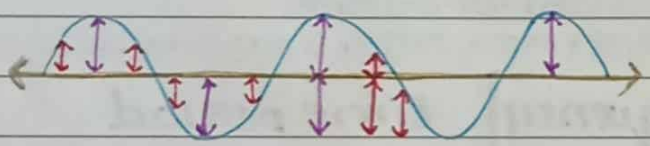
OSCILLATIONS: "When a particle execute repeated movement about a mean position, it is said to be in harmonic motion and if this motion is repeated at regular intervals, it's called periodic motion."

"terminologies"

Vibratory motion	Vibration	Time period	Frequency	Angular frequency
to and fro motion of a body about its mean position	The complete roundtrip of a body about it's meanposition	Time required by a body to complete one vibration	No of vibrations completed in one second	Number of revolutions persecond of a body

Displacement vs amplitude



- red shows displacement x
- purpule shows amplitude x_0

"Amplitude is maximum displacement from mean whereas distance at any instance is called displacement."

"mathematical forms"

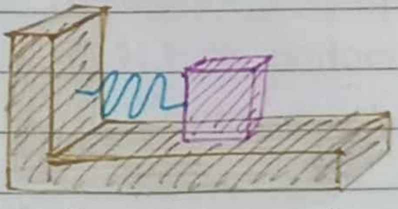
Frequency	Time period	Angular Frequency
$f = \frac{1}{T}$ (its unit is called Hertz) $1Hz = 1cs^{-1}$ ↑ cycles per second.	$T = \frac{1}{f}$	$\omega = \frac{2\pi}{T} = 2\pi f$

SIMPLE HARMONIC MOTION: "The type of motion in which the acceleration of body is always directly proportional to the displacement from mean position and is directed towards mean position is called SHM".

"Conditions"

- System must have inertia
- System must have restoring force
- System should be frictionless.

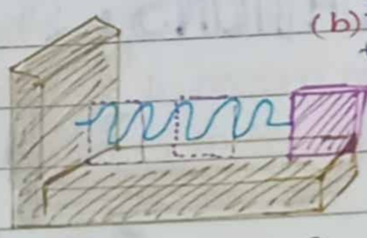
"diagram"



(a) Mass spring system is at rest. No force is applied on the mass. It has covered zero displacement

B O A

$x = 0$



(b) Force F is applied to displace the body to point 'A'. After reaching point 'A' restoring force pulls the body back to 'O' but due to inertia, mass moves to point 'B' compressing the spring. This way vibratory motion proceeds.

B O A

$|\leftarrow x \rightarrow|\leftarrow x \rightarrow|$

"mathematical interpretation"

(in accordance with diagram 'b').

$F_{applied} = -F_{restoring}$

$F_{applied} = kx$

$F_{restoring} = -kx$ (1)

- according to Hooke's law the extension in spring is directly proportional to the force within elastic limit
- The force which brings the body back towards its mean position is called restoring force.
- negative sign shows that Fr is directed opposite to x.

acceleration

$F = ma$ (2)

comparing (1) & (2)

$ma = -kx$

$a = \frac{k}{m}(-x)$ (3)

$\frac{k}{m}$ = constant $\boxed{a \propto -x}$ ★

angular frequency

[reference from pg 157 eq 5.9]

$a = -x\omega^2$ (4)

comparing (3) & (4)

$-\frac{k}{m}x = -x\omega^2$

$\boxed{\omega = \sqrt{\frac{k}{m}}}$ (5)

time period

$T = \frac{2\pi}{\omega}$ (6) $\left[\because f = \frac{\omega}{2\pi}\right]$

comparing (6) & (5)

$T = 2\pi \div \sqrt{\frac{k}{m}}$

$T = 2\pi \times \sqrt{\frac{m}{k}}$

$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$ (7)

and **frequency**

$f = \frac{1}{T}$ (8)

comparing (7) and (8)

