If velocity of body decreases, then ... direction of velocity. If velocity increases then ... will be in the direction of velocity

• max displacement, max Fr, max a from ...

CHAPTER 07

OSCILLATION

"It is to and fro motion of a body about a fixed position in fixed interval of time"

➡ Every periodic motion is not oscillatory motion but every oscillatory motion is periodic motion.

EXAMPLES

1. A mass suspended from spring when pulled down & then released starts oscillating

2. The bob of simple pendulum when displaced from its rest position and released, vibrates.

3. A steel ruler clamped at one end to bench oscillates when the free end is displaced sideways.

4. a steel ball rolling in a curved dish, oscillating about its rest position.

TERMINOLOGY OF OSCILLATORY MOTION

VIBRATORY MOTION To and fro motion of a body about the mean position

VIBRATION The complete round trip of a body about the mean position.

TIME PERIOD The time required by a body to complete one vibration [specific time]

FREQUENCY The no. of vibrations completed by a body in one second. It is the reciprocal of the time period of vibrating body. $f = \frac{1}{T}$ $1Hz = 1Cs^{-1} = cps$ [1 cycle per second]

PERIODIC MOTION Motion which is repeated in equal intervals of time

DISPLACEMENT (x) At any instant, the distance of the oscillating body from mean position.

AMPLITUDE (x_0) The maximum displacement of a body from mean position.

ANGULAR FREQUENCY (ω) The no. of revolution per second of a body. $\omega = \frac{\theta}{T} \Rightarrow \omega = 2\pi \cdot \frac{1}{T} \Rightarrow \omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi}$

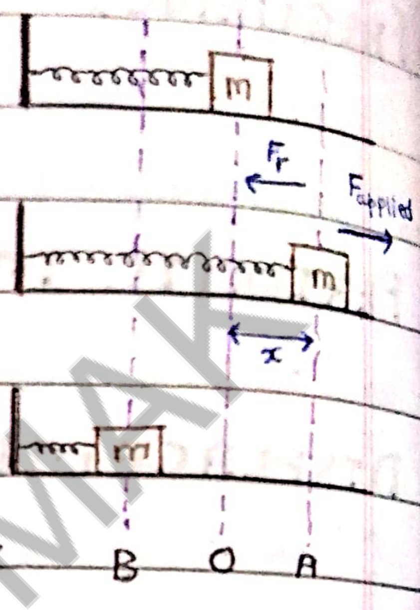
SIMPLE HARMONIC MOTION

DEFINITION Such motion in which acceleration is always directly proportional to the displacement and is always directed toward its mean position is called S.H.M. $a \propto -x$

MOTION OF MASS ATTACHED TO SPRING

Consider a body of mass 'm' attached to one end of an elastic spring which can move freely on a friction-less horizontal surface.

1. Initially, the mass is at Rest position 'O'. When we apply force in right direction, mass will displace to the extreme position 'A'



2. mass will come to mean position due to the **Restoring force**.
3. mass will move to second extreme position 'B' due to **inertia**. The spring will be compressed and start oscillation b/w A & B.

The restoring force is equal and oppositite to the applied force. $F_{applied} = F_{restoring}$

$F_{applied} = ma$ —— (1) $F_{restoring} = -kx$ —— (2)

Comparing eq (1) and (2)

$$ma = -kx$$

$$a = \frac{-k}{m}(x) \quad [\because -\frac{k}{m} = \text{constant}]$$

$$a = -\text{constant}\,(x)$$

$$a \propto -x \quad \text{(acceleration of mass spring system)}$$

TIME PERIOD

$\omega = \frac{\theta}{t}$

$\omega = \frac{2\pi}{t}$

$T = \frac{2\pi}{\omega}$ —— (3)

$a = -x\omega^2$ —— (4)

$a = -\frac{k}{m}x$ —— (5)

comparing eq (4) & (5)

$-x\omega^2 = -\frac{k}{m}x$

$\omega^2 = \frac{k}{m}$

$\omega = \sqrt{\frac{k}{m}}$

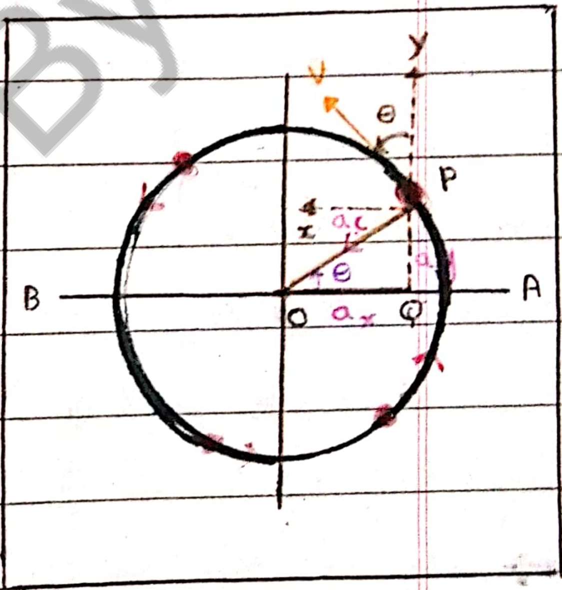
Putting value of 'ω' in eq (3)

$$T = \frac{2\pi}{\sqrt{\frac{K}{m}}} \Rightarrow T = 2\pi \frac{1}{\sqrt{\frac{K}{m}}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

FREQUENCY

$$f = \frac{1}{T} \Rightarrow f = \frac{1}{2\pi\sqrt{\frac{m}{K}}} \Rightarrow \boxed{f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}}$$

CIRCULAR MOTION AND SHM



consider a particle of mass 'm' moving in a circle of radius 'r'.

