Ex 1.3

Q2) **Multiplicative Inverse**

1) $-3i$

$$\frac{1}{-3i}$$

$$-\frac{1}{3i} \times \frac{i}{i}$$

$$-\frac{i}{3i^2}$$

$$-\frac{i}{3(\sqrt{-1})^2} \qquad i = \sqrt{-1}$$

$$-\frac{i}{3(-1)}$$

$$-\frac{i}{-3}$$

$$\frac{i}{3} \rightarrow \text{ans}$$

$(-ai)^4$

$a^4 i^4$

$a^4 (i^2)^2$

$a^4 (-1)^2$

a^4

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i^{-3}

$\frac{1}{i^3}$

$\frac{1}{i^2 \cdot i}$

$\frac{1}{-1 \cdot i} \times \frac{i}{i}$

$\frac{i}{-1 \cdot i^2}$

$\frac{i}{(-1) \times (-1)}$

$\frac{i}{1}$

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i^{-10}

$\frac{1}{i^{10}}$

$\frac{1}{(i^2)^5}$

$\frac{1}{(-1)^5}$

$\frac{\cancel{1}\ -1}{-\cancel{1}\ 1}$

-1.

Prove that

$$\bar{z} = z$$

iff **z is real**

let $z = a$ {which is real}

& $\bar{z} = \bar{a}$

on the other hand $z = \bar{z}$

$z = a + bi$, $\bar{z}$ $a - bi$

$\cancel{a} + bi = \cancel{a} - bi$

$2bi = 0$

$bi = 0/2$

$b = 0$

z is real

Simplify

$5 + 2\sqrt{-4}$

$5 + 2\sqrt{4}i$

$5 + 2 \times 2i$

$5 + 4i$

$(2 + \sqrt{-3})(3 + \sqrt{-3})$

$(2 + \sqrt{3}i)(3 + \sqrt{3}i)$

$\{(2)(3) + 2(\sqrt{3}i) + (\sqrt{3}i)(3) + (\sqrt{3}i)(\sqrt{3}i)\}$

$6 + 2\sqrt{3}i + 3\sqrt{3}i + (\sqrt{3}i)^2$

$6 + 5\sqrt{3}i + 3i^2 \quad \because i^2 = -1$

$6 + 5\sqrt{3}i + 3(-1)$

$6 + 5\sqrt{3}i - 3$

$3 + 5\sqrt{3}i$

$\frac{2}{\sqrt{5} + \sqrt{8}i}$

$$\frac{2}{\sqrt{5}+\sqrt{8}i} \times \frac{\sqrt{5}-\sqrt{8}i}{\sqrt{5}-\sqrt{8}i}$$

$$\frac{2(\sqrt{5}-\sqrt{8}i)}{(\sqrt{5})^2-(\sqrt{8}i)^2} \quad \because i^2=-1$$

$$\frac{2\sqrt{5}-2\sqrt{8}i}{\cancel{2}5-8(-1)}$$

$$\frac{2\sqrt{5}-2\sqrt{8}i}{\cancel{2}5+8}$$

$$\frac{2\sqrt{5}-2\sqrt{8}i}{13}$$

$$\frac{2\sqrt{5}}{13} - \frac{2\sqrt{8}i}{13} \quad \because \sqrt{8}=2\sqrt{2}$$

$$\frac{2\sqrt{5}}{13} - \frac{4\sqrt{2}i}{13}$$

$$\frac{3}{\sqrt{6}-\sqrt{-12}}$$

$$\frac{3}{\sqrt{6}-\sqrt{12}i} \times \frac{\sqrt{6}+\sqrt{12}i}{\sqrt{6}+\sqrt{12}i}$$

$$\frac{3\sqrt{6}+3\sqrt{12}i}{(\sqrt{6})^2-(\sqrt{12}i)^2} = \frac{3\sqrt{6}+3\sqrt{12}i}{6+12}$$

$$\frac{3\sqrt{6}+3\sqrt{12}i}{18}$$

$$\frac{3\sqrt{6}}{18}+\frac{3\sqrt{12}i}{18}$$

$$\frac{\sqrt{6}}{6}+\frac{2\sqrt{3}}{6}i = 1/\sqrt{6} + 1/\sqrt{3}i$$

Show that

$z^2+\bar{z}^2$ is real

let $z = a+bi$

$\bar{z} = a-bi$

$z^2+\bar{z}^2 = (a+bi)^2+(a-bi)^2$

$a^2+\cancel{2abi}+b^2i^2+a^2-\cancel{2abi}+bi^2$

$a^2-b^2 \quad a^2-b^2$

$2a^2-2b^2$

real number

ii)

$$(z-\bar{z})^2$$

$$z-\bar{z} = (a+bi)-(a-bi)$$

$$\cancel{a}+bi-\cancel{a}+bi$$

$$2bi$$

$$(z-\bar{z})^2 = (2bi)^2$$

$$4b^2i^2 \qquad \because i^2=-1$$

$$4b^2(-1)$$

$$-4b^2$$

real.

