ch:07

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@sochbadlobyMAK

Oscillations

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- **Oscillations:**

→ A type of harmonic motion typically periodic motion, in one or more dimensions.

→ **in simple words:**

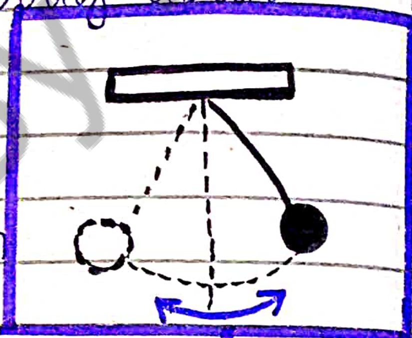
Oscillations are vibrations

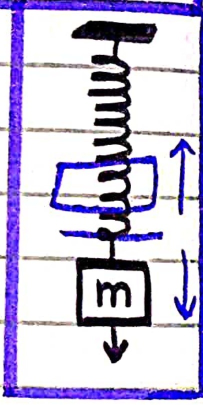
- **Oscillatory motion:**

To and fro motion of a body about a mean position.

→ **examples:**

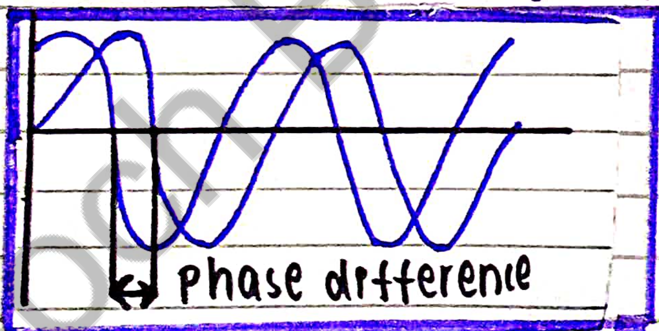
1) motion of bob of simple pendulum
2) motion of mass-spring system

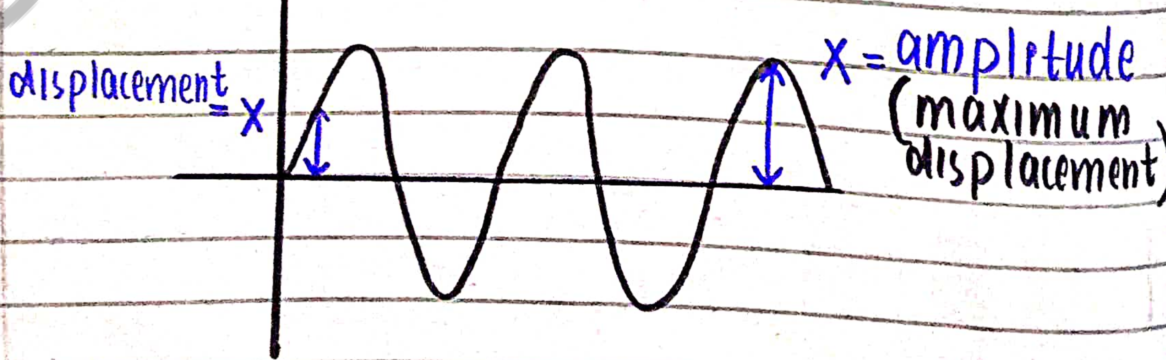




Phase difference:

→ difference in phase angles b/w 2 points of wave.





Simple Harmonic Motion
(S·H·M)

→ **definition:**
The type of motion in which the acceleration of body is always directly proportional to the displacement of body from mean position and is always directed towards the mean position is called SHM

→ **conditions:**

1. The system must have inertia.
2. The system should have restoring force.
3. $a \propto -x$
4. The system should be frictionless

→ **examples:** Mass and spring system, simple pendulum and ball in bowl system.