OP = Representative line showing the motion of a particle in a circle.

V = instantaneous velocity which is tangent to the circle and changing its direction.

Q = projection of a particle vibrating on the diameter $\overline{AB}$

'a_c' is the centripetal acceleration of a body directing toward the mean position 'O'

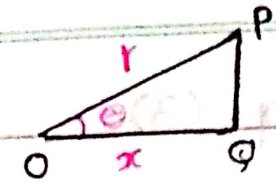
$(a_c)x = a_c \cos\theta$

$a_x = r\omega^2 \cos\theta$ —(1)

$\left[\because a_c = \frac{v^2}{r}\right.$

$\left. a_c = \frac{r^2\omega^2}{r} \Rightarrow a_c = r\omega^2\right]$

In ΔOPQ



Max 'x' when $\theta = 0°$

$x = x_0 \cos\theta$

$x = x_0 \cos 0 \Rightarrow x = x_0$

$\cos\theta = \frac{base}{hypotenuse}$

Min 'x' when $\theta = 90°$

$x = x_0 \cos\theta$

$x = x_0 \cos 90 \Rightarrow x = 0$

$\cos\theta = \frac{x}{r}$

$\boxed{x = r\cos\theta}$ displacement in terms of $\cos\theta$

$x = x_0 \cos\theta \qquad (r = x_0)$

$x = x_0 \cos\omega t$

Putting the value of $\cos\theta$ in eq (1)

$a_x = r\omega^2 \cos\theta$

$a_x = r\omega^2 \frac{x}{r}$

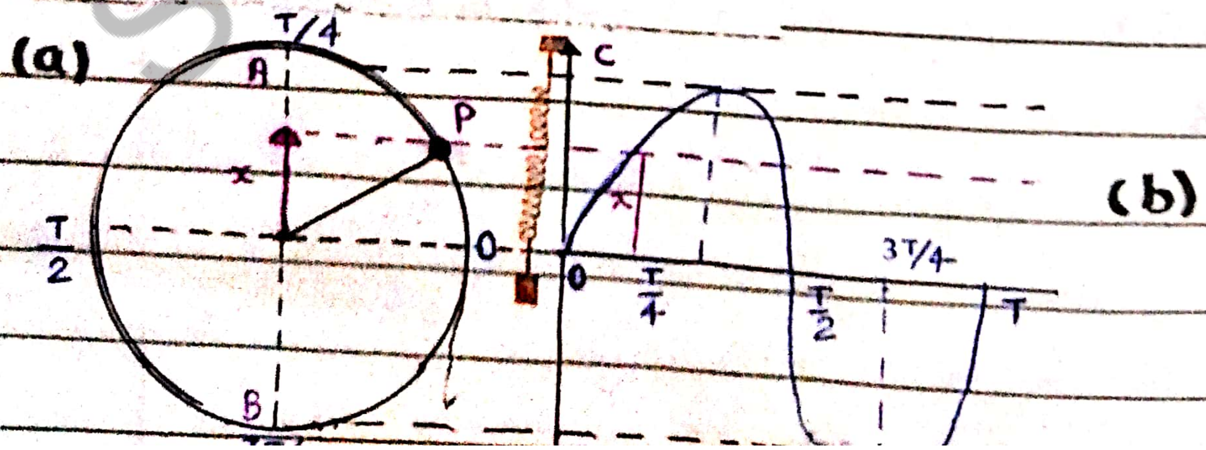
$a_x = \omega^2 x$

$a_x = -\omega^2 x$

$a_x = -\text{constant}\ x$

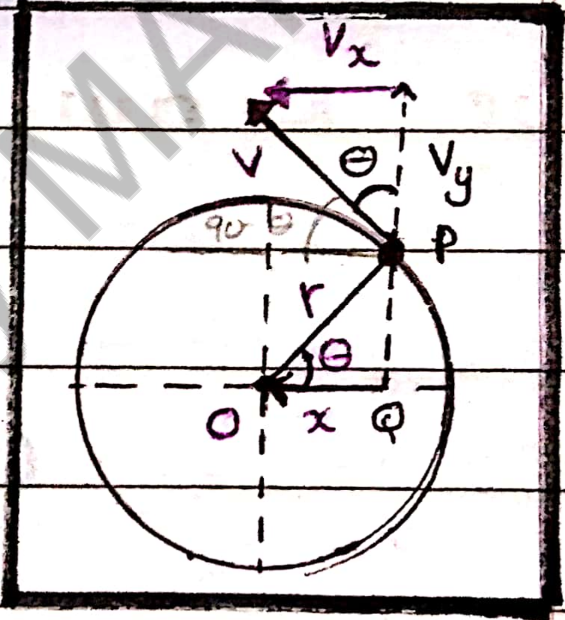
$a_x \propto -x$

→ -ive sign indicates that 'a_x' is directed toward the mean position.