$$\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3$$

$$\left(\frac{\sqrt{3}}{2}i-\frac{1}{2}\right)^3$$

$$\left(\frac{\sqrt{3}}{2}i\right)^3-\left(\frac{1}{2}\right)^3-3\left(\frac{\sqrt{3}i}{2}\right)\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i-\frac{1}{2}\right)$$

$$-\frac{3\sqrt{3}}{8}i-\frac{1}{2}-\frac{9}{8}+\frac{3\sqrt{3}}{8}i = \frac{8}{8} = 1$$

$$\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^3$$

$$\left(-\frac{1}{2}\right)^3-\left(\frac{\sqrt{3}}{2}\right)^3-3\left(-\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)$$

$$-\frac{1}{8}-\frac{3\cancel{\sqrt{3}}i}{\cancel{8}}+\frac{9}{8}+\frac{3\cancel{\sqrt{3}}i}{\cancel{8}}$$

$$-\frac{\cancel{5}}{4} \quad 1$$

$$\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$$

$$\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2+1}$$

$$\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-1}$$

$$\frac{1}{-1/2-\sqrt{3}/2i}$$

$$\frac{-2}{+1+\sqrt{3}i} \times \frac{+1\mp\sqrt{3}i}{+1\mp\sqrt{3}i}$$

$$\frac{-2+2\sqrt{3}i}{(1)^2-(\sqrt{3}i)^2} = \frac{-2+2\sqrt{3}i}{1-(-3)}$$

$$\frac{-2+2\sqrt{3}i}{1-3}$$

$$\frac{-2+2\sqrt{3}i}{-2}$$

$1+\sqrt{3}i$

v) $(a+bi)^2$

$\because (a+b)^2 = a^2+2ab+b^2$

$= (a)^2 + 2(a)(bi) + (bi)^2$

$= a^2 + 2abi + b^2i^2 \quad \because i^2 = -1$

$= a^2 + 2abi - b^2$

v) $(a+bi)^{-2}$