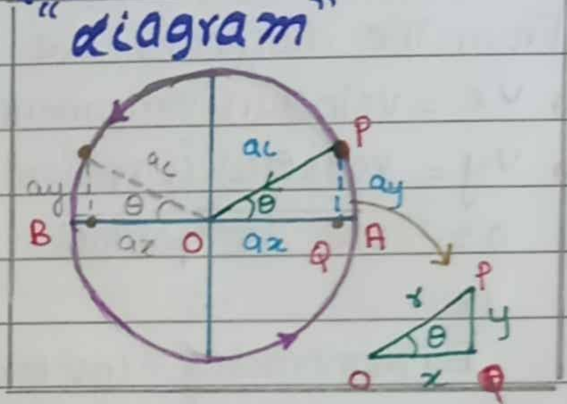
$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$

→ SI unit of 'k' (spring constant) = Nm^{-1}

2, CIRCULAR MOTION AND S.H.M:

A point 'P' moves in a circle having centre at O. Projection 'Q' of point 'P' moves along the diameter $\overline{BA}$ of the circle. As point P moves, the projection 'Q' performs SHM motion along diameter moving from A to O to B and then back to A.

"diagram"



"centripetal acceleration"

centripetal acceleration a_c of point P is directed towards the centre

component along base (or diameter): component of a_c along diameter is a_x

$$\boxed{a_x = a_c\cos\theta} \quad (1)$$

By right angle triangle PQO, $\cos\theta = \frac{\text{Base}}{\text{hyp}} = \frac{x}{r}$

(1) becomes $\boxed{a_x = a_c\left(\frac{x}{r}\right)}$ (2)

we know that $a_c = \frac{v^2}{r} = r\omega^2$ eq 2 becomes

$a_x = r\omega^2\left(\frac{x}{r}\right)$; $a_x = \omega^2 x$

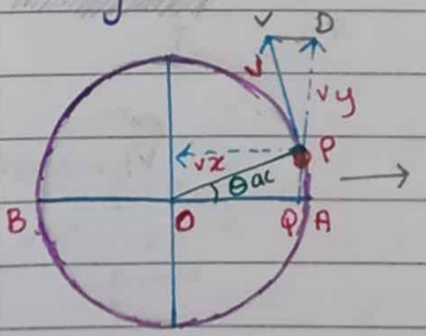
or $\boxed{a_x = \omega^2(-x)}$ minus indicates direction of a_x towards mean position.

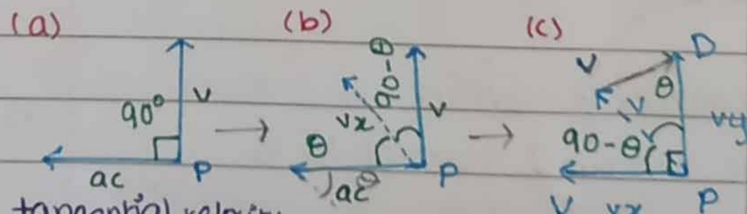
here ω^2 = constant thus $\boxed{a_x \propto -x}$ (3) which is the equation of S.H.M.

Thus projection of 'P' performs SHM along diameter of the circle.

"velocity of projection"

diagram





(a) tangential velocity and ac are at 90° to each other.

(b) By alternate angles, angle between vx and ac is θ.

comparing triangles PVD and QPO

ΔPVD contains velocity and its components.

$\angle POQ = \angle VPD$

$OQ = PD$

which means

$PD = v_y = \text{base}$

$PX = \text{Perp} = v_x$

"mathematical interpretation"

From the diagrams, we have

- $v_x = v\sin\theta$ (1) (component of velocity along perpendicular)
- $v_y = v\cos\theta$ (2) (component of velocity along base)

as v_x is parallel to motion (vibration) of Q, we'll only consider v_x.

By trignometry $\cos^2\theta + \sin^2\theta = 1$, $\sin^2\theta = 1 - \cos^2\theta$ (3)

we know that $\cos\theta = \frac{x}{r}$ so $\cos^2\theta = \frac{x^2}{r^2}$

eq 3 becomes

$$\sin^2\theta = 1 - \frac{x^2}{r^2}, \quad \sqrt{\sin^2\theta} = \sqrt{1 - \frac{x^2}{r^2}}$$

$$\sin\theta = \sqrt{\frac{r^2 - x^2}{r^2}} = \frac{1}{r}\sqrt{r^2 - x^2} \quad (4)$$

putting value of $\sin\theta$ from eq (4) into eq (1)