VELOCITY OF PROJECTION

consider a particle 'P' of mass 'm' moving in a circle of radius 'r'. The velocity of particle P is instantaneous & directed along the tangent to the circle.

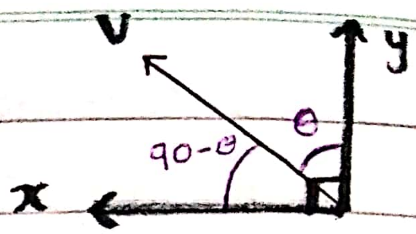


we have resolved the velocity into its components V_x and V_y. V_x is along the x-axis in the direction of displacement x. The projection of a particle P is vibrating in the direction of V_x So we will take only V_x.

Total angle = 90°

1^{st} angle = θ

2^{nd} angle = $90^\circ - \theta$



$V_x = V\cos\theta'$ —— (i)

$V_x = V\cos(90-\theta)$ $\quad (\theta' = 90-\theta)$

As we know,

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$[\alpha = 90^\circ,\ \beta, \theta]$

$\cos(90-\theta) = \cos 90 \cos\theta + \sin 90 \sin\theta$

$\cos(90-\theta) = 0(\cos\theta) + 1(\sin\theta)$

$\cos(90-\theta) = 0 + \sin\theta$

$\cos(90-\theta) = \sin\theta$

Putting in eq (i)

$V_x = V\sin\theta$ —— (ii)

As we know,

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = 1 - \left[\frac{x}{r}\right]^2$ $\quad (\because \cos^2\theta = \frac{x}{r})$

$\sin^2\theta = 1 - \frac{x^2}{r^2}$

$\sin^2\theta = \frac{r^2 - x^2}{r^2}$

$\sin\theta = \sqrt{\frac{r^2-x^2}{r^2}}$

$\sin\theta = \frac{1}{r}\sqrt{r^2-x^2}$

Putting value of $\sin\theta$ in eq (ii)

$V_x = V \times \frac{1}{r}\sqrt{r^2-x^2}$

$V_x = \frac{V}{r}\sqrt{r^2-x^2}$

$V_x = \frac{r\omega}{r}\sqrt{r^2-x^2}$ $\quad (\because V = r\omega)$

$$\boxed{V_x = \omega\sqrt{r^2-x^2}}$$

r = Radius of circle

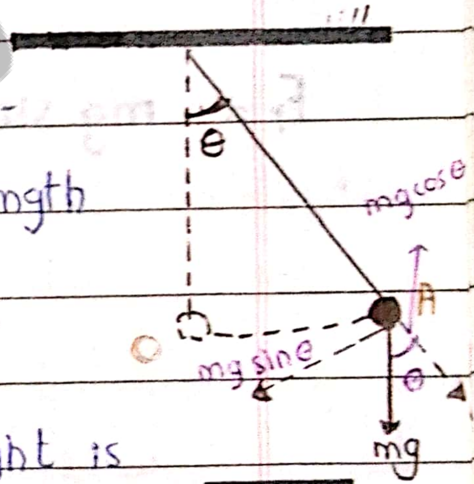
x = displacement of projection Q from M.P 'O'

ω = angular velocity of projection of particle.

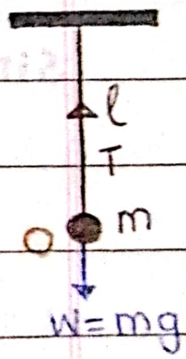
Maximum 'v' (At M.P)	Minimum 'v' (At E.P)
Displacement from M.P $\Rightarrow x = 0$	Displacement from E.P is maximum $\Rightarrow x = x_0$
$v = \omega\sqrt{x_0^2 - 0}$	$v = \omega\sqrt{x_0^2 - x_0^2}$
$v = \omega x_0$	$v = \omega(0) \rightarrow v = 0$

SIMPLE PENDULUM

Simple pendulum consists of small heavy mass '**m**' attached to a weight-less and inextensible string of a length '**l**' fixed at its upper end.



When the bob is at M.P, its weight is acting ~~is~~ downward as an action, as a reaction, Tension is acting in upward direction So weight & tension are equal and opposite in direction.



At M.P 'O'

$F_{net} = 0$

$T = W$

$T = mg$

When we displace the bob from M.P through small angle 'θ', its weight will act in downward direction, in this case tension is not exactly opposite to the weight.

AT E.P : $F_{net} \neq 0$

we have resolved the weight into its components.