→ **Mass-spring system:**

→ Consideration:

→ Consider a mass 'm' attached with one end of spring, (which can move freely on a frictionless horizontal surface)

→ Derivation:

- When mass 'm' is displaced through distance x from mean position by a force F. Then,
- According to Hook's law,

$$F = Kx$$
$$K = \frac{F}{x}$$

∴ K = spring constant
→ force per unit extension

- Due to elasticity, spring opposes applied force. This opposing force is called **restoring force**.

$$F_r = -Kx$$

∴ negative sign shows F_r is directed opposite to x

expressions:

1. acceleration

we know that
$$F = ma \rightarrow (i)$$
$$F = -kx \rightarrow (ii)$$

- comparing (i) & (ii)

$$ma = -kx$$
$$a = -\frac{k}{m}x \qquad \therefore -\frac{k}{m} = \text{constant}$$
$$a = -\text{constant}\ x$$
$$\boxed{a \propto -x}$$

2. angular frequency

we know that;
$$a = -\omega^2 x$$
$$a = -\frac{k}{m}x$$
$$-\omega^2 x = -\frac{k}{m}x$$
$$+\omega^2 = +\frac{k}{m}$$
$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$
i-e angular frequency

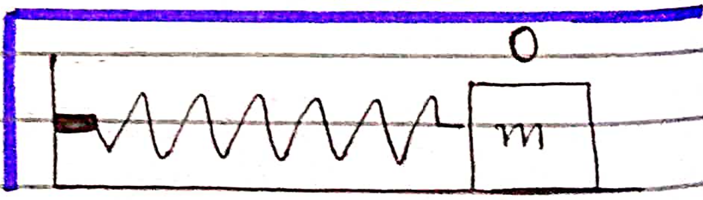
3. time period

$$T = \frac{2\pi}{\omega} \qquad \therefore \omega = \sqrt{\frac{k}{m}}$$
$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$
$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

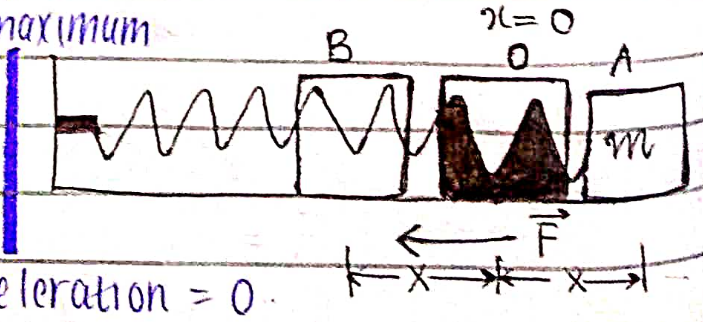
4. Frequency

$$f = \frac{1}{T} \rightarrow (i)$$
$$T = 2\pi\sqrt{\frac{m}{k}} \rightarrow (ii)$$
putting (ii) in (i)
$$f = \frac{1}{2\pi\sqrt{\frac{m}{k}}}$$
$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

→ At extreme positions:
1) K.E is minimum
2) P.E is maximum
3) velocity = 0, acceleration maximum



→ At mean position:
1) K.E is maximum
2) P.E is minimum
3) velocity = maximum, acceleration = 0

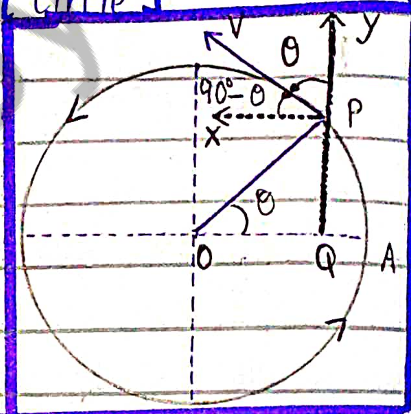


circular motion and S.H.M

→ Consideration:

- Let a particle of mass '**m**' whirling in a **horizontal** circle of radius '**r**' with angular velocity 'ω'.
- A distant light causes a **shadow** of mass **m** on wall

→ particle moves in **vertical** circle] its shadow executes **SHM**
→ particle moves in **horizontal** circle



→ Derivation:

(1) Acceleration of projection
(2) Time period
(3) Frequency
(4) Velocity of projection

(1) Acceleration

→ Let $\overline{AB}$ = diameter ; **O** = centre

When the body moves, its projection '**Q**' vibrates along the diameter of circle

→ When the body is at **P**, its projection **Q** is at distance '**X**' from mean position

→ Resolving centripetal acceleration of body 'a_c' into its components

From ΔPOQ

$$\frac{QO}{PO} = \cos\theta$$

$$\frac{a_x}{a_c} = \cos\theta$$

$$\boxed{a_x = a_c \cos\theta} \rightarrow (1)$$

As, $a_c = r\omega^2$

$$a_x = r\omega^2(\cos\theta) \rightarrow (2)$$

From figure,

$$\frac{x}{r} = \cos\theta \rightarrow (3)$$

$$\boxed{x = r\cos\theta}$$ (instantaneous displacement)

Putting (3) in (2)

$$a_x = r\omega^2\left(\frac{x}{r}\right)$$

$$\boxed{a_x = r\omega^2 x}$$

$$a_x = -\omega^2 x$$

$$a_x = -\text{constant } x$$

$$\because \omega^2 = \text{constant}$$

$$a_x \propto -x$$

$$\boxed{a_x = a}$$

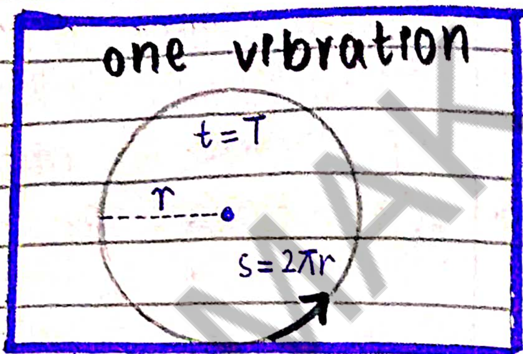
(2) **Time period** = T

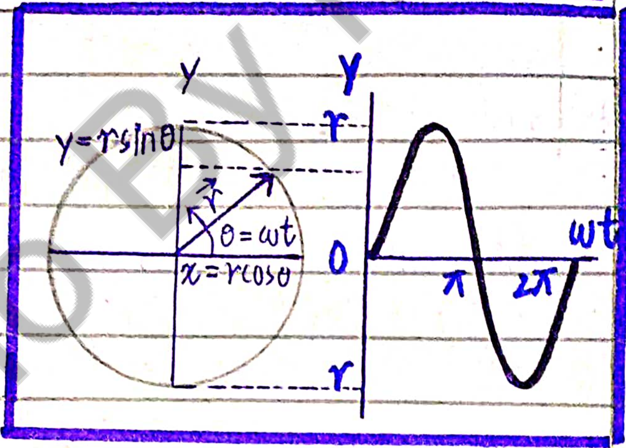
$$T = \frac{2\pi}{\omega}$$

(3) **Frequency** = f = 1/T

$$f = \frac{\omega}{2\pi}$$

$$\boxed{\omega = 2\pi f}$$





$$\boxed{V = \omega\sqrt{r^2 - x^2}}$$

(4) **Velocity of Projection:**

- resolving velocity in rectangular component;

$$V_y = V_p \cos\theta$$

$$V_x = V_p \sin\theta \rightarrow (1)$$

- By trignometry;

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

- From figure; $\cos\theta = \frac{x}{r}$

$$\sin^2\theta = 1 - \frac{x^2}{r^2}$$

$$\sin^2\theta = \frac{r^2 - x^2}{r^2}$$

Taking square root on b.s:

$$\sin\theta = \sqrt{\frac{r^2 - x^2}{r^2}}$$

$$= \frac{1}{r}\sqrt{r^2 - x^2}$$

putting $\sin\theta = \frac{\sqrt{r^2 - x^2}}{r}$ in (1)

