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ELECTROSTATICS

charges at rest

1. Charge: "Charge is an intrinsic property of matter. The amount of charge that an object has can not be changed by changing size shape or form but can be changed by adding or removing electrons or by breaking down atoms." $Q = ne$ Charge is quantized i.e., it is always an integral multiple of elementary charge (unit charge) | PKMZ: $n=3$, $Q = 3\bar{e}$ ← $\bar{e}\bar{e}\bar{e}$ ← body having charge 'n' is always a whole no.

1. Coulomb's law: "The magnitude of the force between two point charges (point charges are such charges which have very small size as compared to the distance by which they are separated) is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance b/w them."

Mathematically:

$F \propto q_1 q_2$ and $F \propto \frac{1}{r^2}$

thus $F \propto \frac{q_1 q_2}{r^2}$ or $\boxed{F = k\frac{q_1 q_2}{r^2}}$

Vectorially

$\vec{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$ [w/respect to diagram(a)]

$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{21}$ [w/ respect to digram (b)]

unit vectors $\hat{r}_{12}$ and $\hat{r}_{21}$ are in opposite directions thus

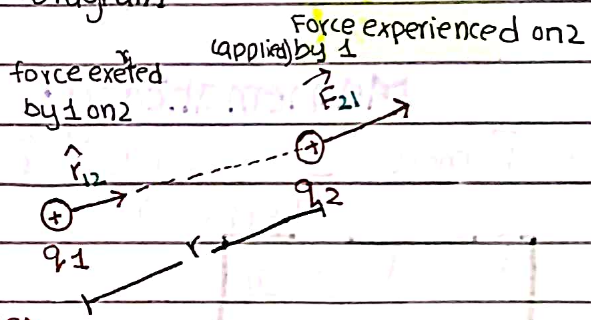
$\hat{r}_{12} = -\hat{r}_{21}$

or (action reaction are equal but opposite: figure c)

$\vec{F}_{21} = -\vec{F}_{12}$

$\frac{kq_1q_2}{r^2}\hat{r}_{12} = -\frac{kq_1q_2}{r^2}\hat{r}_{21}$

Diagram



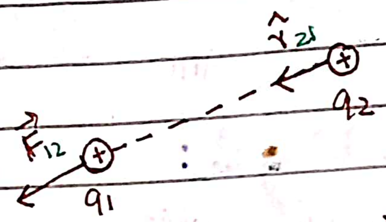
(a)

$\hat{r}_{12}$: Tells us the direction in which force is applied 12 in this case indicates that force is applied in the direction from q_1 to q_2.

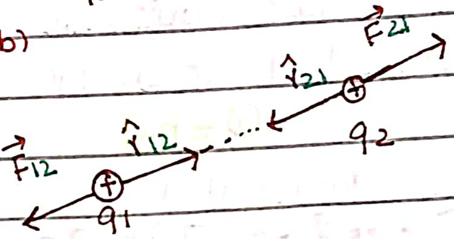
$\vec{F}_{21}$: Tells us by which charge the force is experienced under the influence of the other 21 here indicates that the force is experienced by charge q_2 under the influence of charge q_1.

value of k $K = \frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\ Nm^2C^{-2}$

where $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$



(b)



(c) ~~[combined formofa and b]~~
repulsion b/w similar charges

Coulomb's law in material media:

For material media, ε_0 is replaced by ε

$$\boxed{F_{med} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}}$$

→ ε is the permittivity of medium

→ ε_0 is the permittivity of free space

→ The concept of **permittivity** is that the material between the charges decides wether or not the field lines of charges will pass.

→ $\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}}$ ε_r is relative permittivity or dielectric constant

Important point: 'Free space' applies to vaccum. The reason why ε_0 is used even for air although it is a matrial media is that the values of ε_r ~~for~~ space and air have very close values i.e 1 and 1.0006.

Mathematically:

$$\vec{F}_{med} = \frac{1}{4\pi\varepsilon_r\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

$$\boxed{\vec{F}_{med} = \frac{\vec{F}_{vac}}{\varepsilon_r}}$$

↳ Electric Field and it's intensity:

شدت

→ An electric field is defined as any region around a charge in which an electric test charge would experience an electric force.

→ The intensity of an electric field at any point is the force per unit positive test charge placed at that point

→ Test charge: A unit positive charge which is so small that it's not able to distort the original field.

→ Electric field intensity is a vector quantity denoted by $\vec{E}$

→ **SI unit** of electric field intensity is NC^{-1} or Vm^{-1} [multiply and divide NC^{-1} by 'm' ; $NC^{-1} \times \frac{m}{m}$, $Nm \times \frac{C^{-1}}{m}$, $J \times \frac{C^{-1}}{m}$, $(JC^{-1} = V)$, $Vm^{-1} = NC^{-1}$]

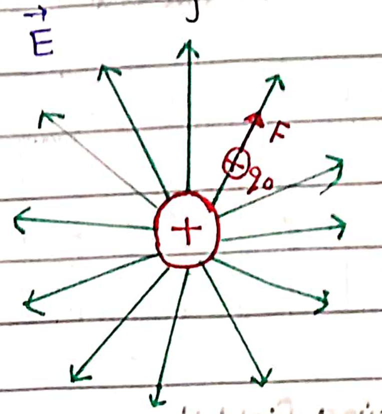
→ **Mathematical form** is $\vec{E} = \frac{\vec{F}}{q_0}$ [force experienced by test charge in field]

subsituting the value for $\vec{F}$ from coulomb's law we get

$$\vec{F} = \frac{kqq_0}{r^2}\hat{r}$$

$$\vec{E} = \frac{kqq_0}{r^2}\hat{r} \times \frac{1}{q_0} \quad , \quad \boxed{\vec{E} = \frac{kq}{r^2}\hat{r}}$$

Diagram



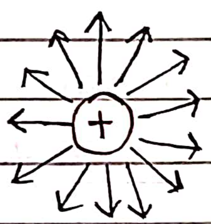
• Electric field intensity due to a point charge

Important Point: $\vec{E}$ being a vector has magnitude and direction both. The direction of $\vec{E}$ depends upon the sign of source charge.