$W_x = T = mg\cos\theta$

$W_y = mg\sin\theta$

only $mg\sin\theta$ is acting & responsible for the motion of pendulum. and also act as restoring force

$$F_r = mg\sin\theta$$

$$F_{applied} = -F_{restoring}$$

$$ma = -mg\sin\theta$$

$$a = -g\sin\theta \quad \text{—— (1)}$$

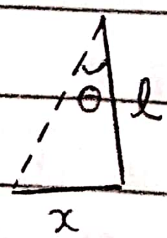
As amplitude is very small or for small angle

$\sin\theta \approx \theta$ so eq (1) become :

$$a = -g\theta$$

$$a = -g\left(\frac{x}{l}\right) \qquad \because S = r\theta$$

$$a = \frac{-g}{l}x \qquad \frac{S}{r} = \theta$$



$$a = -\text{constant}\ x$$

$\boxed{a \propto -x}$ **General Expression**

TIME PERIOD

$\omega = \frac{\theta}{t}$

$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega}$

$a = \frac{-gx}{l}$

$-x\omega^2 = \frac{-gx}{l}$

$\omega^2 = \frac{g}{l}$

$\omega = \sqrt{g/l}$

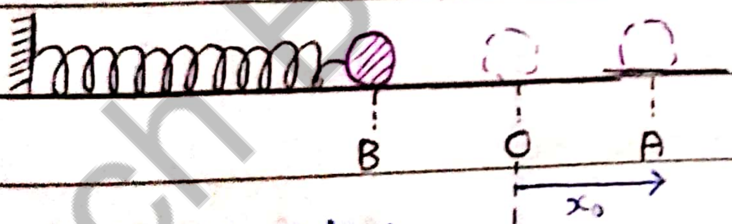
$T = \frac{2\pi}{\sqrt{g/l}}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

(1) $T \propto \sqrt{l}$
(2) independent of mass.

ENERGY CONSERVATION IN SHM

"Total Energy of vibrating spring system remains constant"



consider a vibrating mass-spring system ~~let mass is pulled thr-~~ ough displacement x_0 and released. The mass will oscillate with amplitude x_0. Let at certain instant of time the oscillating mass is at displacement x from M.P

According to Hook's Law

$F_r = -kx$

$F_{app} = -F_r$

$F = -(-kx)$

$F = kx$

$F \propto x$

At M.P	$x = 0$	$F = 0$
At E.P	$x = x_0$	$F = kx_0$
At any position	$x = x$	$F = kx$

K.E OF MASS SPRING SYSTEM

$K.E = \frac{1}{2} mv^2$

Velocity for SHM:

$v = \omega\sqrt{x_0^2 - x^2}$

$K.E = \frac{1}{2} m\left(\omega\sqrt{x_0^2 - x^2}\right)^2$

$K.E = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$

$K.E = \frac{1}{2} m \frac{k}{m} (x_0^2 - x^2)$

$$\boxed{K.E = \frac{1}{2} k (x_0^2 - x^2)}$$

P.E OF MASS SPRING SYSTEM

$P.E = W$

$P.E = F_{av} d$

$P.E = F_{av} x$

$F_{av} = \frac{F_1 + F_2}{2}$

$F_{av} = \left[\frac{0 + kx}{2}\right] x$

$F_{av} = \frac{kx^2}{2}$

$$\boxed{P.E = \frac{1}{2} kx^2}$$

K.E At M.P

$x = 0$

$K.E = \frac{1}{2} k (x_0^2 - 0)$

$K.E = \frac{1}{2} kx_0^2$

P.E At M.P

$P.E = \frac{1}{2} kx^2$

$P.E = \frac{1}{2} k(0)^2$

$P.E = 0$

K.E At E.P

$x = x_0$

$K.E = \frac{1}{2} k (x_0^2 - x_0^2)$

$K.E = 0$

P.E At E.P

$x = x_0$

$P.E = \frac{1}{2} k(x_0^2)$

$P.E = \frac{1}{2} kx_0^2$

TOTAL ENERGY AT M.P

$(E)_{M.P} = (K.E)_{M.P} + (P.E)_{M.P}$

$(E)_{M.P} = \frac{1}{2}Kx_0^2 + 0$

$(E)_{M.P} = \frac{1}{2}Kx_0^2$

TOTAL ENERGY AT E.P

$(E)_{E.P} = (K.E)_{E.P} + (P.E)_{E.P}$

$(E)_{E.P} = 0 + \frac{1}{2}Kx_0^2$

$(E)_{E.P} = \frac{1}{2}Kx_0^2$

TOTAL ENERGY AT ANY POSITION

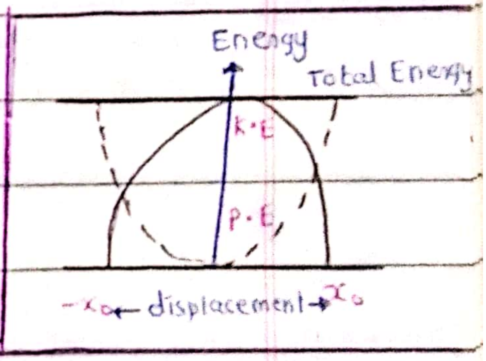
$(T.E)_{\text{Any position}} = (K.E)_{A.P} + (P.E)_{A.P}$

$(T.E)_{A.P} = \frac{1}{2}K(x_0^2 - x^2) + \frac{1}{2}Kx^2$

$(T.E)_{A.P} = \frac{1}{2}Kx_0^2 - \cancel{\frac{1}{2}Kx^2} + \cancel{\frac{1}{2}Kx^2}$

$(T.E)_{A.P} = \frac{1}{2}Kx^2$

CONCLUSION

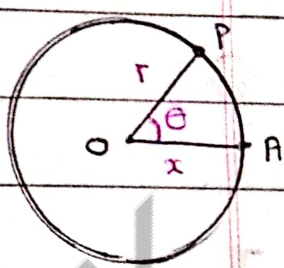


The total energy of SHM remains constant everywhere. At M.P, P.E is zero and the whole energy is kinetic. At E.P, then energy is wholly is potential and K.E is zero. The interchange occurs continuously from one form to the other as the spring is compressed and released alternatively. The energy oscillates back and forth b/w K.E & P.E but the total energy remains same.