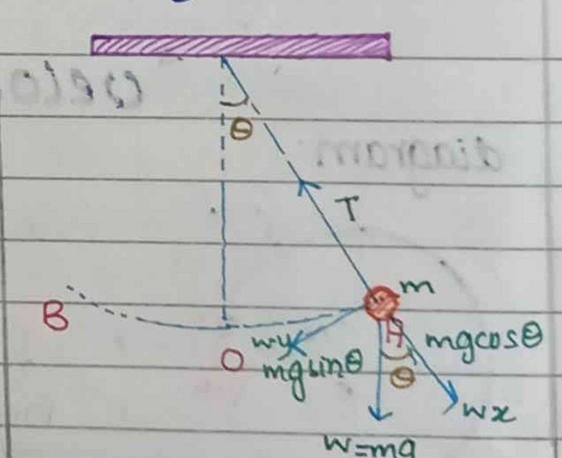
$$v_x = v\frac{1}{r}\sqrt{r^2 - x^2} \quad (\text{where } v = r\omega)$$

$$v_x = \frac{\cancel{r}\omega}{\cancel{r}}\sqrt{r^2 - x^2}, \quad \boxed{v_x = \omega\sqrt{r^2 - x^2}} \quad (5)$$

1. SIMPLE PENDULUM :-

- When the bob is displaced to position A, component of weight $mg\cos\theta = T$.
- component of force which causes oscillation is $mg\sin\theta$
- Bob executes oscillatory motion as it moves from point A to mean O from O to B. and the motion reverses.

"diagram"



"mathematical interpretation"

$F_{applied} = -F_{restoring}$

$F_{restoring} = -mg\sin\theta$

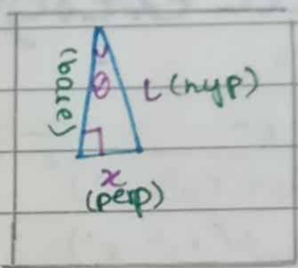
$ma = -mg\sin\theta$ (1) ∵ $F_{applied} = ma$

Special condition

when θ is very small, $\sin\theta \approx \theta$ eq (1) becomes $\cancel{m}a = -\cancel{m}g\theta$

$a = -g\theta$ (2)

when θ is very small, point 'O' will be very near to point A and arc OA ≈ x (straight line) then ΔAOC will be a right angle triangle.



$\sin\theta = \frac{Perp}{hypotenuse} = \frac{x}{l}$

$x/l = \sin\theta \approx \theta$ (eq 2 becomes)

$a = -g\,x/l$, $a = -\left(\frac{g}{l}\right)x$

as g and l are constant

$a = -\text{constant}\, x$ $(g/l = \omega^2)$

or $a \propto -x$ (3)

This is the equation of SHM so motion of simple pendulum is a SHM.

"time Period"

$T = \frac{2\pi}{\omega}$ (1)

$\omega^2 = \frac{g}{l}$, $\sqrt{\omega^2} = \sqrt{g/l}$

$\boxed{\omega = \sqrt{g/l}}$ (2)

Putting eq (2) in (1)

$T = 2\pi\sqrt{l/g}$ (3)

(i) longer the pendulum, greater the 'T'

(ii) 'T' is independent of mass of body

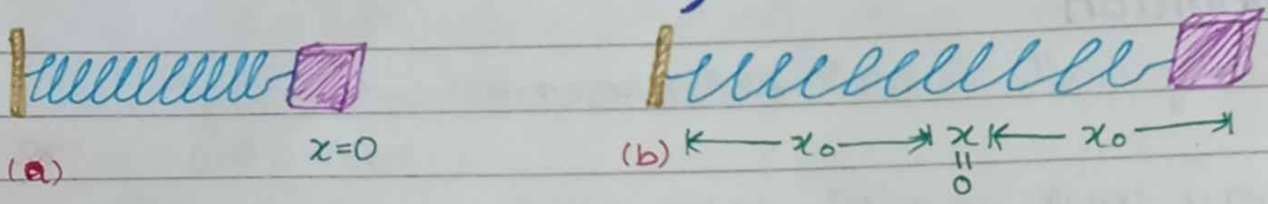
Q(FBISE) Can we realize an ideal simple pendulum?

Ans. No, we can't realize an ideal simple pendulum.

Reason: Ideal simple pendulum consists of a heavy point mass suspended by a massless and inextensible string. In practise it's not possible.