$$= \frac{1}{(a+bi)^2}$$

$$= \frac{1}{(a)^2+2(a)(bi)+(bi)^2}$$

$$= \frac{1}{a^2+2abi+b^2i^2}$$

$$\frac{1}{a^2+2abi-b^2}$$

$$\frac{1}{a^2+2abi-b^2} \times \frac{a^2-b^2-2abi}{a^2-b^2-2abi}$$

$$\frac{a^2-b^2-2abi}{(a^2-b^2)^2-(2abi)^2}$$

$$\frac{a^2-b^2-2abi}{(a^2-b^2)(a^2+b^2)+4a^2b^2}$$

$$\frac{a^2-b^2-2abi}{(a^2)^2+(b^2)^2-2ab^2-4a^2b^2i^2}$$

$$\frac{a^2-b^2-2abi}{a^4+b^4+2a^2b^2}$$

$$(a+bi)^3$$

$$(a)^3+(bi)^3+3(a)(bi)(a+bi)$$

$$a^3+b^3i+3abi(a+bi)$$

$$a^3+b^3i+3a^2bi+3ab^2(-1)$$

$$a^3+b^3i+3a^2bi-3ab^2$$

$$a^3-3ab^2+b^3i+3a^2bi$$

$$a^3-3ab^2\ i(-b^3+3a^2b)$$

$(a)^3-(bi)^3-3(a)(bi)(a-bi)$

$a^3-b^3i-3abi(a-bi)$

$a^3-b^3i-3a^2bi+3ab^2(-1)$

$a^3-b^3i-3a^2bi-3ab^2$

$a^3-3ab^2-b^3i-3a^2bi$

$a^3-3ab^2-(b+3a^2b)i$

$(3-\sqrt{-4})^3$

$(3-\sqrt{+4}i)^{-3}$

$$\frac{1}{(3-2i)^3}$$

$$\frac{1}{(3)^3-3(3)(2i)(3-2i)-(2i)^3}$$

$$\frac{1}{27-8i-3(6i)(3-2i)}$$

$$\frac{1}{27-8i-18(3-2i)}=\frac{1}{9-8i-54+36i}$$

$$\frac{1}{-9-46i} \times \frac{-9+46i}{-9+46i}$$

$$\frac{-9+46i}{(-9)^2-(46i)^2}$$

$$\frac{-9+46i}{81-2116i^2}$$

$$\frac{-9+46i}{81+2116}$$

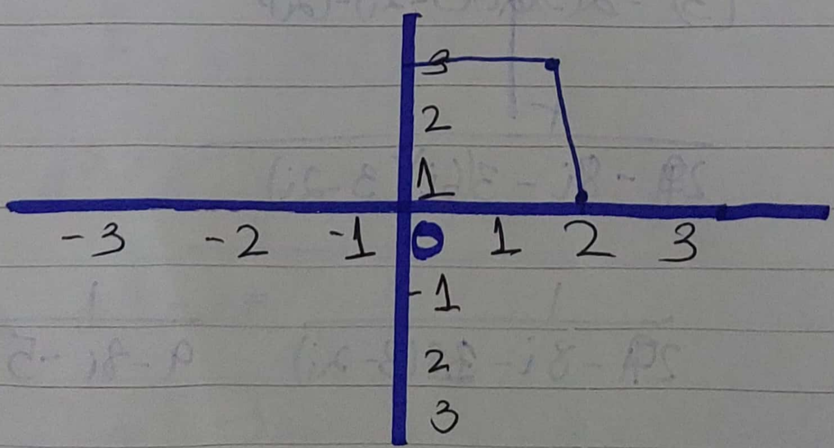
$$\frac{-9+46i}{2197}$$

$$\frac{-9}{2197} + \frac{46}{2197}i$$

Graph

$2+3i$

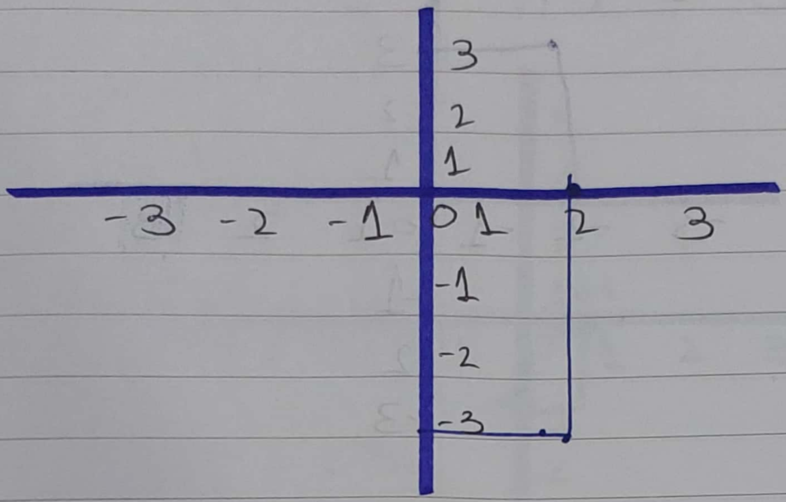
$x = 2$ units
$y = 3$ units



i)

$2 - 3i$

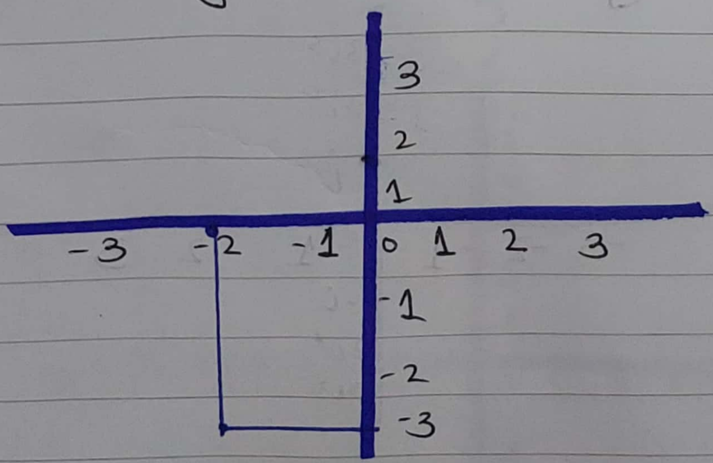
$x = 2$
$y = -3$



i)