FREE OSCILLATION	FORCED OSCILLATI[ON]
EXTERNAL FORCE	
Free oscillations takes place without being acted upon by external force.	Forced oscillations takes place under the influence of external force.
FREQUENCY	
The body of a system oscillates with its natural frequency	The body of a system oscillat[es] with provided frequency.
AMPLITUDE	
Amplitude decreases with time due to aerodynamic force	Amplitude throughout vibratio[n] is constant due to constant fo[rce]
ENERGY LOSS	
energy loss will exist due to aerodynamic force	energy loss doesn't exist (ideal case) when amplitude remains constant by providing constant force
EXAMPLE	
simple pendulum vibrates freely with its natural frequency	• Vibration of factory floor caused by running heavy machinery

WAVEFORM OF SHM

The $(x-t)$ graph of simple harmonic oscillator is known as wave form of SHM.

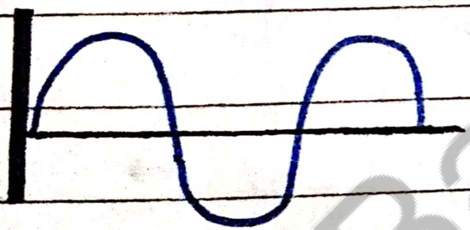


In ΔAOP $\cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$

As circular motion is the replica of vibratory motion so r is replaced by x_0

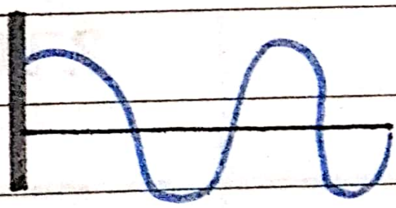
$$x = x_0 \cos\omega t$$

SINE WAVE



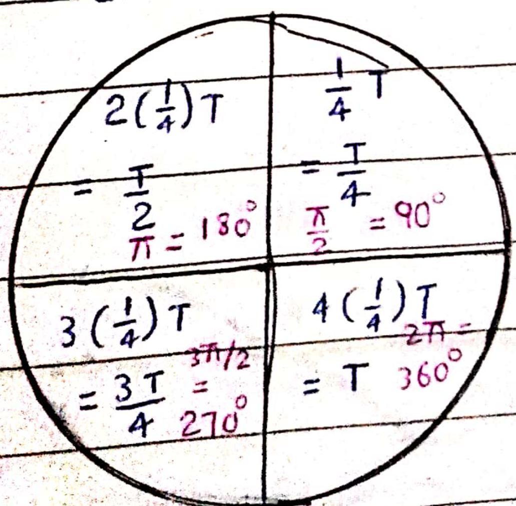
start from M.P

COS WAVE



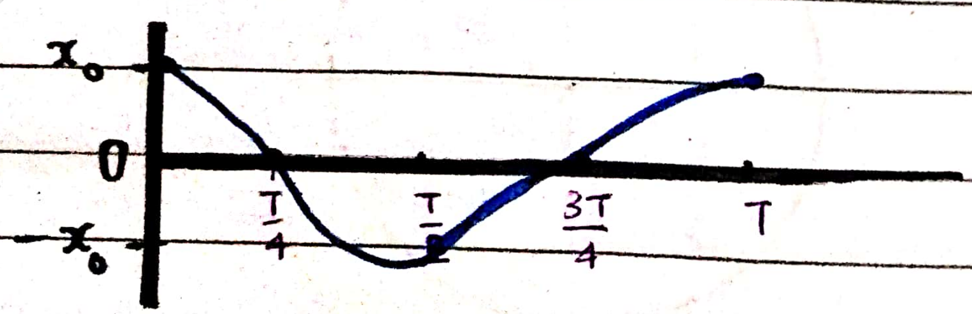
start from E.P

TIME INTERVAL



DISPLACEMENT IS FUNCTION OF TIME

Vibration in time	angular vibration	$x = x_0 \cos\left[\frac{2\pi}{T}\right] t$
0	0°	$x = x_0 \cos\left(\frac{2\pi}{T}\right) \times 0$ $x = x_0 \cos 0 \rightarrow x = x_0$
$\frac{T}{4}$	90°	$x = x_0 \cos\left(\frac{2\pi}{T} \times \frac{T}{4}\right)$ $x = x_0 \cos\left(\frac{\pi}{2}\right)$ $x = x_0 (0) \rightarrow x = 0$
$\frac{T}{2}$	180°	$x = x_0 \cos\left(\frac{2\pi}{T} \times \frac{T}{2}\right)$ $x = x_0 \cos(\pi)$ $x = x_0 (-1) \rightarrow x = -x_0$
$\frac{3T}{4}$	270°	$x = x_0 \cos\left(\frac{2\pi}{T} \times \frac{3T}{4}\right)$ $x = x_0 \cos\left(\frac{3\pi}{2}\right)$ $x = x_0 (0) \rightarrow x = 0$
T	360°	$x = x_0 \cos\left(\frac{2\pi}{T} \times T\right)$ $x = x_0 \cos(2\pi)$ $x = x_0 \cos(1) \rightarrow x = x_0$



PHASE ($\theta = \omega t$)

- direction of vibrating body
- displacement.