$$V_x = V_p \sin\theta$$

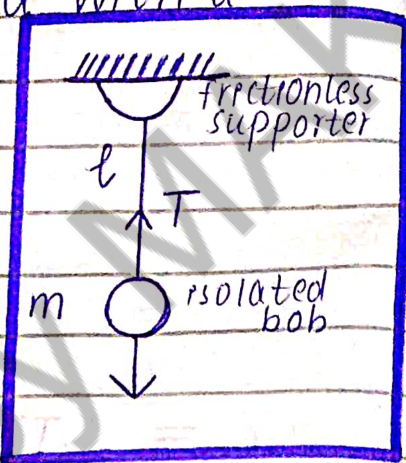
$$V_x = V_p\left(\frac{\sqrt{r^2 - x^2}}{r}\right)$$

$$V = r\omega\left(\frac{\sqrt{r^2 - x^2}}{r}\right)$$

$$\boxed{V = \omega\sqrt{r^2 - x^2}}$$

→ Simple Pendulum

definition:
A simple pendulum consists of a small heavy mass attached with light and inextensible string suspended with a frictionless support.



→ Working:
When the simple pendulum is displaced from its mean position through θ and released, then it starts to oscillate about mean position O.

→ Consideration:

- Let the bob of pendulum of mass 'm' having weight 'W' is displaced from mean position 'O' towards 'A'
- Weight 'W' acts vertically in downward direction.

'l' is length of pendulum.
↳ sum of length of ~~length~~ string + radius 'r' of bob
'T' is the tension in string.

→ Components of weight:
Resolving the weight 'W' into two rectangular components: $W\cos\theta$; $W\sin\theta$
Tension in spring $= W\cos\theta$

$$\boxed{T = mg\cos\theta}$$

→ Restoring force:
Restoring force $F = -W\sin\theta$

$$\boxed{F = -mg\sin\theta} \longrightarrow (1)$$

-ve sign shows, restoring force always directed toward mean position

→ Acceleration α:

→ direction of restoring force is opposite to displacement

- As we know,

$$F = ma \longrightarrow (1)$$
$$F = -mg\sin\theta \rightarrow (2)$$

- comparing (1) & (2)

$$ma = -mg\sin\theta$$
$$a = -g\sin\theta$$

- For small value of θ

$\therefore \sin\theta \approx \theta$

$$a = -g\theta \longrightarrow (3)$$

- From figure;

$$\theta = \frac{\text{arc } OA}{l} \qquad \therefore \theta = \frac{S}{r}$$
$$\theta = \frac{X}{l}$$

- θ is small so arc $OA \approx X$

Putting $\theta = X/l$ in (3)

$$a = -g\left(\frac{X}{l}\right)$$
$$a = \left(-\frac{g}{l}\right)x$$
$$a = (-\text{constant})\,x$$
$$\boxed{\vec{a} \propto -\vec{x}}$$

→ Time Period (T)

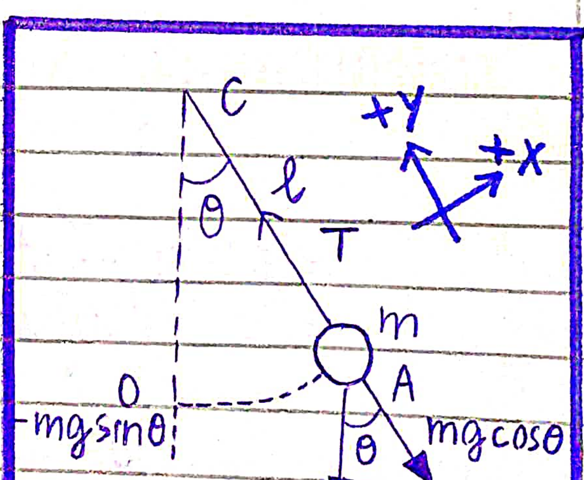
As, $T = \dfrac{2\pi}{\omega}$

but $\omega^2 = \dfrac{g}{l}$

$$\omega = \sqrt{\frac{g}{l}}$$

(1) becomes,

$$T = \frac{2\pi}{\sqrt{g/l}}$$
$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$



