CHAPTER 6 FLUID DYNAMICS

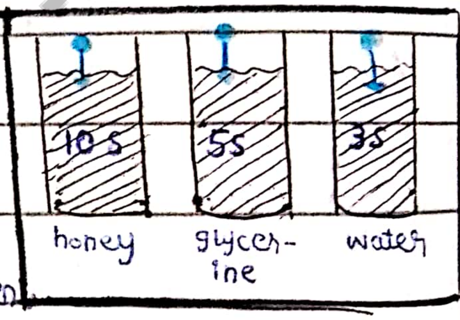
VISOSITY

Viscosity is the resistance to flow of a fluid.

* Viscosity of a fluid is the measure of its **resistance to flow**
* It measure that how much force is required to slide one layer of the liquid over another layer

EXPLANATION

consider 3 different containers filled equally (volume is same) with honey, glycerine and water. If we throw glass pebbles in each of the container



honey will offer high resistance due to strong intermolecular forces and hence it will take more time to flow. Glycerine will offer less resistance and will take less time to flow as compared to honey. In case of water, it offers least resistance and will take less time to flow.

$(\text{Viscosity})_{honey} > (\text{Viscosity})_{glycerine} > (\text{Viscosity})_{H_2O}$

- Viscosity depends on movement of molecules
- $v \propto \frac{1}{t}$ $(S = vt)$ distance covered is same

Soch Badlo By MAK

CS CamScanner

- Viscosity of liquids decreases with increase in temperature
- Viscosity of gases increases with increase in temperature

- Viscosity ∝ intermolecular forces
- Viscosity ∝ $\frac{1}{\text{Temperature}}$ [T ↑, intermolecular forces ↓, viscosity ↓]

COEFFICIENT OF VISCOSITY (η)

→ The numeric value of resistance to flow of liquid

UNITS

* SI unit is PaS or $kg\,m^{-1}s^{-1}$ or $Nm^{-2}s$
* Non SI unit is Poise or dyne second/cm^2
* 1 PaS = 10 Poise
* 1 centipoise = 1 millipascal second

Viscosity of water = 8.91×10^{-4} PaS
Viscosity of honey = 1.42 PaS
Viscosity of blood = 1.6×10^{-3} PaS

DIMENSION

$$\eta = \frac{F}{rv}$$
$$= \frac{kg\,ms^{-2}}{m \cdot ms^{-1}}$$
$$= kg\,ms^{-2}\,m^{-2}\,s$$
$$= kg\,m^{-1}s^{-1}$$
$$= [ML^{-1}T^{-1}]$$

Soch Badlo By MAK

CS CamScanner

FLUID FRICTION & STOKES LAW

DRAG FORCE

The force of friction acting on a body while moving through the fluid is called drag force or fluid friction.

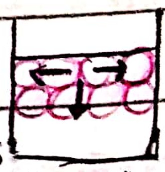
Drag force depends on:

1. size, shape and orientation of the object
2. properties of the fluid (viscosity and density)
3. speed of the object relative to the fluid

STATEMENT OF STOKES LAW

" The force that retards a sphere moving through a viscous fluid is directly proportional to the velocity 'v', radius 'r' and coefficient of viscosity 'η'."

Tension present within the layers of fluid due to intermolecular forces is called surface tension



MATHEMATICALLY

$F_D \propto v$

$F_D \propto r$

$F_D \propto \eta$

→ $F_D \propto \eta r v$

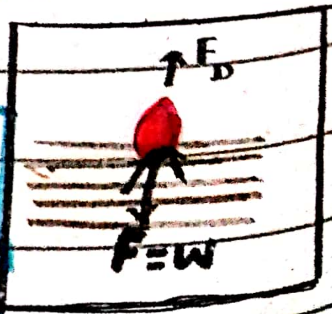
$F_D = A \eta r v$

$$F_D = 6\pi \eta r v$$

A = constant of proportionality

$A = 6\pi$

Drag force acts opposite to the direction of motion of object



Soch Badlo By MAK

CS CamScanner

TERMINAL VELOCITY (V_t)