→ **Conclusion** $\vec{E} \propto \frac{1}{r^2}$ (strength of field decreases as test charge moves away from source charge)

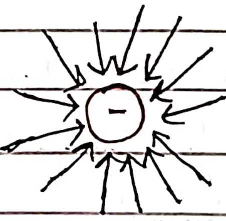
$\vec{E} \propto q$ (strength of field is directly proportional to magnitude of the charge)

Electric field Lines : "Pictorial representation of electric field around a charge are called electric field lines."



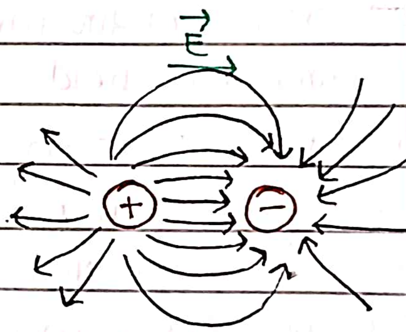
The arrows pointing outwards show the direction in which +ve test charge will accelerate upon placing it in field.

▵ Positive charge



Arrows pointing inward show +ve test charge will accelerate towards the -ve charge

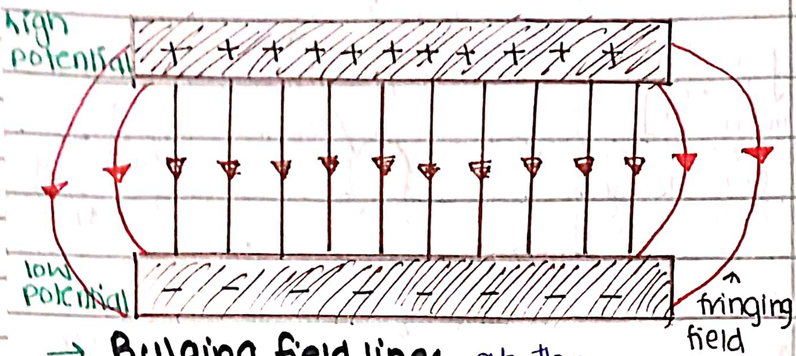
▵ negative charge



arrows in this case show that test charge will move from +ve charge towards -ve charge

▵ Two equal and opposite charges

▾Uniform electric field between charged plates



→ **Bulging field lines** at the corners of the plates represents weakend electric field strength at this area.

→ **To overcome bulging** plates of infinite length can be used.

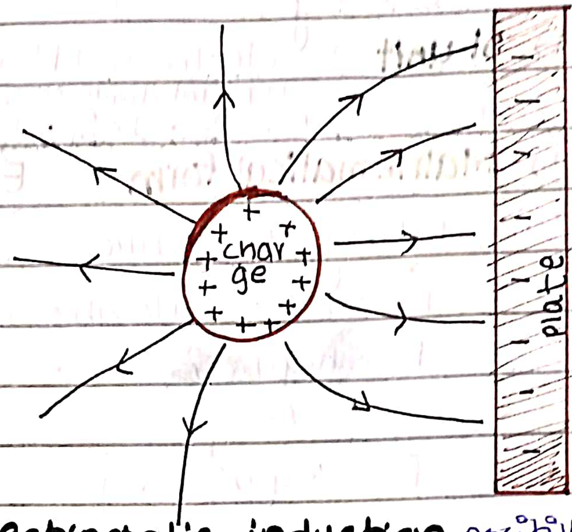
properties of field lines

→ The electric field lines initiate from +ve charge and terminate at -ve charge surface

→ The number of lines drawn is directly proportional to the magnitude of charge

→ curved lines show non uniform electric field and straight parallel lines show uniform electric field.

▾Electric lines of force on the surface of metal plate



→ **Electrostatic induction** positive charge will attract the free electrons present inside the metal plate. This way positive and negative charges in ~~metal plate will be separated~~.

→ **zero electric field inside conductor** when the positive charge engages all the negative charges with it self, field lines will not be able to enter the conductor rather they'll be outside between the +ve charge and negative charge on outer surface of the metal plate.

→ The tangent at any point on a curved line gives the direction of field. intensity at that point. → Where field lines are close to each other the strength of field is greater and vice versa.

→ field lines never cross each other, the propagate in straight lines and move away from each other as distance increases. If the cross each other it would mean electric field intensity has numerous directions but for a given point, it has only 1 direction

→ Field lines do not exist inside charged body.

Electric Flux: "The measure of number of lines of force that pass through a vector area placed in the electric field is called electric flux ϕ_E"

Flux to pass through vector area

Vector area $\vec{A}$ if area is considered with unit vector $\hat{n}$, it is vector area

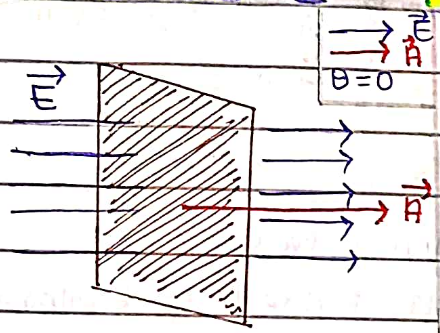
Mathematically: $\phi_E = \vec{E}\cdot\vec{A}$ (dot product)

SI unit: volt metres

$\boxed{\phi_E = EA\cos\theta}$ (scalar quantity)

Depends On: (i) surface area (ii) electric field intensity (iii) The orientation of surface area A with respect to field Lines E.