→ Angular f:

{ $T\uparrow \; l\uparrow$ }

we know that;

$$a = -\omega^2 x \rightarrow (1); \quad a = -(g/l)\,x \rightarrow (2)$$

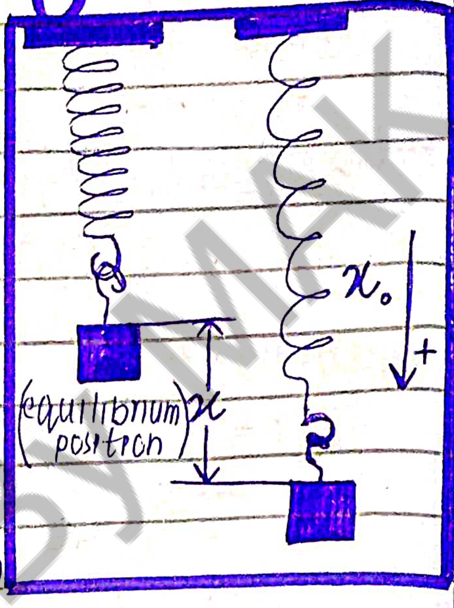
comparing (1) and (2), we have

$$-\omega^2 x = -(g/l)\,x$$
$$\omega^2 = g/l \longrightarrow \boxed{\omega = \sqrt{g/l}}$$

Energy Conservation in S.H.M

→ consideration:

- Consider a mass **m** suspended from a strong support by a spring of spring constant **k** as shown.
- Let the mass is pulled through displacement X_0 and released
- The mass will oscillate with amplitude X_0.



1. Instantaneous K.E in SHM:

$$K.E = \frac{1}{2} m v^2$$

$$V = \omega\sqrt{x_0^2 - x^2}$$

$$K.E = \frac{1}{2} m \left(\omega\sqrt{x_0^2 - x^2}\right)^2$$

$$K.E = \frac{1}{2} \underbrace{m\omega^2}_{=k} (x_0^2 - x^2) \rightarrow (1)$$

Putting $m\omega^2 \cancel{(x_0^2 - x^2)}$

$$\boxed{K.E = \frac{1}{2} k (x_0^2 - x^2)} \rightarrow (2)$$

→ Maximum K.E

→ K.E is maximum at 'O' mean position where $\boxed{x = 0}$

$$K.E = \frac{1}{2} k \{x_0^2 - (0)^2\}$$

$$\boxed{(K.E)_{max} = \frac{1}{2} k x_0^2}$$

→ **Minimum K.E:**

→ K.E is minimum at ~~mean~~ extreme position

where $\boxed{X = X_0}$

$$K.E = \frac{1}{2} K\{x_0^2 - (x_0)^2\}$$

$$\boxed{(K.E)_{min} = 0}$$

2. Instantaneous P.E in S.H.M:

The restoring force of simple harmonic oscillator at displacement x is:

$$F_r = -Kx \qquad \because K = m\omega^2$$

$$F_r = -m\omega^2 x$$

As,

$$F = -F_r$$

$$F = -(-m\omega^2 x)$$

$$F = m\omega^2 x$$

☆ **Average Force:**

$$F_{av} = \frac{F_i + F_f}{2}$$

$$F_{av} = \frac{0 + m\omega^2 x}{2}$$

$$F_{av} = \frac{1}{2} m\omega^2 x$$

☆ **Work done against restoring force:**

$$W = F_{av}(X)$$

$$W = \frac{1}{2} m\omega^2 x (X)$$

$$\boxed{W = \frac{1}{2} m\omega^2 x^2}$$

→ Work done is stored as elastic P.E :