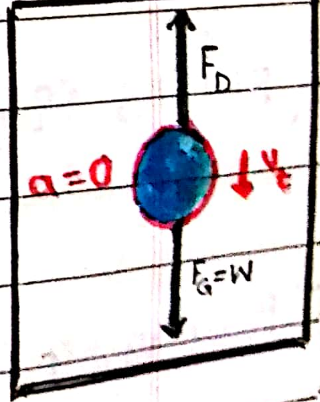
DEFINITION

The constant maximum velocity that is attained and maintained by an object while falling through a resistive medium is called terminal velocity

EXPLANATION

When a drag force acts on the droplet, it will increases the velocity of the droplet. A point will come that the droplet's weight and the drag force become equal ($F_{upward} = F_{downward}$) Both forces are equal in magnitude and opposite in direction. The net force become zero and the droplet will fall with ~~with~~ constant velocity which is known as terminal velocity.

MATHEMATICALLY



Consider a droplet of mass 'm' and radius 'r'. During its suspension, there are 2 forces acting on it:

1. Weight of the droplet 'W' (↓)
2. Drag force 'F_D' (↑)

Sooch Badlo By MAK

CamScanner

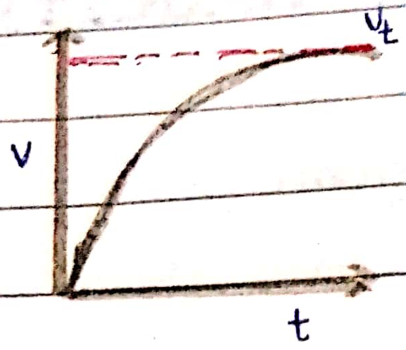
$F_{net} = F_1 + F_2$

$F_{net} = -W + F_d$

when $F_{net} = 0$

$0 = -W + F_d$

$W = F_d$



According to Stokes Law

$mg = 6\pi\eta r v_t$

$$v_t = \frac{mg}{6\pi\eta r} \quad (*)$$

Terminal velocity in terms of mass

To find 'm'

As density is equal to mass over volume

$\therefore \rho = \frac{m}{V}$

$\rho V = m$ —— (1)

Volume of sphere

$= \frac{4}{3}\pi r^3$ —— (2)

Putting eq (2) in eq (1)

$m = \rho\left[\frac{4}{3}\pi r^3\right]$

$m = \frac{4}{3}\pi r^3 \rho$ —— (3)

Putting eq (3) in eq (*)

$v_t = \frac{\left(\frac{4}{3}\pi r^3 \rho\right) g}{6\pi\eta r}$

$v_t = \frac{4\rho r^3 g\pi}{8\eta r\pi}$

$$v_t = \frac{2}{9}\frac{\rho r^2 g}{\eta}$$

$v_t \propto r^2$

Terminal velocity in terms of density

To Find 'r'

$$r^2 = \frac{v_t\, 9\eta}{\rho g\, 2}$$

V_t depends on:

1. size 2. Shap 3. orientation

4. η 5.

MCQ:

A person falling through air on earth reaches 'V_t' after about **12s** covering distance of about **450 m**