Case I $\vec{E} \parallel \vec{A}$



$\phi_E = EA\cos 0$

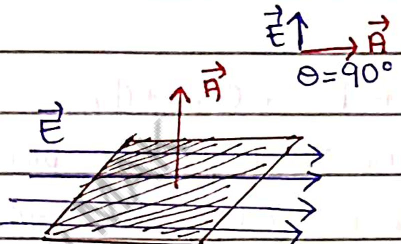
$EA\cos 0 = EA$

$(\cos 0 = 1)$

$\phi_E = EA$

maximum flux

Case II $\vec{E} \perp \vec{A}$



$\phi_E = EA\cos 90°$

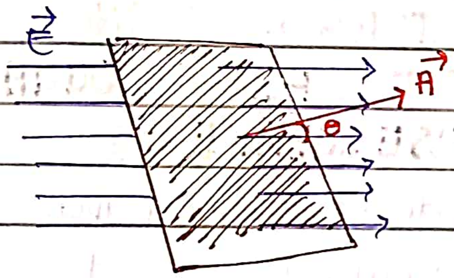
$\cos 90 = 0$

$\phi_E = EA(0)$

$\phi_E = 0$

minimum flux

Case III arbitrary angle



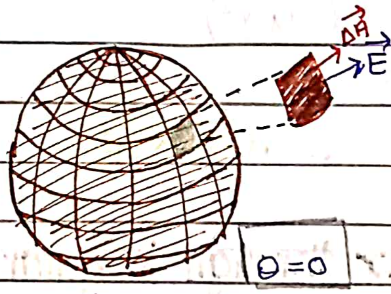
$\phi_E = EA\cos(\theta)$

$\phi = EA\cos\theta$

flux in between max and min flux

θ is less than 90° and more than 0°

Case IV closed surface



we divide the whole sphere in small patches having area ΔA thus

$\phi_E = E\cdot\Delta A$ (one patch)

$\phi_E = \Sigma\vec{E}\cdot\Delta\vec{A}$ (total patches)

$\phi_E = E\Delta A$ $(\cos\theta = 1)$

maximum flux

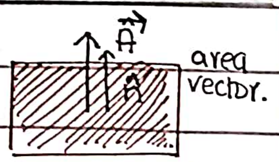
Important Points

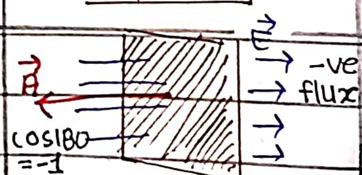
→ Area vector $\hat{n}$ is always perpendicular to the surface

→ If $\vec{E}$ is perpendicular to 'surface' flux is NOT zero (see case I) it's maximum but if $\vec{E}$ is perpendicular to 'area vector' (case II) flux is minimum.

→ Flux can be negative if $\vec{E}$ and $\vec{A}$ are antiparallel $(\theta = 180°)$

→ Through a closed surface, electric flux is ; zero – if no. of field lines entering are equal to those leaving, positive – if no. of field lines entering are less than those leaving and negative – if no. of field lines entering are more than those leaving.





1) Gauss's Law : "Guass's law states that the net electric flux through a closed surface is equal to the total charge q enclosed by the surface divided by the permittivity of free space."

Mathematically

$\phi_1 = \vec{E_1} \cdot \Delta \vec{A_1} = E_1 A_1 \cos 0 = E_1 \Delta A_1$

$\phi_2 = \vec{E_2} \cdot \Delta \vec{A_2} = E_2 \Delta A_2 \cos 0 = E_2 \Delta A_2$

$\phi_3 = \vec{E_3} \cdot \Delta \vec{A_3} = E_3 \Delta A_3 \cos 0 = E_3 \Delta A_3$

$\phi_n = \vec{E_n} \cdot \Delta \vec{A_n} = E_n \Delta A_n \cos 0 = E_n \Delta A_n$

Total flux

$$\phi_E = \phi_1 + \phi_2 + \phi_3 + \dots + \phi_n$$

since $\vec{E}$ is same for all patches placed at equal radius but ΔA is different we can state

$$\boxed{\phi_E = E \sum_{surface} \Delta A}$$

where $E = \frac{q}{4\pi\varepsilon_0 r^2}$

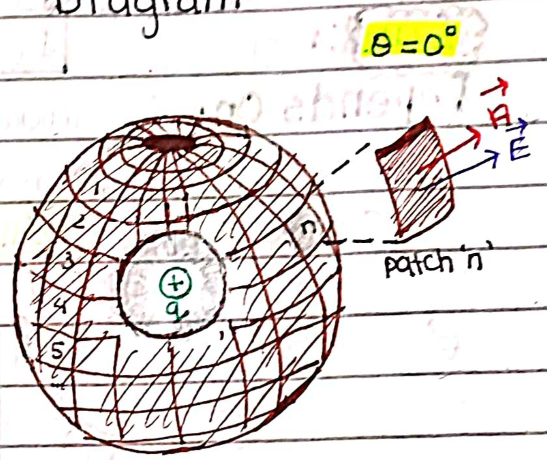
$\phi_E = \frac{q}{4\pi\varepsilon_0 r^2} \sum_{surface} \Delta A$

total area $= 4\pi r^2$

$\phi_E = \frac{q}{4\pi\varepsilon_0 r^2} (4\pi r^2)$

$$\boxed{\phi_E = \frac{q}{\varepsilon_0}}, \quad \boxed{\phi_E \propto q}$$

Diagram



→ suppose +ve charge is place inside the sphere with radius 'r' and all the area is divided in small patches.

FACTORS AFFECTING ELECTRIC FLUX THROUGH CLOSE SURFACE

→ It does not depend upon the geometry of a closed surface.

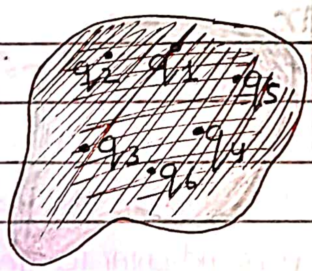
→ Reason for upper point is that all field lines pass through the surface

→ It depends upon the medium

→ It is directly proportional to charge enclosed by the closed surface.