"The angle $\theta = \omega t$ which specifies the displacement as well as direction of point executing SHM is called phase."

IN PHASE

- When 2 waves start from same position
- When they start from same M.P or E.P
- When phase difference ($\Delta\theta$) is zero.

OUT OF PHASE

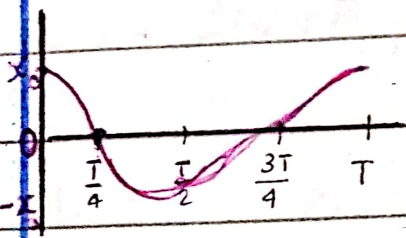
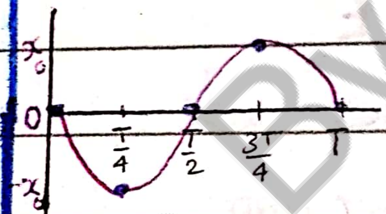
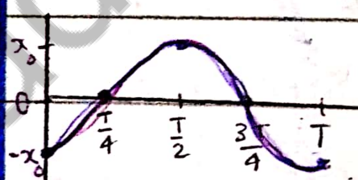
- When one wave start from crest & other from a trough.
- When one wave start from one E.P & the other from 2nd E.P
- When $\Delta\theta$ is equal to $180°$

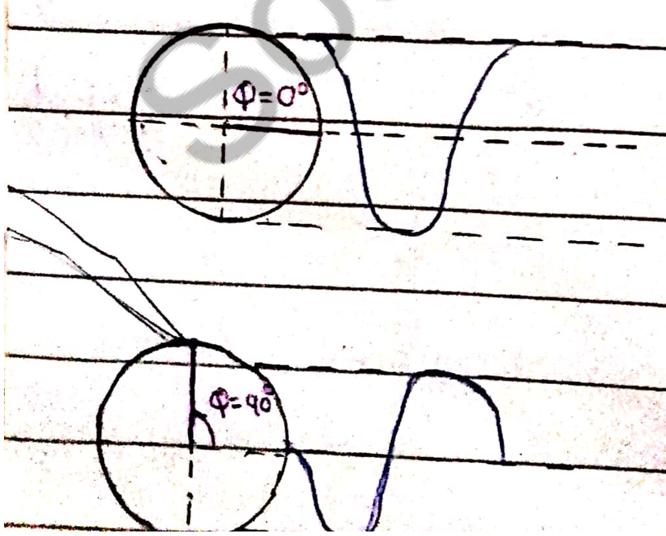
- θ is the phase discussed as quantity giving variation according time
- ϕ in the article is constant phase angle which is supposed for condition.

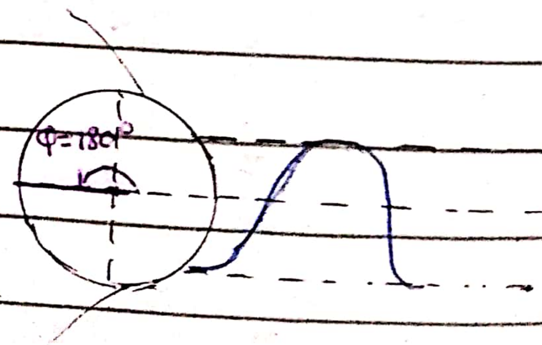
1) If $\Delta\theta = \theta_f - \theta_i = 0$ then waves are "in phase"

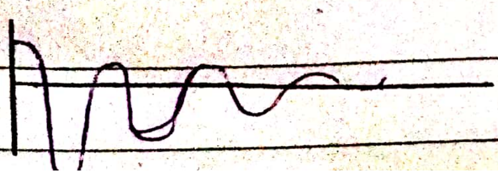
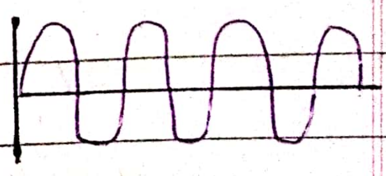
2) If $\Delta\theta = \theta_f - \theta_i = 90^\circ$ then one wave leads the other by 90° or

3) If $\Delta\theta = \theta_f - \theta_i = 180^\circ$ then waves are out of phase.