ENERGY CONSERVATION IN CASE OF S.H.M :-

"diagram"



"Kinetic energy"

$K.E = \frac{1}{2} m v^2$

$v = \omega \sqrt{x_0^2 - x^2}$

$K.E = \frac{1}{2} m (\omega \sqrt{x_0^2 - x^2})^2$

$K.E = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$

$m\omega^2 = \text{constant} = k$

$$\boxed{K.E = \frac{1}{2} k (x_0^2 - x^2)}$$

"Potential energy"

(Elastic potential energy will be considered).

$F_{restoring} = -kx$

$F_r = -m\omega^2 x$

$F_a = -F_r$

$F = -(-m\omega^2 x$

$F = m\omega^2 x$ when displacement is x

$F = m\omega^2 x_0$ when displacement is x_0

$\therefore F_{av} = \frac{F_{(x=0)} + F_{(x=x_0)}}{2} = \frac{0 + kx_0}{2}$

$F_{av} = \frac{1}{2} kx = \frac{1}{2} m\omega^2 x$

$W = F \times x$

$$\boxed{PE = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} kx^2}$$

maximum

When $x = 0$ [i.e., when mass passes through mean position]

$$\boxed{K.E = \frac{1}{2} kx_0^2}$$

when $x = x_0$ [when mass is at extreme position]

$$\boxed{P.E = \frac{1}{2} kx_0^2}$$

minimum

when $x = x_0$ [i.e when mass is at extreme position]

$$\boxed{K.E = \frac{1}{2} k(x_0^2 - x_0^2) = 0}$$

when $x = 0$ [i.e., when mass is at mean position]

$$\boxed{P.E = \frac{1}{2} k(0) = 0}$$

"Total Energy"

Total energy = KE + P.E

$$E_t = \frac{1}{2} m\omega^2 (x_0^2 - x^2) + \frac{1}{2} m\omega^2 x^2$$ [Taking general values of KE & PE]

$$= \frac{1}{2} m\omega^2 x_0^2$$

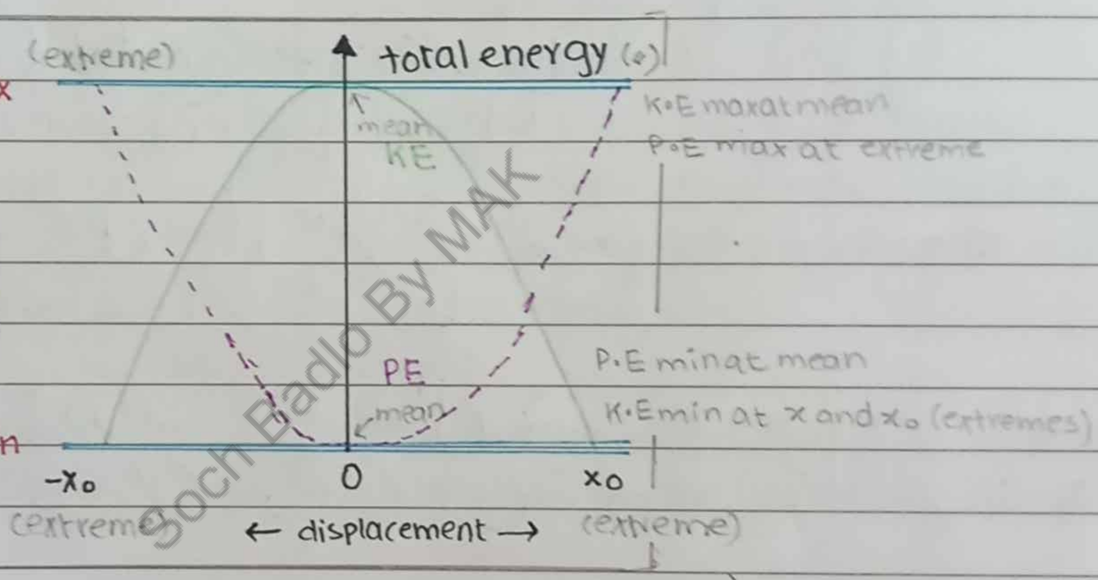
$$\boxed{E_t = \frac{1}{2} K x_0^2}$$

Energy conservation

- Total energy of SHM remains constant everywhere.
- At mean position P.E is zero and energy is wholly kinetic.
- At extreme position K.E is zero and energy is wholly potential.