$\phi_E \propto q$

→ Electric flux through irregular closed surface



$\phi_E = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \frac{q_3}{\varepsilon_0} + \dots + \frac{q_n}{\varepsilon_0}$

$= \frac{1}{\varepsilon_0}(q_1 + q_2 + q_3 + \dots + q_n)$

$$\boxed{\phi_E = \frac{1}{\varepsilon_0}\sum_{i=1}^{n} q_i} \quad \text{or} \quad \boxed{\phi_E = \frac{Q}{\varepsilon_0}}$$ (Q = total charge)

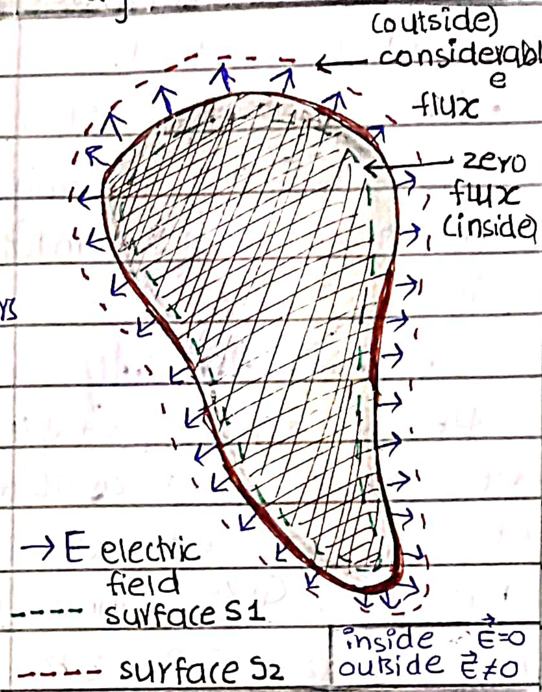
→ Thus Guass's law shows that the electric flux through any closed surface is $\frac{1}{\varepsilon_0}$ times the total charge Q.

II) applications :

Guassian Surface : such a surface on which guass's law is appliciable.

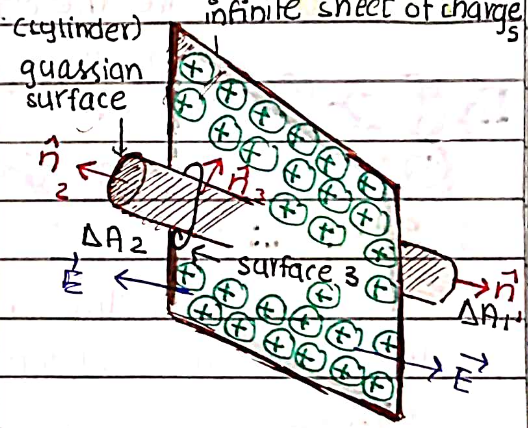
→ Electric Flux Inside A Conductor

Diagram



- A conductor contains free electrons and +ve ions
- Field lines are due to free charges i.e., electrons in this case and not the metal cations as they are fixed.
- Skin effect can be seen in the case of conductors which is that the charge resides on the outer surface of the conductor (even when current flows it's through the outer surface).
- Thus guassian surface S1 will have zero flux due to $\boxed{\text{zero value of } \vec{E}}$ where as S2 will have a $\boxed{\text{considerable value of } \vec{E}}$

→ Electric field Intensity due to An Infinite sheet of Charges



- Surface charge Density tells us that in a given area, how much charge is present $\boxed{\sigma = Q/A}$
- For an ifinite sheet of charges, we will consider a cylinderical guassian surface as this shape covers all dimensions of the infinite sheet.

Mathematically Guassian surfaces : = 3

we know $\sigma = Q/A$ and $Q = \sigma A$ so $\boxed{\phi_E = \frac{\sigma A}{\varepsilon_0}}$ ①

$\phi_1 = \vec{E}_1 \Delta \vec{A}_1 \cos\theta_1 = EA_1 = EA$ ②

$\phi_2 = \vec{E}_2 \Delta \vec{A}_2 \cos\theta_2 = EA_2 = EA$ ③

$\phi_3 = \vec{E}_3 \Delta \vec{A}_3 \cos\theta_3 = 0$ ④

ϕ_1	$\vec{E}$ $\hat{n}$ $(\vec{A}_1)$	$\theta_1 = 0°$
ϕ_2	$(\vec{A}_2)$ $\hat{n}$ $\vec{E}$	$\theta_2 = 0°$
ϕ_3	$\vec{A}_3$ $\vec{E}$	$\theta_3 = 90°$

Total flux $\phi_{EA} = \phi_1 + \phi_2 + \phi_3$ (putting 2,3,4)

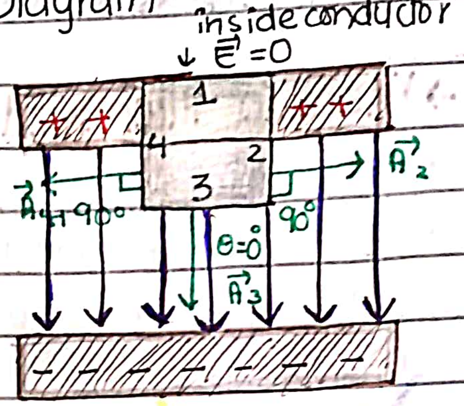
$\phi_{EA} = EA + EA + 0 = 2EA$ (comparing w/ ①)

$2EA = \sigma A/\varepsilon_0$, $\boxed{E = \frac{\sigma}{2\varepsilon_0}}$ or $\boxed{\vec{E} = \sigma/2\varepsilon_0 \, \hat{r}}$

→ Electric Field Intensity B/W Two Oppositely Charged Parallel Plates

- Plates are considered to be of infinite length so as to avoid fringing fields.
- The cube's upper side is assumed to be inside the plate as that is the origin of field lines.
- In order to cover all sides in between the plates, a cube shape is chosen.

Diagram



▲ a pical view of the plates and the box making the box appear 2d

Mathematically

$\phi E = \phi 1 + \phi 2 + \phi 3 + \phi 4 = E_1A_1\cos\theta_1 + E_2A_2\cos\theta_2 + E_3A_3\cos\theta_3$

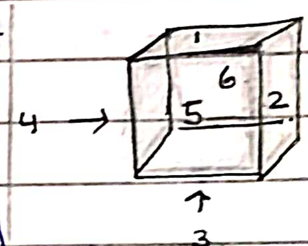
$\phi E = 0 \times A_1\cos\theta + E_2A_2\cos 90° + E_3A_3\cos 0 + E_4A_4\cos 90 + E_4A_4\cos\theta_4$

$\phi E = 0 + 0 + EA + 0$, $\phi E = EA$ ①

But $\phi E = \Phi/\varepsilon_0$ where $\Phi = \delta A$ thus $\phi E = \delta A$ ②

By ① and ②, $E\cancel{A} = \frac{\delta \cancel{A}}{\varepsilon_0}$, $\boxed{E = \frac{\delta}{\varepsilon_0}}$ or $\boxed{\vec{E} = \frac{\delta \hat{r}}{\varepsilon_0}}$ ε_0

$\phi 6$ and $\phi 5$ will be 0 as $\theta = 90°$. This is why we don't consider them.