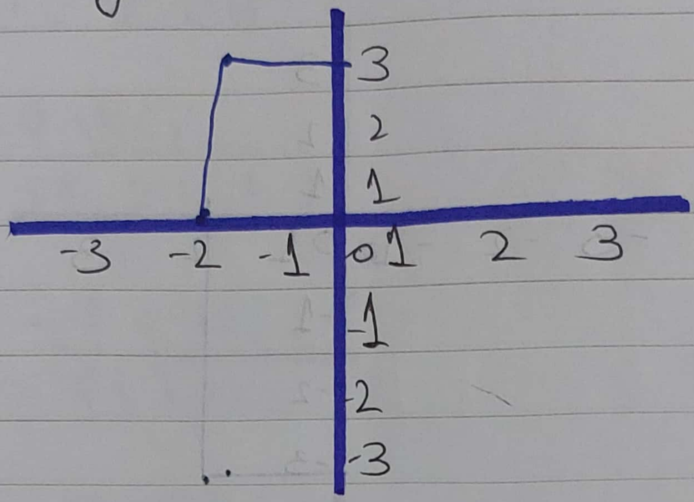
$-2 - 3i$

$x = -2$
$y = -3$



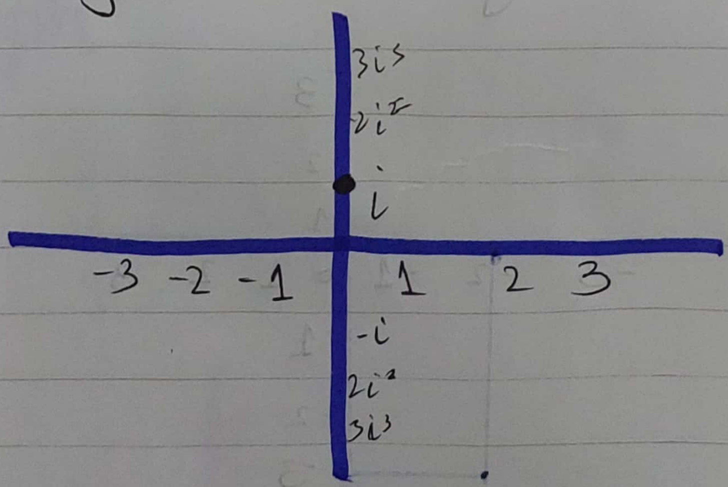
$-2 + 3i$

$x = -2$
$y = 3$



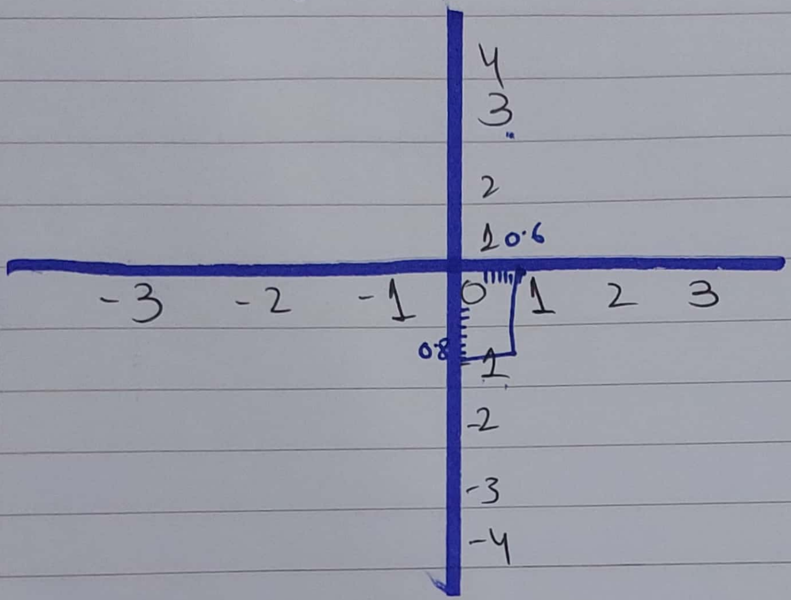
i

$x = 0$
$y = i$



$\frac{3}{5} - \frac{4}{5}i$

$x = 3/5 = 0.6$ unit
$y = 4/5 = 0.8$ units



$-5-6i$
$x = -5$
$y = -6$