Graph

Important concept: Periodic exchange of energy is the property of all oscillatory systems but total energy is conserved.



Table

[Important mcq wise]

Physical Quantity	Mean Position	Extreme Position
Displacement	Zero	Maximum
velocity	Maximum	zero
Acceleration	Zero	Maximum
Restoring force	Zero	Maximum
Momentum	Maximum	zero
K.E	Maximum	Zero
P.E	Zero	Maximum
T.E	Equal to max K.E	Equal to max P.E

FREE VIBRATIONS & FORCED VIBRATIONS

	Free Vibrations	Forced Vibrations
Production (i)	Free vibrations are produced when a body is disturbed from its equilibrium position and released	Forced vibrations are produced by an external periodic force of any frequency
Requirement of Force (ii)	To start free vibrations only, the force is required initially.	Continuous external periodic force is required. If external periodic force is stopped, the forced vibrations also stop
Dependence of Frequency (iii)	The frequency of free vibrations depends on the natural frequency.	The frequency of forced vibrations depends on the frequency of the external periodic force
Energy of Body (iv)	Energy of the body remains constant in absence of friction, air resistance etc.	Energy of the body is maintained constant by the external periodic force.
Inhibition (v)	These are self-sustained.	These stop as soon as external force is ceased
Examples (vi)	• oscillation of simple pendulum	• Vibrations of pendulum in a clock. • loud music produced by sounding wooden boards of string instruments.

Date:........./........./20.........

1, RESONANCE :- "The increase in amplitude of oscillation of oscillating system exposed to a periodic force whose frequency is equal to the natural frequency of the system is called resonance."

"main points"

- Resonance occurs when applied force has frequency an integral multiple of the natural frequency of body
- In resonance phenomena, energy absorption is maximum
- Resonance causes an increase in vibration

"examples" [important]

→ Collapse of suspended bridge: If frequency of force due to the steps of soldiers becomes equal to the natural frequency of the bridge, thene the bridge may collapse due to resonance.

Fun way to learn : [PKMZ] fiziks kahani mani zubani
Resonance is just like the bond between two best friends when natural frequencies of 2 friends become equal, best friends are made.

→ Radio: When we turn the knob of a radio to tune a station, we are changing the natural frequency of the electric circuit to make it equal to the transmition frequency of the radio station.

→ M.R.I (Magnetic Resonance Image): Strong radio frequency radiations are used to cause nuclei of atoms to ~~(collide)~~ oscillate. When resonance occurs, energy is absorbed by the molecules. The pattern of energy absorption can be used to produce a computer enhanced photograph.

→ Micro wave oven (heating food): Microwave with a frequency similar to the natural frequency of vibration of water or fat molecules is used in ovens. When food containing water molecules is placed in the oven, the water molecules resonates, absorbing energy from the microwaves and consequently gets heated. The plastic and glass container do not heat up since they donot contain water molecules

1. WAVE FORM OF SHM :- "The displacement - time $(x-t)$ graph of a simple harmonic motion is known as the wave form of SHM."

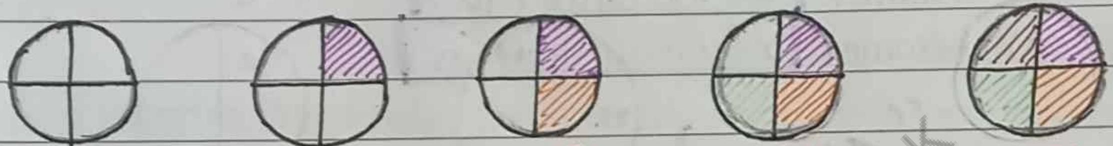
"mathematical interpretation"

instantaneous displacement of SHM is given by $x = r\cos\theta$ (1)

$r = x_0$ and $\theta = \omega t$, eq (1) becomes $x = x_0\cos\omega t$ (2)

$\omega = \frac{2\pi}{T}$, eq (2) becomes $\boxed{x = x_0\cos\left(\frac{2\pi}{T}\right)t}$ (3)

➡ now we'll put different values of 't' to get different points on the graph which we'll join to get waveform of SHM.