Conclusion

$\boxed{E \propto \delta}$ more the charge, more the electric field intensity

⇒ Calculating Flux Through Arbitrary Guassian surface:

→ **Step 1:**

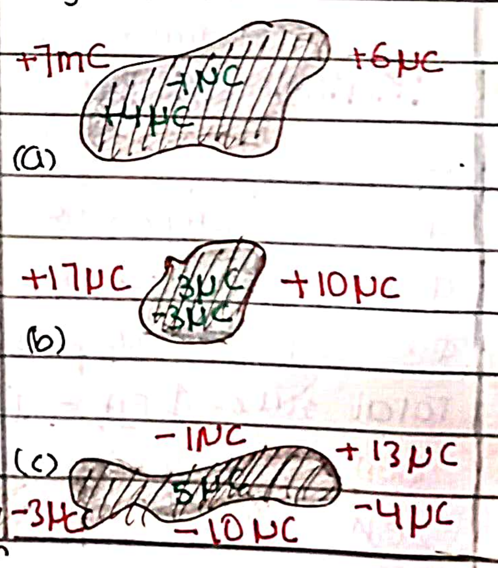
Only consider the charges present inside the guassian surface and **not** those on the outside

→ **Step 2:**

Find the net charge present inside and use the equation : $\boxed{\phi E = \Phi/\varepsilon_0}$

- $\Phi E(a) \rightarrow \quad \phi E = \frac{+4\mu C - 1\mu C}{8.85 \times 10^{-12}\, C^2N^{-1}m^{-2}} = 0.3 \times 10^6\, Vm$
- $\Phi E(b) \rightarrow \quad \phi E = \frac{0}{8.85 \times 10^{-12}} = 0\, Vm$
- $\Phi E(c) \rightarrow \quad \phi E = 5\mu C / 8.85 \times 10^{-12} = 0.5649 \times 10^6\, Vm$

Diagram



Electric Potential :- "Electric Potential is the ability possessed by a charge to perform work"

→ **Electric Potential Energy** is the potential energy stored by a unit positive charge q_o when work is done on it to displace it against the electric field.

→ **Work Energy theorem and Electric Potential**

consider figure (1) and (2). When charge $+q_o$ is displaced against the field from point (b) to (a) it faces opposition due to positive charge at plate (1). Thus energy will be required to displace it. This energy, in the form of work done against the field, will be stored in charge as electric potential energy. "This electric potential energy per unit charge is basically electric potential."

Mathematically:

Electric Potential at point A $\boxed{\uparrow V_A = \uparrow W/q_o}$ ①

" at point B = $\boxed{\downarrow V_B = \downarrow W/q_o}$ ②

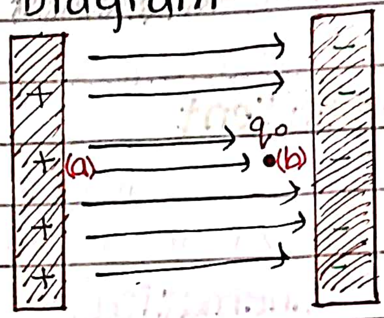
(↑ shows more work is done to displace charge from (b) to (a) and ↓ shows less work required for charge to move from (a) to (b))

but according to work energy theorem,

Work = energy thus $\boxed{W = U}$ (where U is electric P.E)

thus $\boxed{V_A = U_A/q_o}$ ③ and $\boxed{V_B = U_B/q_o}$ ④

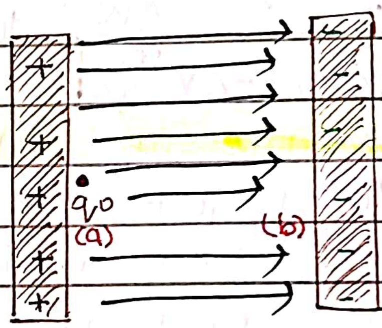
Diagram



(1) high potential (2) low potential

▲ suppose charge is at point (b) near plate (2)

figure 1 ↑



▲ In order to displace the charge towards plate (1) work will be done because this movement is against the electric field. Thus energy will be required

figure 2 ↑

Potential Difference b/w (a) and (b)

$\Delta V = V_A - V_B$, $\Delta V = U_A/q_o - U_B/q_o$

$\boxed{\Delta V = 1/q_o\,(U_A - U_B)}$ or $\boxed{\Delta V = \frac{\Delta U}{q_o}}$ ⑤

where $\Delta U = W_{B\to A}$ (work from b→a)

thus eq (5) becomes $\boxed{\Delta V = \frac{W_{B\to A}}{q_o}}$

if 'B' is at infinity then $\boxed{V_A = \frac{W_{\infty A}}{q_o}}$

→ Units

$V = \frac{W}{q_o}$ → Joule / → Coulomb

Volts

$\boxed{\text{Volt} = JC^{-1}}$ mV, KV, GV

$V = \frac{U}{q_o}$ ← energy to do work; V ↑ ability; q_o ← unit charge

Field And Potential Gradient

"The rate of change of electric potential ΔV with respect to displacement Δr is known as potential gradient"

→ **Gradient** tells us how one physical quantity changes with respect to other quantity. (ΔV's variation with Δr in this case)

Mathematically

we know work done in displacing charge equals

$W = F \cdot d$ or $W = F\Delta r$ but $F = q_o E$ [by electric field intensity]

Thus $\boxed{W = q_o E \times \Delta r}$ (1)

we also know that $\Delta V = \frac{W_{B \to A}}{q_o}$ [electric potential's change from previous topic]

or $\Delta V = \Delta W / q_o$ thus $\Delta V \times q_o = \Delta W$

$\boxed{\Delta W = -\Delta V \times q_o}$ (2)

Negative sign's reason we put negative sign because the charge is moving from point 'b' to 'a' against the electric field's direction (from plate 1→2) electric ←, charge →

combining (1) and (2) we get $\cancel{q_o} E \Delta r = -\Delta V \cancel{q_o}$

or $\boxed{E = -\frac{\Delta V}{\Delta r}}$ → **Conclusion** strength of field equals rate of change of electric potential with displacement.

