}=\pm\sqrt{\frac{49a^2}{4}}$

$$x - \frac{a}{2} = \pm \frac{7a}{2}$$

$$x = \frac{a \pm 7a}{2}$$

$$\frac{a+7a}{2} \quad , \quad \frac{a-7a}{2}$$

$$\frac{8a}{2} \quad , \quad \frac{-6a}{2}$$

$$4a \quad , \quad -3a$$

Sol set $= \{4a, -3a\}$