Soch Badlo By MAK

CS CamScanner

APPLICATION OF TERMINAL VELOCITY

MOTION OF PARATROOPER

In free fall fall, the paratrooper attains his terminal velocity **twice**

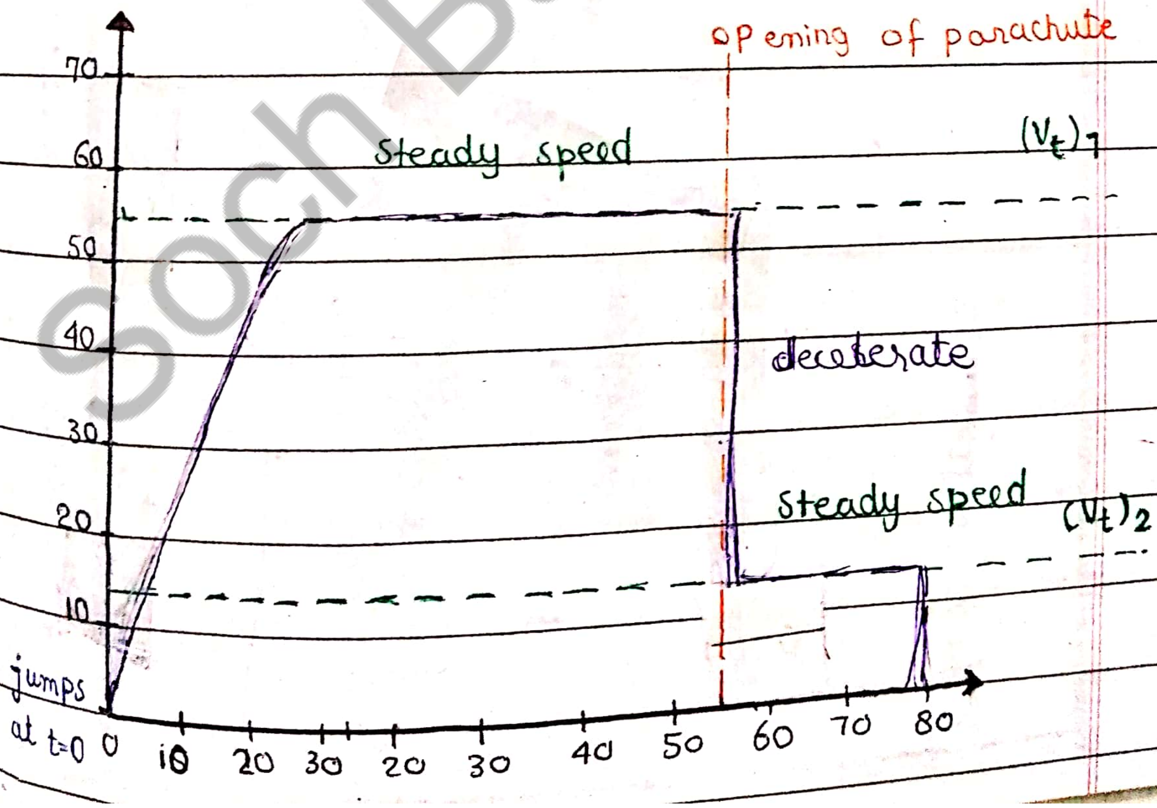
1. Without opening the chute

- offers lower radius to air
- has a high terminal speed

2. After opening the chute

- has large radius
- have low terminal speed

GRAPHICAL EXPLANATION



Soch Badlo By MAK

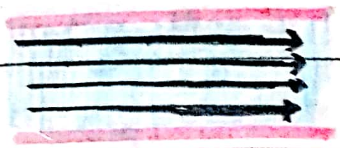
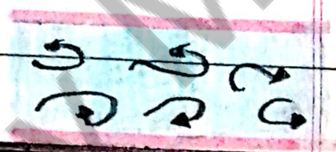
CamScanner

STAGE-1	STAGE-2	STAGE-3	STAGE-4
t = 0 • After just jumping from the plane the skydiver is not moving fast • weight is bigger force than air resistance W > Air Resistance • skydiver accelerate downward	t = 19s • Force of air resistance has increased $(F)_{\text{air Resistance}} = (W)_{\text{sky-diver}}$ • Forces are balanced • speed remains constant Terminal Velocity - 1	t = 54 s • The chute opens • air Resistance increases $(F)_{\text{air Resistance}} > W$ • Skydiver slows down	t = 58 s • Skydiver slows • Air resistance from chute is Reduced $(F)_{\text{air Resistance}} = W$ • Forces are balanced • speed remains constant Terminal velocity - 2