• Key point {PKMZ}

(with respect to figure 1 and 2 from electric field)

$\uparrow V_A = \frac{W\uparrow}{q_o}$

$\downarrow V_B = W/q_o \downarrow$

electric potential is higher at point 'a' as compared to point 'b'. This difference is due to the displacement 'Δr' of charge q_o from point b.

• Less displacement from point b → less electric field intensity

• More displacement from point b → more electric field intensity

"This rate of change of ΔV is potential gradient".

Equipotential surfaces

: "The surfaces which have equal electrostatic potential at each point are known as Equipotential surfaces"

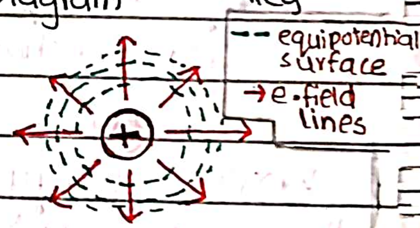
→ **Work done** on a charge moving along an equipotential surface is zero because there is no change in Potential Energy

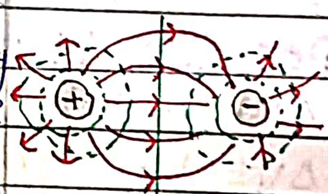
→ **Perpendicular to field lines**; equipotential surfaces are always perpendicular to electric field lines.

→ **Interior of conductor** is equipotential region due to no field lines.

→ **Parallel plates**: between two parallel plates, equipotential

Diagram





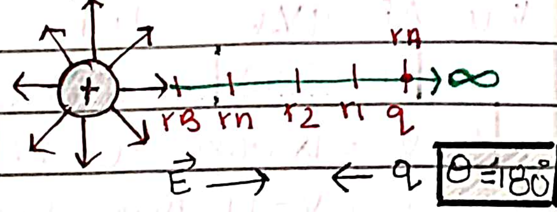
• suppose +ve charge has potential +5 and -ve charge has potential -2 then the equipotential surface everywhere will have potential of $+5-2=3$

surfaces are evenly spaced and are parallel to the plates. The potential difference between adjacent surfaces is constant.

→ **Irregular shapes** have complex electric field configuration. Equipotential surfaces can take various shapes and need not be symmetric.

Electric Potential Energy and Potential Due to a point charge:

Diagram



∴ work done on a test charge moving towards a source charge

→ Division of displacement

Displacement is divided into small patches so that electric field intensity remains constant in each patch.

→ Introduction

Suppose a charge Q (source) with electric field intensity $\vec{E}$ which extends upto infinity. A unit positive test charge q is brought close to it but due to mutual positive charge it will face repulsion thus external work will be done to displace the charge q towards Q. This work done will be electric potential energy.

Mathematical calculations

work $\Delta W = \vec{F} \cdot \Delta\vec{r} = F\Delta r\cos\theta$

$\Delta W = F\Delta r\cos 180° = F\Delta r(-1)$

$\Delta W = -F\Delta r$ where $F = qE$

thus $\Delta W = -qE\Delta r$ but since work is done against the field $\Delta W = -(qE\Delta r) \Rightarrow \boxed{\Delta W = qE\Delta r}$

$\Delta W = q \times \frac{1 \times Q}{4\pi\varepsilon_0 r^2} \times \Delta r$, $\boxed{\Delta W = \frac{qQ\Delta r}{4\pi\varepsilon_0 r^2}}$

where Δr equals $r_x - r_y$ & $r^2 = r_x r_y$

→ 1st negative sign for work

ΔW gets a negative sign due to the factor $\cos 180$.

→ 2nd negative sign for ΔW

• ΔW gets another -ve sign as work is done against the electric field of source charge thus as an end result ΔW will have a positive sign. $(- \cdot - = +)$

$\Delta W_{r_A \to r_1} = \frac{qQ}{4\pi\varepsilon_0}\left(\frac{r_A - r_1}{r_A r_1}\right)$	$\Delta W_{r_1 \to r_2} = \frac{qQ}{4\pi\varepsilon_0}\frac{r_1 - r_2}{r_1 r_2}$	$\Delta W_{r_n \to r_B} = \frac{qQ}{4\pi\varepsilon_0}\left(\frac{r_n - r_B}{r_n r_B}\right)$

$\Delta W_{r_A \to r_B} = \Delta W_{r_A \to r_1} + \Delta W_{r_1 \to r_2} + \dots + \Delta W_{r_n \to r_B}$

$\Delta W_{r_A \to r_B} = \frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$

suppose r_A lies at infinity then $\Delta W_{r_\infty \to r_B} = \frac{qQ}{4\pi\varepsilon_0 r}$

$\boxed{\text{Electric Potential Energy } (U) \text{ at distance } r \text{ from } Q = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]}$, $\boxed{\text{Electric Potential at distance } r \text{ from } Q \ (V) = \frac{W}{q} = \frac{Q}{4\pi\varepsilon_0 r}}$

The Electron Volt: "the amount of energy acquired or lost by an electron when it is displaced across two points between which potential difference is one volt."