$$P.E = \frac{1}{2} m\omega^2 x^2$$

Putting $m\omega^2 = k$

$$P.E = \frac{1}{2} kx^2$$

→ **P.E maximum**
at extremes, $x = x_0$

$$\boxed{(P.E)_{max} = \frac{1}{2} kx_0^2}$$

→ **P.E minimum**
at mean, $x = 0$

$$\boxed{(P.E)_{min} = \frac{1}{2} k(0)^2}$$

→ Total energy in SHM

$$E = P.E + K.E$$

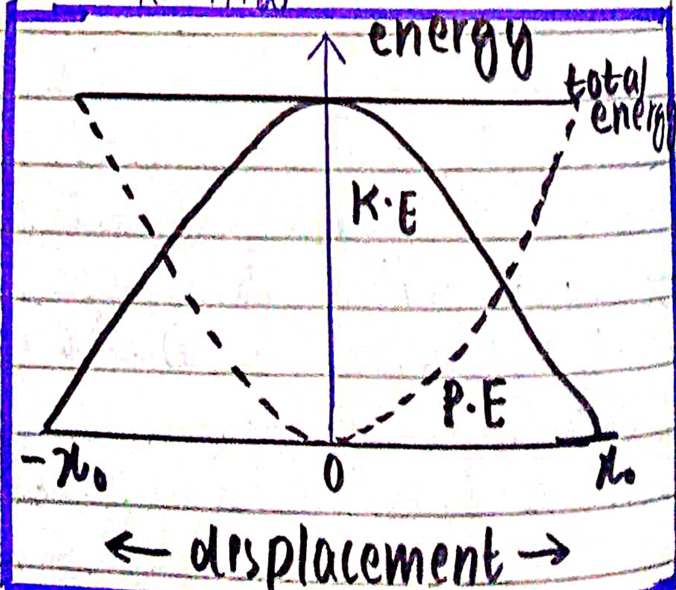
$$E = \frac{1}{2} m\omega^2(x_0^2 - x^2) + \frac{1}{2} m\omega^2 x^2$$

$$E = \frac{1}{2} m\omega^2 x_0^2 - \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m\omega^2 x^2$$

$$E = \frac{1}{2} m\omega^2 x_0^2$$

$\because k = m\omega^2$

$$\boxed{E = \frac{1}{2} kx_0^2}$$



Free oscillations	Forced oscillations
definition	
A body which has tendency to oscillate itself	A body that can not oscillate itself and requires external force
external force	
Doesn't require an external force	Requires an external force.
manner of oscillation	
Free oscillations diminish gradually due to resisting forces called damping force.	Forced oscillations persist as long as the body is acted upon by an external force
example	
Oscillations of simple pendulum.	vibration of pendulum in clock

Resonance:

"Object's oscillation with maximum amplitude, when the frequency of the applied force becomes equal to natural frequency of the object is called resonance.

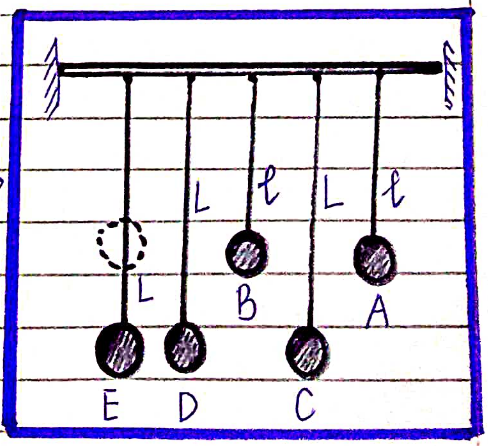
→ **Alternate definition:**

If f_1 is the natural frequency, then resonance takesplace at: $f_2 = 2f_1$, $f_3 = 3f_1$, $f_4 = 4f_1$