Soch Badle By MA

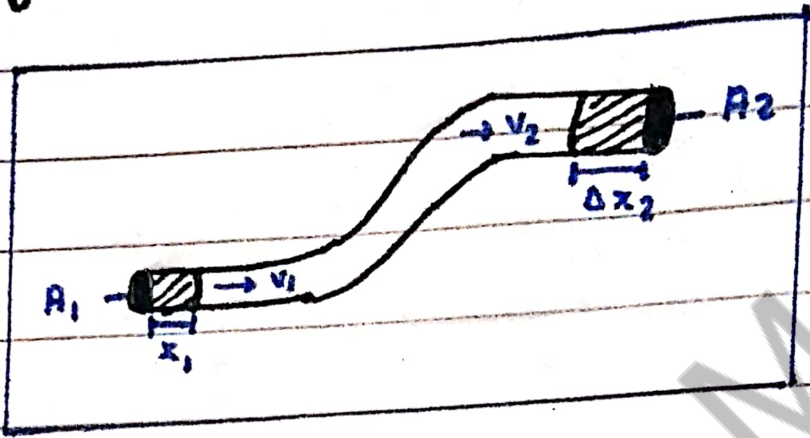
CS CamScanner

FLUID FLOW

Stremline Flow	Turbulent Flow
Definition	
The flow of fluid in which every particle of fluid moves along a smooth path. 	The irregular or unsteady flow of fluid is called turbulent flow. 
Path	
Regular and smooth	irregular, not smooth
Crossing	
streamlines donot cross each other and fluid layers move ll to each other	fluid layers cross each other and don't move parallel to each other
Velocity	
constant (low)	changes abruptly (high)
Viscosity	
tends to occur at high viscosity	tends to occur at low viscosity
Position	
can be determined properly	can't be determined properly
Example	
when air is passing through pipe with constant speed	• blood flow in arteries • oil transport in pipelines

Sooch Badlo By MAK

EQUATION OF CONTINUITY
(Law of conservation of mass)



STATEMENT

The product of crossectional area and the speed of the fluid at any point along the pipe is constant.

$$A_1 v_1 = A_2 v_2$$

$$Av = \text{constant}$$

DERIVATION

Consider a pipe of non-uniform crossectional area and fluid flow is ideal in it. As there is no source or sink in the pipe so **equal mass** will flow through each end of pipe.

CONDITIONS

We are considering that the fluid is

1. Incompressible (density is constant)
2. Non-viscous (internal friction is neglected)
3. Flow is steady (velocity at each point is constant)

Soch Badlo By MAK

CS CamScanner

A_1	cross sectional area of pipe at lower end
A_2	cross sectional area of pipe at upper end
Δx_1	position of Δm_1 in lower end of pipe
Δx_2	position of Δm_2 in upper end of pipe
V_1	speed of Δm_1 in lower end of pipe
V_2	speed of Δm_2 in upper end of pipe
Δm_1	mass of water considered for lower end of pipe
Δm_2	mass of water consider for upper end of pipe.

Δm_1 IN LOWER END OF PIPE	Δm_2 IN UPPER END OF PIPE
$\rho = \frac{\Delta m_1}{\Delta V_1}$	$\rho = \frac{\Delta m_2}{\Delta V_2}$
$\Delta m_1 = \rho \Delta V_1$	$\Delta m_2 = \rho \Delta V_2$
$\Delta m_1 = \rho (l \times b \times h)$	$\Delta m_2 = \rho (l \times b \times h)$
$\Delta m_1 = \rho (Ah)$	$\Delta m_2 = \rho (Ah)$
$\Delta m_1 = \rho A_1 \Delta x_1$	$\Delta m_2 = \rho A_2 \Delta x_1$
$(\because \Delta x = V \Delta t)$	
$\boxed{\Delta m_1 = \rho A_1 V_1 \Delta t}$	$\boxed{\Delta m_2 = \rho A_2 V_2 \Delta t}$