$t=0$ $\quad$ $t = T/4$ $\quad$ $t = 2T/4$ or $T/2$ $\quad$ $t = 3T/4$ $\quad$ $t = 4T/4$ or T

"values for x-axis"

"values for y-axis" [using eq 3]

- when $t=0$

$x = x_0\cos\frac{2\pi}{T}\times 0$

$x = x_0\cos 0$

$\boxed{x = x_0}$

- when $t = T/4$

$x = x_0\cos\frac{2\pi}{T}\times\frac{T}{4}$

$x = x_0\cos\left(\frac{\pi}{2}\right)$

$x = x_0\cos(90) = 0$ $\boxed{x=0}$

- when $t = T/2$

$x = x_0\cos\frac{2\pi}{T}\times\frac{T}{2}$

$x = x_0\cos 180°$

$x = x_0(-1)$ $\boxed{= -x_0}$

- when $t = 3T/4$

$x = x_0\cos\frac{2\pi}{T}\times\frac{3T}{4}$

$x = x_0\cos\frac{3\pi}{2}$

$x = x_0\cos 270° = x(0)$ $\boxed{=0}$

- when $t = T$

$x = x_0\cos\left(\frac{2\pi}{T}\right)\times T$

$x = x_0\cos 2\pi$

$x = x_0\cos 360°$

$x = x_0(1)$

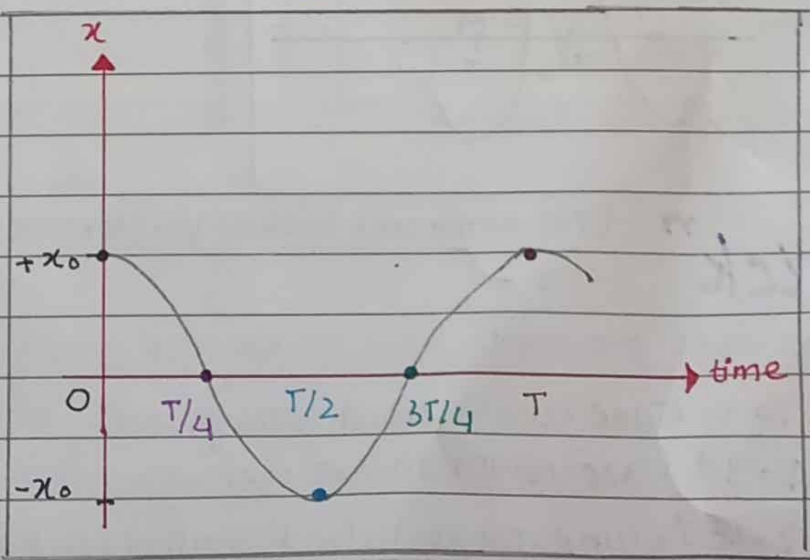
$\boxed{x = x_0}$

"table"

d	x	0	$-x$	0	x
T	0	$T/4$	$T/2$	$3T/4$	T

Note: colour coordination will help you to understand the plotting of values on the graph.

"graph"



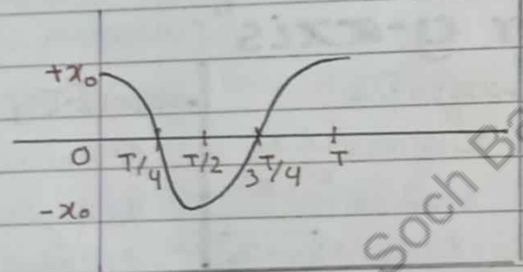
1, PHASE :- "The angle $\theta = \omega t$ which specifies the displacement x as well as the direction of motion of the point oscillating with SHM is called phase."