Hence, $\boxed{f_n = nf_1}$

→ **experiment:**

$A, B \rightarrow \ell$; $C, D \rightarrow L$



→ **case (a)**

when 'E' = 'A' & 'B'

if 'E' → set into vibrations

- 'C' & 'D' → rest
- 'A' & 'B' → vibrates

case (b)

when 'E' = 'C' & 'D'

if 'E' → set into vibrations

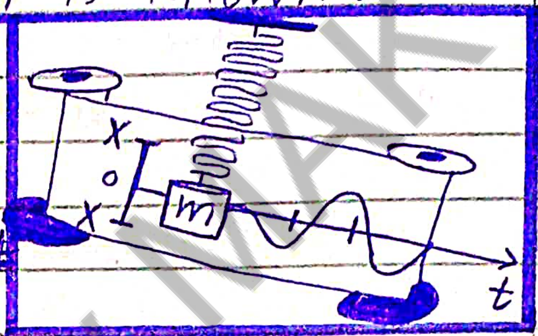
- 'C' & 'D' → vibrate
- 'A' & 'B' → rest

→ **examples:** Radio, M.R.I, microwave oven etc.

→ Waveform of SHM:

definition:

→ The displacement – time **(x-t)** graph of a simple harmonic oscillator is known as the wave form of SHM.



We know that;

$$x = r\cos\theta$$

$$\boxed{x = x_o \cos \omega t}$$

$\because r = x_o$
$\because \theta = \omega t$

$$\boxed{x = x_o \cos\left(\frac{2\pi}{T}\right)t}$$

$\omega = 2\pi/T$

a) When $t = 0$

$$x = x_o \cos\left(\frac{2\pi}{T}\right)\times(0)$$

$$x = x_o \cos 0°$$

(extreme) $\boxed{x = x_o \text{ (maximum)}}$

b) When $t = \frac{T}{4}$

$$x = x_o \cos\left(2\pi/T\right)\times T/4$$

$$x = x_o \cos(\pi/2)$$

$$x = x_o \cos(90°)$$

$\boxed{x = 0}$ (mean)

c) When $t = T/2$

$$x = x_o \cos\left(2\pi/T\right)\times T/2$$

$$x = x_o \cos(\pi)$$

$$x = x_o \cos 180°$$

$$x = x_o(-1)$$

$\boxed{x = -x_o}$ (extreme)

d) When $t = 3T/4$

$$x = x_o \cos\left(2\pi/T\right)\times 3T/4$$

$$x = x_o \cos(3\pi/2)$$

$$x = x_o \cos 270°$$

$\boxed{x = 0}$ (mean)

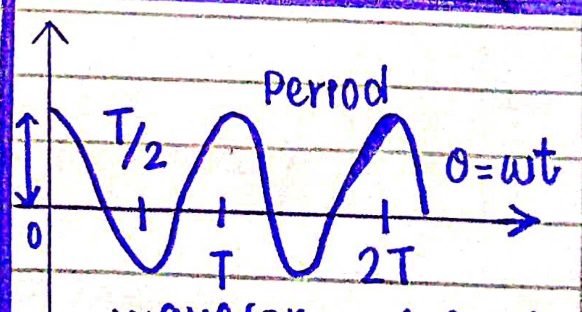
e) When $t = T$

$$x = x_o \cos\left(2\pi/T\right)\times T$$

$$x = x_o \cos 2\pi$$

$$x = x_o(1)$$

$\boxed{x = x_o}$ (extreme)



waveform of SHM

Phase

definition :

The angle $\theta = \omega t$ which specifies the displacement x as well as the direction of motion of the point oscillating with S·H·M is called phase.

→ **In other words :**

Phase is the quantity which shows the <u>state of motion of an oscillator</u>

Explanation :

As we know, the displacement/projection of the body moving in circle, executing S·H·M

$$\boxed{X = X_0 \cos\theta = X_0 \cos \omega t}$$

∴ $X =$ instantaneous displacement ; $X_0 =$ maximum displacement ; $\omega =$ angular velocity

$$X = X_0 \cos(\omega t + \Phi)$$

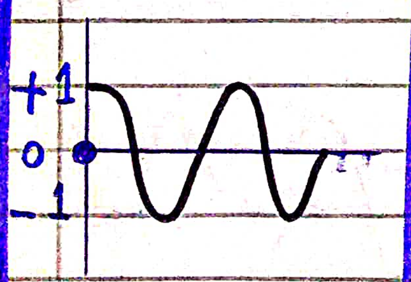
where $\theta = (\omega t + \Phi)$ is phase angle.