Soch Badlo By MAK

CamScanner

According to Law of Conservation of mass

$$\Delta m_1 = \Delta m_2$$

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2 \qquad \left[A \propto \frac{1}{v}\right]$$

$$A v = \text{constant}. \qquad A_1 > A_2$$

$$V_1 < V_2$$

VOLUME FLOW RATE

- $Av = \text{constant}$
- $A\left[\frac{\Delta x}{\Delta t}\right] = \text{constant} \qquad (\because v = \frac{\Delta x}{\Delta t})$

$$\boxed{\frac{\Delta V}{\Delta t} = \text{constant}} \qquad (\because A\Delta x = \Delta V)$$

$A_1 v_1 = A_2 v_2$ (in terms of cross sectional area)

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

$r_1^2 v_1 = r_2^2 v_2$ (in terms of radius)

$$\frac{d_1^2}{4} v_1 = \frac{d_2^2}{4} v_2$$

$d_1^2 v_1 = d_2^2 v_2$ (in terms of diameter)

$$\left[\begin{array}{l} d = 2r \Rightarrow r = \frac{d}{2} \\ \left(\frac{d_1}{2}\right)^2 v_1 \Rightarrow \frac{d_1^2 v_1}{4} \end{array}\right]$$

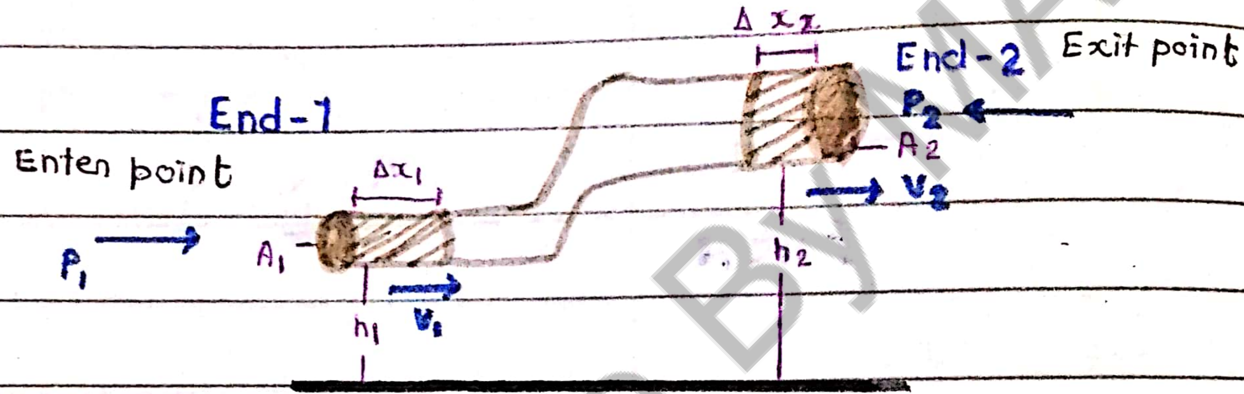
Soch Badlo By MAK

CS CamScanner

BERNOLLI'S EQUATION

[Law of conservation of Energy In Fluid Dynamics]

➡ Bernolli's equation relates Pressure, flow speed (v), and height of flow of an ideal fluid.