"main points"

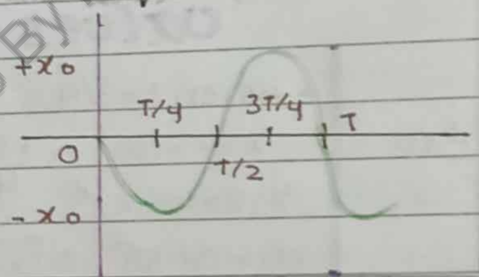
- $\boxed{x = x_0 \cos(\omega t + \Phi)}$ (1) where $\theta = (\omega t + \Phi)$ is the phase angle.
- Φ is called starting or initial phase of oscillator
- Φ also represents the phase difference between the states of motion of two oscillators
- If $\Phi = 0$ then eq (1) becomes $\boxed{x = x_0 \cos\omega t}$ a
- If $\Phi = 90°$ then eq (1) becomes $\boxed{x = x_0 \cos(\omega t + 90°)}$ b
- If $\Phi = 180°$ then eq (1) becomes $\boxed{x = x_0 \cos(\omega t + 180°)}$ c

"graph"

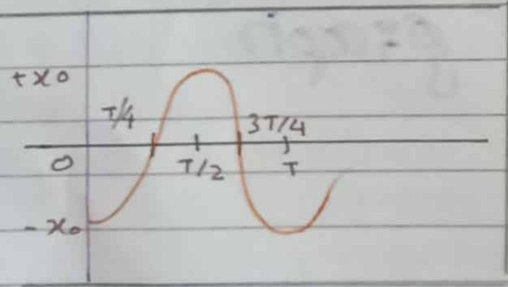
for eq a



for eq b



for eq c



{Note:} By putting values of Φ in eq (1) and then the respective values of $t = 0, T/4, T/2, 3T/4, T$, graphs 'a' 'b' and 'c' are obtained.

"Trick"

[This trick is only for finding initial point on graph when $t = 0$]

1, How to remember the points from which the graph starts for different values of Φ

For graph a $\Phi = 0$ (Take $\cos(0)$ to find starting point) $x = x_0(\cos 0) \boxed{= x_0}$

For graph b $\Phi = 90°$ (Take $\cos(90)$ to find starting point) $x = x_0(\cos 90) = 0$

For graph c $\Phi = 180°$ (Take $\cos(180)$ to find starting point) $x = x_0(\cos 180) \boxed{= -x_0}$

1, DAMPED OSCILLATIONS:- "Oscillations which have a decrease in amplitude due to energy dissipation with time are known as damped oscillations"

1, Damping

Damping is the process by which energy is lost by the oscillating system.

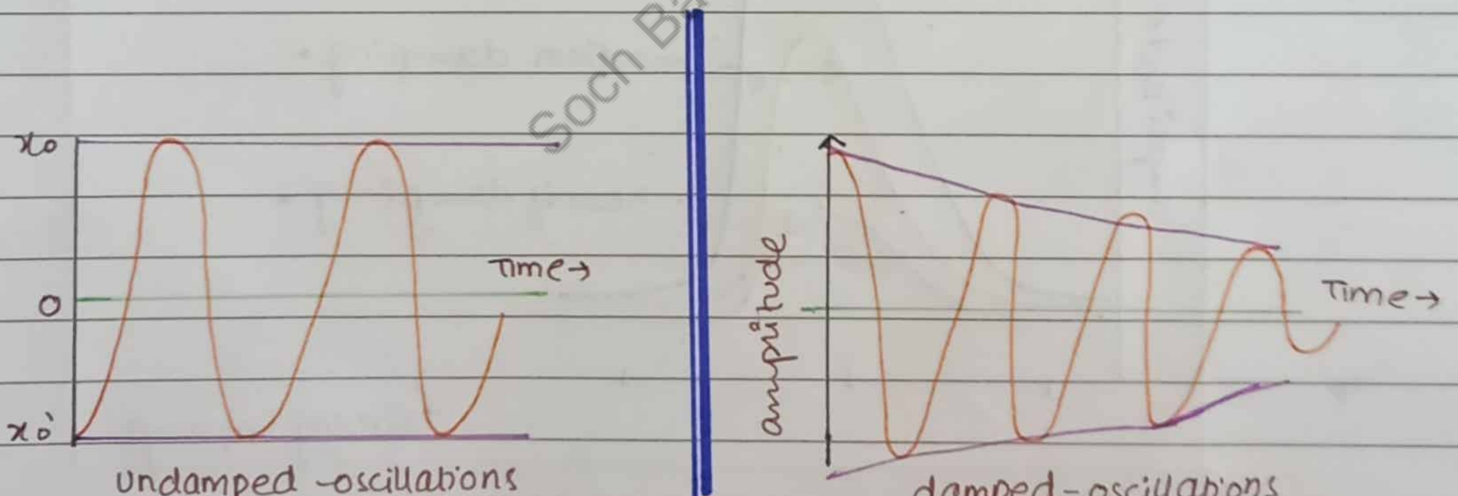
"application"

1, Shock absorbers are installed in a car which provides a damping force to absorb or to stop the excessive oscillations.