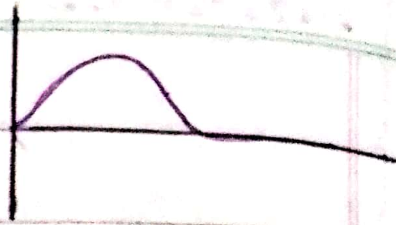
Phase Angle	Equation	Wave form	
$\phi = 0^\circ$	$x = x_0\cos(\omega t + 0)$ $t=0$, $x = x_0$ $t = T/4$, $x = 0$ $t = T/2$, $x = -x_0$ $t = 3T/4$, $x = 0$ $t = T$, $x = x_0$		wave form is showing the vibration of the particle from E.P
$\phi = 90^\circ$	$x = x_0\cos(\omega t + 90)$ $t=0$, $x = 0$ $t = T/4$, $x = -x_0$ $t = T/2$, $x = 0$ $t = 3T/4$, $x = x_0$ $t = T$, $x = 0$		wave form 1 leads the wave form-2 by 90°
$\phi = 180^\circ$	$x = x_0\cos(\omega t + 180)$ $t=0$, $x = -x_0$ $t = T/4$, $x = 0$ $t = T/2$, $x = x_0$ $t = 3T/4$, $x = 0$ $t = T$, $x = -x_0$		displacement is -ive max value as compared to W.F-1 so W.F-1 & W.F-2 are out of phase





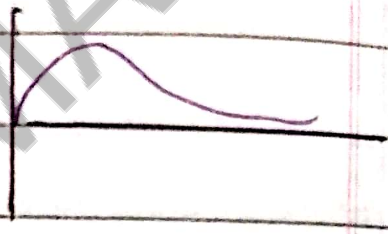
DAMPED OSCILLATION	UNDAMPED OSCILLATION
AMPLITUDE	
Amplitude decreases with the passage of time	Amplitude remains constant.
ENERGY LOSS	
Energy loss will occur	No loss of energy.
OSCILLATIONS	
oscillations die with time	oscillations never die.
SYSTEM	
It is real system	It is an ideal system
WORK AGAINST FRICTION	
Work is done against the friction	No work is done against the friction.
TYPE OF OSCILLATION	
Free oscillations	forced oscillation
EXAMPLE	
Swinging pendulum, shock absorber 	optical fibres. 

OVER DAMPING



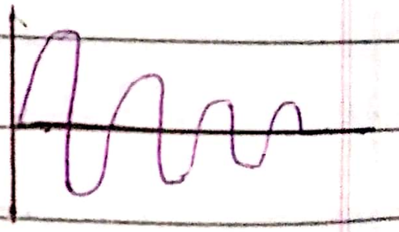
- When damping force become more than the oscillating force. $F_D > F_o$
- system will immediately come to M.P

CRITICAL DAMPING



- When damping force is equal to oscillating force. $F_D = F_o$
- system will slowly come to its M.P with uniform velocity.

UNDER DAMPING



- When damping force is less than the oscillating force. $F_D < F_o$
- Body will oscillate and its amplitude will gradually decrease.

Spring In Series

$$x = x_1 + x_2 \qquad \left[F = kx,\ x = F/k\right]$$

$$\frac{F}{K_{eff}} = \frac{F}{K_1} + \frac{F}{K_2}$$

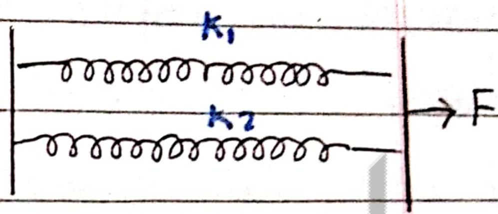
$$\frac{1}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\frac{1}{K_{eff}} = \frac{1}{K} + \frac{1}{K}$$

$$\frac{1}{K_{eff}} = \frac{2}{K}$$

$$K_{eff} = \frac{K}{2}$$

Spring In Parallel



$$F = F_1 + F_2$$

$$K_{eff}\,x = K_1 x + K_2 x$$

$$K_{eff} = K_1 + K_2$$

RESONANCE

"Increase in amplitude of a body when a force having time period equal to natural time period of the body acts on it is called resonance"

Changes that occur during resonance :

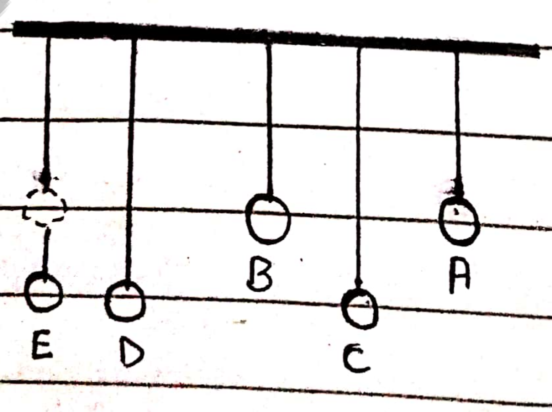
(1) Maximum energy is absorbed

(2) Amplitude increases

→ Resonance also occurs when the applied has frequency an integral multiple of natural frequency of a body. If f_1 is the natural frequency of a body then resonance occurs at

$$f_2 = 2f_1, \quad f_3 = 3f_1 \quad f_4 = nf_1 \quad \ldots\ldots \quad f_n = nf_1$$

EXPERIMENT



1. length of A = length of B = l
2. length of C = length of D = L

CASE-1	CASE-2
1. pendulum E is kept equal in length to pendulum A & B E = A & B	1. pendulum E is kept equal in length to pendulum C & D E = C & D
2. E is set to vibration, pendulum A & B start vibrations automatically	2. E is set to vibration, pendulum C & D start vibrations automatically.
3. pendulum C & D do not vibrate because of different length.	3. pendulum A & D do not vibrate because of different length.