A_1 = Cross section area at end-1

A_2 = Cross section area at end-2

h_1 = hieight from the surface of end-1

h_2 = height from surface of end-2

V_1 = velocity of the fluid at end-1

V_2 = velocity of the fluid at end-2

Δx_1 = The distance moved by the fluid at end-1

Δx_2 = The distance moved by the fluid at end-2

P_1 = Pressure at end-1

P_2 = Pressure at end -2

Assuming that fluid is incompressible, non viscous, flows in steady state manner

Soch Badlo By MAK

CamScanner

Work Done 'W_1' By The Fluid At The Lower End Of Pipe	Work Done 'W_2' On The Fluid At The Upper End Of Pipe
	
$W_1 = F_1 \cdot \Delta x_1$	$W_2 = F_2 \cdot \Delta x_2$
$W_1 = F_1 \Delta x_1 \cos\theta_1 \ (\theta = 0)$	$W_2 = F_2 \Delta x \cos\theta_2$
$W_1 = P_1 A_1 \Delta x_1 \quad [P = \frac{F}{A}]$	$W_2 = F_2 \Delta x_2 \cos 180$
$W_1 = P_1 A_1 v_1 \Delta t \quad (S = vt)$	$W_2 = -F_2 \Delta x_2$
$W_1 = P_1 \frac{V_1}{\Delta t} \Delta t$	$W_2 = -P_2 A_2 v_2 \Delta t$
$W_1 = P_1 V_1$	$W_2 = -P_2 \frac{V_2}{\Delta t} \Delta t$
	$W_2 = -P_2 V$

According to Law of conservation of Energy

$$\Delta W = \Delta K.E + \Delta P.E$$

$$W_1 - W_2 = \frac{1}{2} m_2 v_2^2 - \frac{1}{2} m_1 v_1^2 + m_2 g h_2 - m_1 g h_1$$

$$m_1 = m_2 = m$$

$$W_1 - W_2 = \frac{1}{2} m_2 v_2^2 - \frac{1}{2} m_1 v_1^2 + m g h_2 - m g h_1$$

$$(P_1 - P_2) V = m \left[\frac{1}{2} v_2^2 - \frac{1}{2} v_1^2 + g h_2 - g h_1\right]$$

$$(P_1 - P_2) \frac{m}{\rho} = m \left[\frac{1}{2} v_2^2 - \frac{1}{2} v_1^2 + g h_2 - g h_1\right] \qquad \rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$$

$$P_1 - P_2 = \rho \left[\frac{1}{2} v_2^2 - \frac{1}{2} v_1^2 + g h_2 - g h_1\right]$$

$$P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 + \rho g h_2 - \rho g h_1$$

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

GENERALLY

$$P + \frac{1}{2} \rho v + \rho g h = \text{constant}$$

Sooch Badlo By MAK

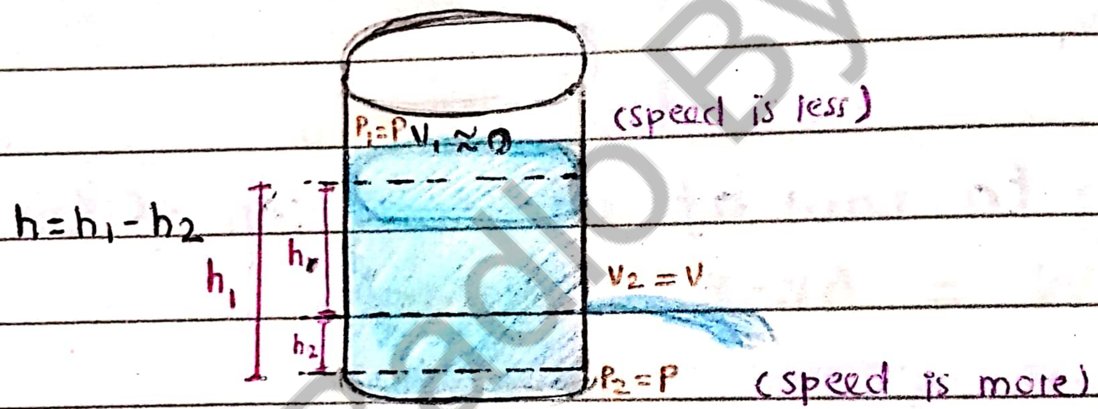
CamScanner

APPLICATIONS OF BERNOLLI'S EQUATIO

TORRICELLI'S THEOREM (speed of efflux)

STATEMENT

"The speed of efflux is equal to the speed gained by fluid while falling through height 'h' under action of gravity"



EXPLANATION

consider a large storage tank which develop leak at bottom.