"undamped oscillations"

1, Ideal oscillations are undamped oscillations whose amplitude remains same with time i-e no energy dissipation takes place.

"graph"



undamped -oscillations

damped-oscillations

"definitions"

1, Overdamped when the damping force is greater than oscillating force then it's called over damping

2, Critical damping when damping force is ~~smaller~~ equal to the oscillating force

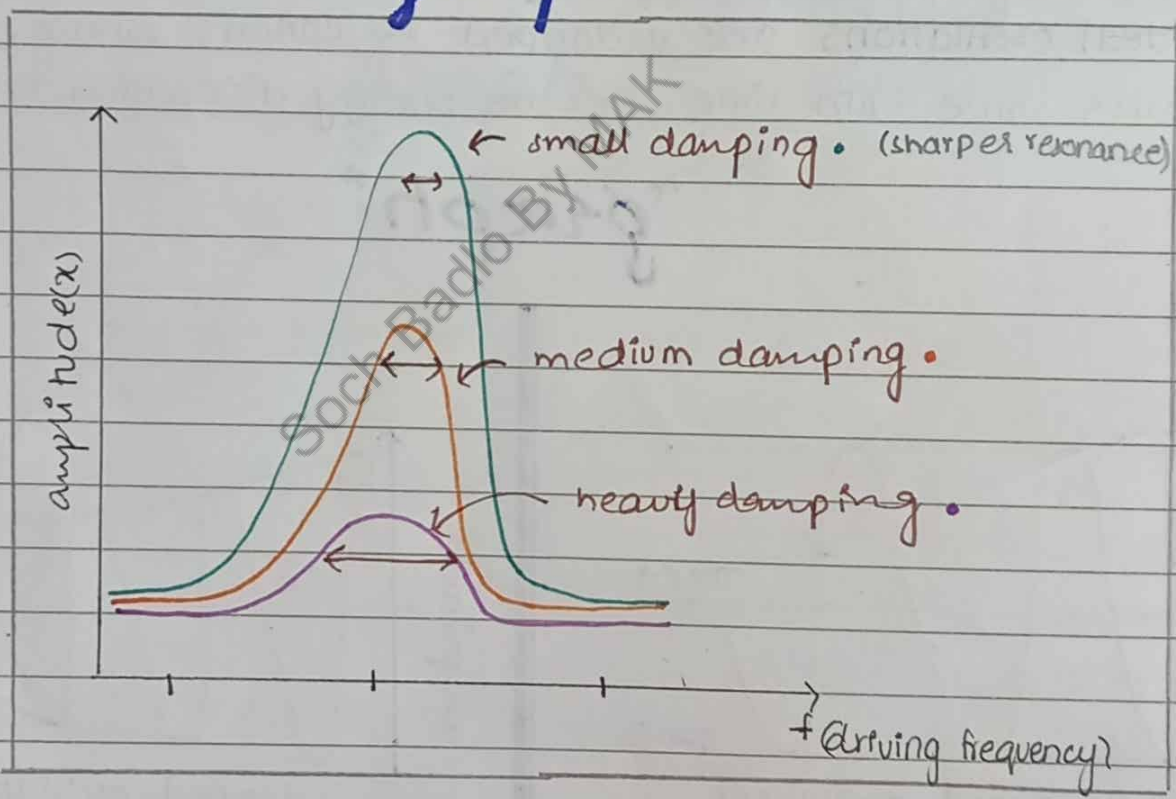
3, Under damping When damping force is smaller than the oscillating force then the motion of the body is called under-damping.

"Sharpness of resonance"

Main points

- If damping is small, amplitude of vibration becomes very large at resonance.
- Increase in damping decreases the amplitude of vibration at resonance
- $\boxed{\text{sharpness of resonance} \propto 1/\text{damping}}$
- Shock absorbers are built on this principle i.e they reduce the sharpness of resonance by increasing damping.

"graph"



- The sharpness of the resonance curve depends on the frictional loss of energy.