Φ → initial phase / starting phase

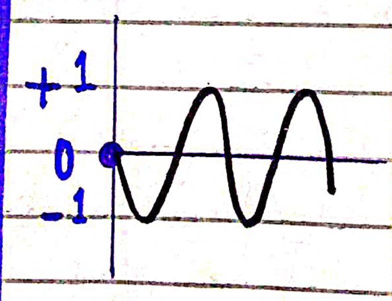
Φ → phase difference b/w states of motion of two oscillators

→ **Phasor diagrams :**

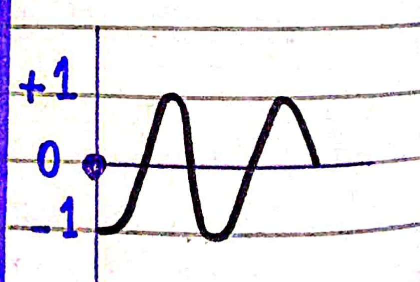
If $\Phi = 0°$



If $\Phi = 90°$



If $\Phi = 180°$



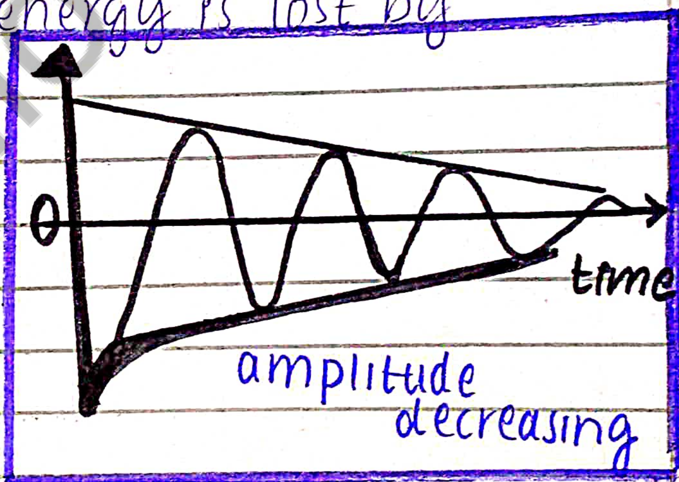
→ Damped Oscillations

→ **definition:**
Oscillations in which amplitudes decreases with time due to energy dissipations.

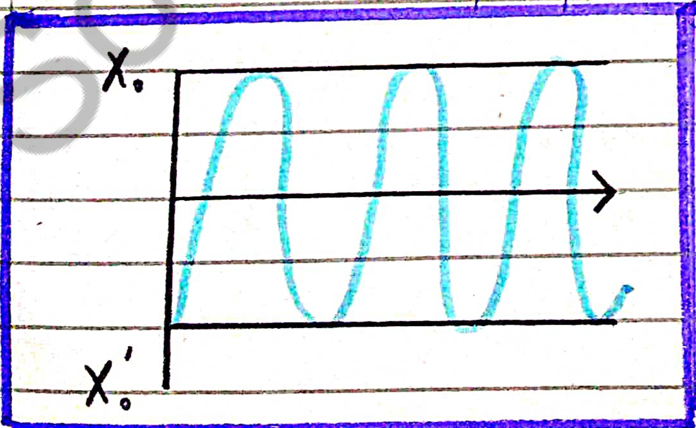
→ **Explanation:**
The amplitude of the oscillating body becomes smaller and smaller because of friction / air resistance

→ **Application:**
Shock Absorbers in cars provide a damping force to stop excessive oscillations

→ **Damping:**
A process by which energy is lost by oscillating system



→ **Undamped Oscillations:**
Oscillations in which amplitude remains the same with time.
e.g: Oscillations of an ideal simple pendulum



damping e.g:
- suspension systems of cars, bikes
- when vehicles move on a bumpy road, shock absorbers are used for comfortable ride.