1. Pressure at both ends is same $P_1 = P_2 = P$
2. velocity at the top is considered zero $(V_1 \approx 0)$
3. Bottom velocity is to be determined $(V_2 = V)$
4. height of water level from ground (h_1)
5. height of ~~bottle~~ from ground (h_2)

Soch Badlo By MAK

CamScanner

6. height of water level above orifice $(h = h_1 - h_2)$

According to Bernulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$v_1 \approx 0 \qquad P_1 = P_2 = P$$

$$P + \frac{1}{2}\rho (0)^2 + \rho g h_1 = P + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\rho g h_1 - \rho g h_2 = \frac{1}{2}\rho v^2$$

$$\rho g (h_1 - h_2) = \frac{1}{2}\rho v^2$$

$$\rho g h = \frac{1}{2}\rho v^2$$

$$g h = \frac{1}{2} v^2$$

$$2 g h = v^2$$

$$v^2 = 2gh$$

$$\boxed{v = \sqrt{2gh}}$$

Soch Badlo By MAK

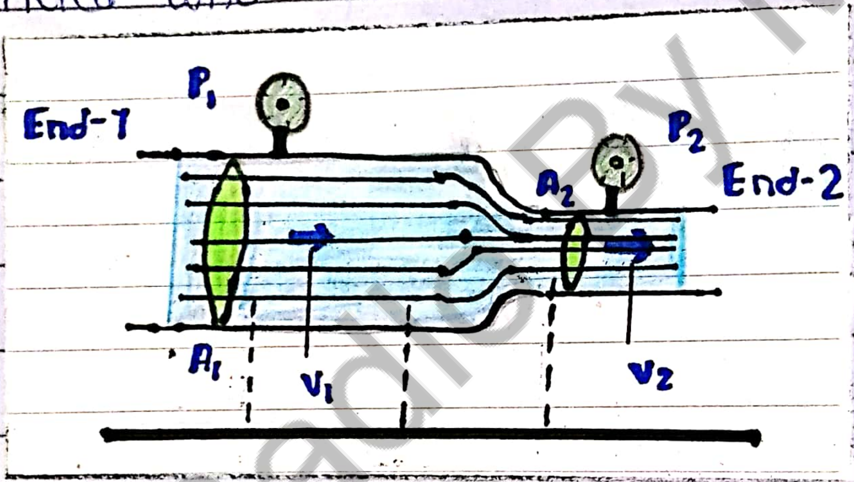
CS CamScanner

VENTURI METER (FLOW METER)

"Venturi meter is a device used to measure the flow speed or flow rate through piping system"

PRINCIPLE

It works on the principle of pressure difference b/w restricted and unrestricted flow region.



consider a pipe of Non-uniform cross sectional area placed horizontally. The part of pipe having wide cross sectional area have area A_1, pressure P_1 and velocity V_1. The narrow ~~end~~ have cross-sectional area A_2, velocity V_2 and pressure P_2.

CONDITIONS

We are considering that the fluid is incompressible, has negligible internal frictional and its flow is steady (ideal fluid)

Soch Badlo By MAK

CamScanner

According to equation of continuity, when fluid flows through a narrow cross sectional area, then its velocity will increase ($A \propto \frac{1}{v}$)

When velocity will increase then According to Bernoulli's equation, the pressure will decrease ($P \propto \frac{1}{v}$)

MATHEMATICALLY

According to Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

As the pipe is placed horizontally with same height, therefore

$$h_1 = h_2 = h$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 \quad \text{——— (1)}$$

