Unit no:-05 Rotational and Circular motion

Date: Day:

Relation b/w S, r and θ: $\theta = S/r$ (rad)

Angular velocity:-

$(\omega) = \theta/t \quad \omega = 2\pi f \quad \omega = rpm/60 \times 2\pi \quad \omega = rph/3600 \times 2\pi$

Average angular velocity $\vec{\omega}_{av} = \Delta\vec{\theta}/\Delta t$

Ins. angular velocity $\vec{\omega} = \lim_{\Delta t \to 0} \Delta\vec{\theta}/\Delta t$

Average angular acceleration

$\vec{\alpha}_{av} = \Delta\vec{\omega}/\Delta t$

Instantaneous angular acceleration

$\vec{\alpha}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t}$

$v = r\omega$

$a = r\alpha$

Equation of angular motion:- $\omega_f = \omega_i + \alpha t$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$ (•) $2\alpha\theta = \omega_f^2 - \omega_i^2$

Linear motion	Angular motion
$S = vt$	$\theta = \omega t$
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$v_f^2 - v_i^2 = 2aS$	$\omega_f^2 - \omega_i^2 = 2\alpha\theta$
$S = v_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$
Interia $= m$	Moment of interia $I = mr^2$
$F = ma$	$\tau = I\alpha$
$P = mv$	$L = I\omega$
$K.E_{lin} = \frac{1}{2}mv^2$	$K.E_{rot} = \frac{1}{2}I\omega^2$

Moment of interia of a thin rod of length L $I = \frac{1}{12}mL^2$

Moment of interia of hoop $I = mr^2$

Moment of interia of a disc:- $I = \frac{1}{2}mr^2$

Moment of interia of sphere → $I = \frac{2}{5}mr^2$

Critical orbital velocity:-

$v = \sqrt{2gR} \quad v = \sqrt{\frac{GM}{R}}$

Time period of close orbiting satellite: $T = 2\pi R/v$

Apparent weight

$T = mg$ ★

$T = mg + ma$ ★

$T = mg - ma$ ★

$T = 0$ ★

Rotational K.E → $K.E_{rot} = \frac{1}{2}I\omega^2$

" " of disc → $K.E_{rot} = \frac{1}{4}I\omega^2$

" " of hoop → $K.E_{rot} = \frac{1}{2}I\omega^2$

Velocity of disc → $v = \sqrt{4/3\, gh}$

" " " **sphere** → $v = \sqrt{10/7\, gh}$

" " " **hoop** → $v = \sqrt{gh}$

Orbital velocity:- $v = \sqrt{\frac{GM}{R}}$

Spinning frequency of a satellite:-

$f = \frac{1}{2\pi}\sqrt{g/R}$

▷ **SI unit of angular momentum**

Js. (kgm^2s^{-1})

3 satellites form GPS in geostationary orbit.

Orbital radius of geostationary satellite:- $r = \left(\frac{GmT^2}{4\pi^2}\right)^{1/3}$

K.E total = K.E trans + K.E rot

Critical velocity to put an satellite in an orbit close to earth

---

Angular velocity
- vector quantity
- $[T^{-1}]$
- SI unit rad/s

Angular acceleration
- vector quantity
- $[T^{-2}]$
- rad/s²

Height of geostationary satellite above equator is 3.6×10^7 m

Centripetal force is not a axial vector.

Rest	$T = mg$
Moving up	$T = mg + ma$
Moving down	$T = mg - ma$
Freely fall	$T = 0$

Relation between escape velocity and orbital velocity

$v_{esc} = \sqrt{2gR}$

$v_{orb} = \sqrt{gR}$

$v_{es} = \sqrt{2}(\sqrt{gR})$

$v_{es} = \sqrt{2}\,v_{orb}$

is 7.9km/s

Centripetal acceleration

$a_c = v^2/r$

$a_c = v\omega$

$a_c = r\omega^2$

Centripetal force:-

$F_c = \frac{mv^2}{r}$

$F_c = mr\omega^2$ (In angular measure).

Banking of road

$v = \sqrt{rg\tan\theta}$

Moment of interia ($I = mr^2$)

SI unit kgm^2

Dimension $[ML^2]$

angular momentum and torque:-

$\Delta\vec{L} = \vec{r} \times \Delta\vec{P}$

Divide both sides by Δt

$\frac{\Delta\vec{L}}{\Delta t} = \vec{r} \times \frac{\Delta\vec{P}}{\Delta t}$

$\frac{\Delta\vec{L}}{\Delta t} = \vec{r} \times \vec{F}$

$\vec{\tau} = \Delta\vec{L}/\Delta t$

law of conservation of angular momentum

$\tau = 0$

$\frac{\Delta L}{\Delta t} = 0$

$\Delta L = 0$

$L_2 - L_1 = 0$

$L_2 = L_1$

L = constant