Mathematically

since $V = \frac{W}{q}$ [from electric Potential]

$qV = W$ [But work = energy]

thus $\Delta E = q\Delta V$ ['where q = electron]

$$\boxed{\Delta E = 1e\Delta V}$$

$$\boxed{1e.V = 1 \times 1.6 \times 10^{-19} C \times 1V}$$

where $CV = 1J$ thus $1eV = 1.6 \times 10^{-19} J$

→ **Bigger Units**

1 Mega electron volt $= 1MeV = 10^6 eV$

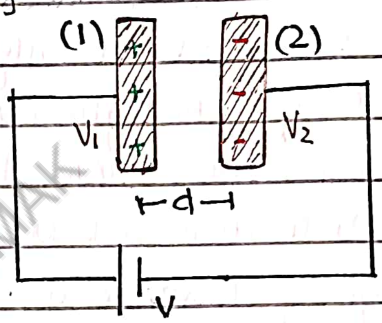
1 Giga electron volt $= 1GeV = 10^9 eV$

Capacitor: "Capacitors are devices which store charge."

→ **Construction**

A simple capacitor consists of two plates; one negative (connected to the -ve terminal) and the other positive (connected to the +ve terminal) and some dielectric between the two.

Diagram (a)



▲ a simple capacitor

higher the voltage of battery, higher the charge depositing at plates

$V \propto Q$

→ **Working**

The plate connected to the positive terminal will acquire positive charge. The plate infront of this +ve plate will become negatively charged due to electrostatic induction.

→ **Potential difference b/w Two plates and capacity of capacitor**

Higher the voltage of the battery, higher the +ve charge acquired on 1st plate and higher the -ve charge induced on the 2nd plate. Consequently higher will be the potential difference developed between the capacitor plates. The capacitor of the capacitor to store charges will be as much as the potential difference of the battery. $\boxed{\uparrow Q \propto \uparrow V}$

→ **When will the capacitor be fully charged?**

When +ve charges at +ve plate begin to get repelled by the +ve charges coming from the +ve terminal, capacitor is said to be fully charged.

→ **Capacitance of a capacitor and its unit**

Capacitance simply refers to capacity. Capacitance of capacitor is its ability to store charge (related with the 3^{rd} point)

Mathematically $Q = CV$, $\boxed{C = \frac{Q}{V}}$

SI unit Farad (F) ; capacity of the capacitor which stores a charge of 1C having 1V P.difference b/w the plates

(μF, pF are other units)

→ **Capacitance of a parallel plate capacitor**

Consider the diagram (a). The plates are connected with a voltage source V and charges on plate (1) and (2) are $+Q$ and $-Q$. Plate (1) is at potential V_1 and plate (2) is at potential V_2. In this case electric field strength between the plates is explained below:

Mathematically $E = -\Delta V / \Delta r$ where $\Delta V = V_2 - V_1$ thus

$E = -(V_2 - V_1)/\Delta r = \frac{V_1 - V_2}{\Delta r}$ but $\Delta r = d$ (separation b/w plates)

here we can write $V_1 - V_2$ as V thus, $\boxed{E = \frac{V}{d}}$ ①

Electric field is also dependent on charges on the plates therefore

as $\sigma = Q/A$ (charge density) By Guass's law we have : $E = \frac{\sigma}{\varepsilon_o}$

or $\boxed{E = \frac{Q}{A\varepsilon_o}}$ ② [By ① and ② we get] $\boxed{\frac{V}{d} = \frac{Q}{A\varepsilon_o}}$

$Q = \frac{\varepsilon_o A V}{d}$ ③ we know that $C_{vac} = \frac{Q}{V}$ ④

[By ③ and ④ we get]

$\boxed{C_{vac} = \frac{Q}{V} = \frac{\varepsilon_o A}{d}}$ if dielectric is present $\boxed{C_{med} = \frac{\varepsilon_o \varepsilon_r A}{d}}$ ⑤

→ **Ways to increase capacitance**

- Increasing area of plates increases capacitance $\boxed{\uparrow C \propto A \uparrow}$ by eq 5
- Increasing number of plates $\boxed{C \propto \text{no. of plates}}$
- Decreasing distance b/w plates $\boxed{\uparrow C \propto 1/d \downarrow}$ by eq 5
- By using a dielectric $\boxed{C_{med} \propto \varepsilon}$ by eq 5
- By connecting capacitor in parallel $\boxed{C_e = C_1 + C_2 + C_3}$

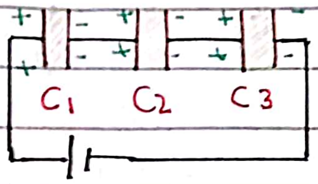
→ **What's a dielectric?**

Dielectric is an insulator which upon exposure to electric field becomes polarized (turns into a dipole)

Every dielectric is an insulator but NOT every insulator is a dielectric.

→ Combination of capacitors

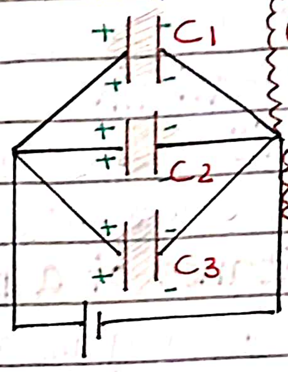
Series



Q same
V different
Through each capacitor

- $C_e < C_{individual}$
- $\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

Parallell



Q different
V same
through each capacitor.

- $C_e = C_1 + C_2 + C_3$
- $C_e > C_{individual}$

↳ Electric Polarization & dipole

:- "The procedure in which upon exposure to electric field the dielectric turns into a dipole is called electric polarization. On the other hand dipole is a system in which two charges of equal magnitude but of opposite sign are separated by the distance d, are present is termed as a dipole."

→ Procedure

[electric polarization of dielectric into dipole]

External constant electric field will cause the atom to stretch. The atom will become polar due to shifting of electron cloud. Greater the extent of stretch, greater will be the polarity.

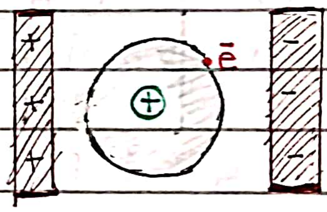
Important point: Every dielectric is an insulator but not every insulator is dielectric.

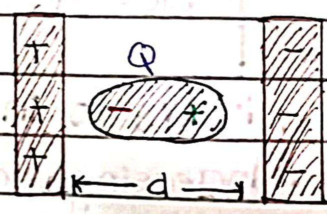
Mathematically

Electric dipole moment is represented by 'p' $\boxed{P = |Qd|}$ (charge x displacement)

- Dipole moment tells us how polar our dipole is
- Dipole moment is a vector quantity.