ADVANTAGE	DISADVANTAGE
Resonance can be used when a body is to be vibrated at certain frequency e.g tuning of a radio station	Resonance can increase the amplitude of body to a large extent and it can collapse e.g marching of army on a bridge.

TYPES OF RESONANCE

1- ELECTRICAL RESONANCE (Tuning a Radio)

i. When we turn the knob of a radio, its natural frequency changes.

ii. When natural frequency of electrical circuit becomes equal to transmission frequency of the radio station, the energy absorption will be maximum and this is the only station we hear.

2. MECHANICAL RESONANCE (Swing)

It is like a pendulum with single natural frequency depending on its length. If a series of regular pushes are given to the swing, its motion can be built up enormously. If pushes are given irregularly, the swing will hardly vibrate.

3. MRI

i. Strong radio frequency radiations are used to cause nuclei of atoms to oscillate.

ii. When resonance occurs, energy is absorbed by molecules.

iii. Pattern of energy absorption can be used to produce a computer enhanced photograph.

4. COOKING OF FOOD

microwave oven uses
→ microwave with frequency similar to natural frequency of vibration of water or fat molecules

i. ~~Steng to~~ Food that contains water molecules is placed in microwave oven

ii. When water molecules resonate, they absorb energy and are heated up.

iii. The plastic or glass container do not heat up since they do not contained water molecules.

5. COLLAPSE OF SUSPENDED BRIDGE

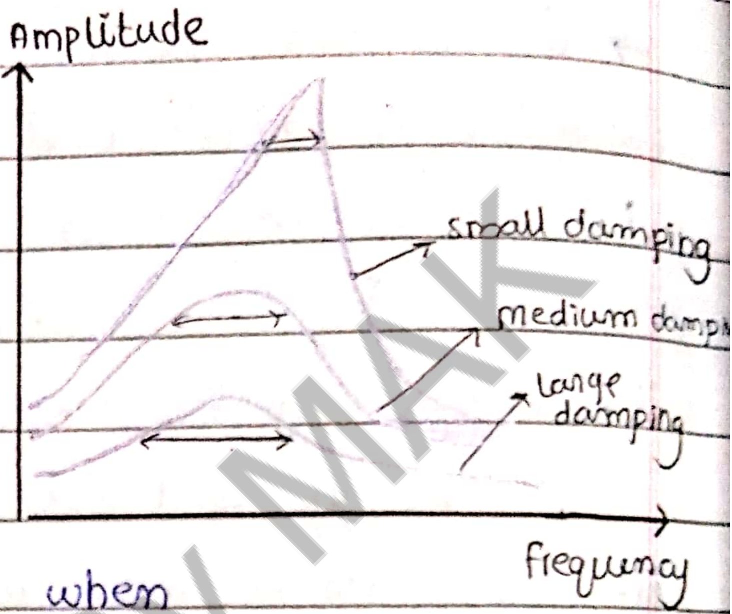
If frequency of force due to steps of soldiers become equal to natural frequency of the bridge, then the bridge may collapse due to resonance.

SHARPNESS OF RESONANCE

Sharpness $\propto$ Amplitude

Sharpness $\propto \dfrac{1}{\text{damping}}$

Amplitude $\propto \dfrac{1}{\text{damping}}$



- The amplitude will be large when damping is small
- Amplitude decreases rapidly at frequency slightly different different from resonant frequency.
- Amplitude and damping sharpness depends on damping
- heavily damped system has fairly flat resonance curve.

EXPERIMENT

1. When a pith bob and lead bob of same length are attached to a rod
2. When they are set to vibration by 3rd pendulum of same length
3. The lead bob will oscillate with high amplitude as compared to pith ball

$$(\text{Amplitude})_{Pb} > (\text{Amplitude})_{pith}$$

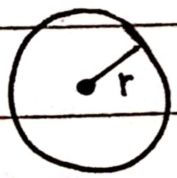
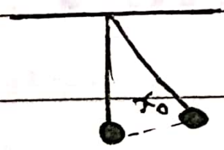
4. Damping effect for pith bob due to air Resistance is much greater than for lead bob.

CONCLUSION

(Amplitude)$_{Pb}$ > (Amplitude)$_{pith}$

(Damping)$_{Pb}$ < (Damping)$_{pith}$

(Sharpness)$_{Pb}$ > (Sharpness)$_{pith}$.

CIRCULAR MOTION	VIBRATORY MOTION
	
Radius remains constant	amplitude remains constant
F_c is responsible to keep the object in circular motion	F_r is responsible to keep the object toward its M.P
a_c is directed toward centre $a_c = v^2/r$	acceleration is directed toward M.P $a \propto -x$