Diagram





- constant external electric field will stretch the atom. This resultant will be called as a dipole.
- Extent of stretch is directly proportional to electric field.

Energy stored in a capacitor :

- Initially : Zero energy stored in capacitor as no charge is deposited on the plates and the voltage is zero
- Finally : Charges +Q and -Q get deposited on the plates. The potential difference between the plates becomes V.

→ **Why do we take average voltage?**

we have two values for voltage i.e, initial → 0 , final → V

Thus we take average $\boxed{V_{av} = \frac{0+V}{2} = V/2}$

Mathematically

we know that $V = U/Q$ or $U(\text{energy}) = QV$ where $V = V/2$

thus $\boxed{U = QV/2}$ ① where $Q = CV$ thus $U = \frac{CV \times V}{2}$ → $\boxed{U = \frac{CV^2}{2}}$ ②

we also know that $E = V/d$ or $V = Ed$. putting in ②

$U = \frac{1}{2} \times C \times (Ed)^2$ where $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$ thus $U = \frac{1}{2} \times \frac{\varepsilon_0 \varepsilon_r A}{\cancel{d}} \times E^2 d^{\cancel{2}}$

$U = \frac{1}{2} \times \varepsilon_0 \varepsilon_r A \times E^2 d$ or $U = \frac{1}{2} \times \varepsilon_0 \varepsilon_r \times E^2 \times Ad$ where $A \times d = \text{Volume}$

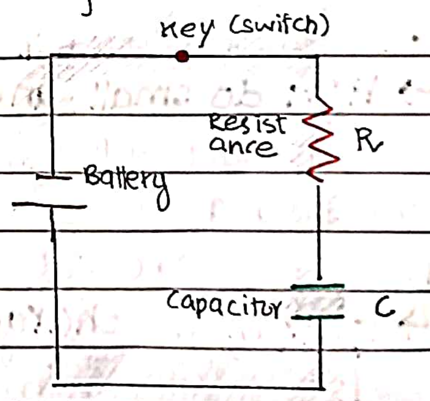
thus $\boxed{U = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 V}$ ③ (Ad is volume b/w the plates)

energy density 'u' $= \frac{\text{Energy}}{\text{Volume}} = \frac{U}{Ad}$ (energy contained by volume b/w plates)

putting eq ③ we get $\boxed{u = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2}$

Charging & discharging of a capacitor through a resistor :

Diagram



→ **Arrangement of circuit**

Capacitor C and Resistor R are joined in series with a battery of voltage V through a key K.

When key is open : (initial conditions) $t=0$, $i=0$ (current), $q=0$

When key is closed : at certain time t : $i = I$, $q = q^{\circ}$

(where q is charge on capacitor and I is current)

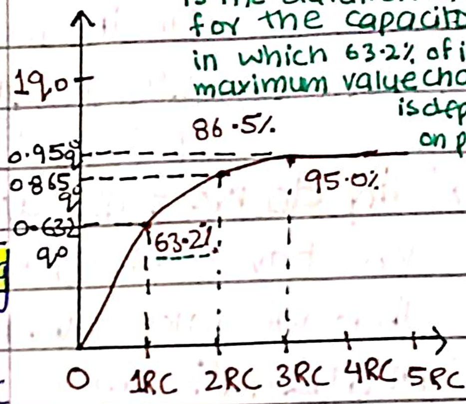
→ Charging

Time constant(t) = R×C

- Initially; plates have no charges, after time 't' plates acquire charges but the process gradually slows down due to repulsion b/w similar charges
- $\boxed{0 \longrightarrow q^\circ}$ (zero to maximum charge q°)
- Higher the value of R×C, higher will be the charging time and viceversa $\boxed{\text{value of } R\times C \propto \text{time for charging or discharge}}$
- Total time required to charge a capacitor fully is equal to five time constants i.e, 5t or 5R×C
- As in the beginning there are no charges on the capacitor plates, charging is very fast and greater curve is observed but as charges accumulate on the plates charging slows and curve flattens the greater curve in beginning shows exponential growth.
- equation for graph ; $\boxed{q = q^\circ(1 - e^{-t/RC})}$
- We can say that a capacitor stores charges due to opposite charges attracting eachother.

Graph

Time constant RC or t is the duration of time for the capacitor in which 63.2% of its maximum value charge is deposited on plates

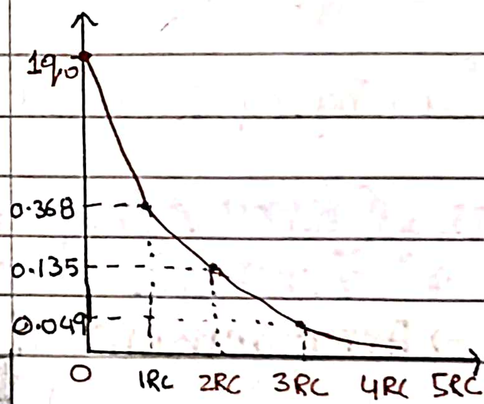


Put values for 't' in equation and get points for x-axis.

→ Discharging

- When the battery is removed, the capacitor starts acting as the battery and resistor works as a load.
- Due to the resistor, charges flow out of capacitor and discharging begins.
- $\boxed{q^\circ \longrightarrow 0}$ (maximum charge to zero.
- equation for graph ; $\boxed{q = q^\circ \times e^{-t/RC}}$

Graph



put values for 't' in equation to get corresponding points for graph.

→ How do small and large RC differ?

Small product of R×C indicates lesser time required for charging aswell as discharging whereas large product of R×C indicates viceversa.

→ Prove that R·C = t

$\boxed{R = V/I}$ and $\boxed{C = Q/V}$ thus

$\frac{V}{I} \times \frac{Q}{V} = \frac{Q}{I}$ but we know

$\boxed{Q = It}$ therefore $\frac{Q}{I} = t$

→ Maximum charge on capacitor

$\boxed{\text{Capacitance} \times \text{emf of battery}}$