According to law of conservation of mass

$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2}$$

Putting the value of 'V_2' in eq (1)

$$P_1 - P_2 = \frac{1}{2}\rho\left[\frac{A_1 V_1}{A_2}\right]^2 - \frac{1}{2}\rho V_1^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left[\frac{A_1^2 V_1^2}{A_2^2}\right]^2 - \frac{1}{2}\rho V_1^2$$

Soch Badlo By MAK

CamScanner

$$P_1 - P_2 = \frac{1}{2} \rho V_1^2 \left[\frac{A_1^2}{A_2^2} - 1 \right]$$

$$P_1 - P_2 = \frac{1}{2} \rho V_1^2 \left[\frac{A_1^2 - A_2^2}{A_2^2} \right]$$

$$V_1^2 = \frac{2(P_1 - P_2) A_2^2}{\rho (A_1^2 - A_2^2)}$$

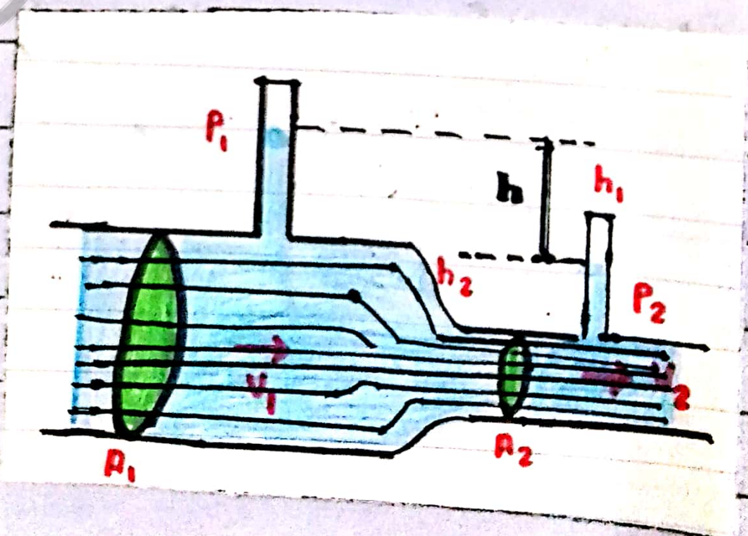
Taking square root

$$\sqrt{V_1^2} = \sqrt{\frac{2(P_1 - P_2) A_2^2}{\rho (A_1^2 - A_2^2)}}$$

$$\boxed{V_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho (A_1^2 - A_2^2)}}}$$

velocity of pipe when barometer is used

WHEN NO BAROMETER IS USED :



Soch Badlo By MAK

CS CamScanner

$P_1 = \rho g h_1$, $P_2 = \rho g h_2$

$$V_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

$$V_1 = A_2 \sqrt{\frac{2(\rho g h_1 - \rho g h_2)}{\rho(A_1^2 - A_2^2)}}$$

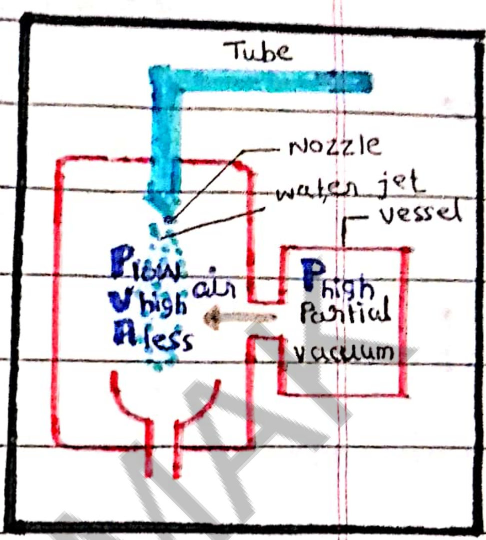
$$V_1 = A_2 \sqrt{\frac{2\rho g(h_1 - h_2)}{\rho(A_1^2 - A_2^2)}}$$

$$\boxed{V_1 = A_2 \sqrt{\frac{2gh}{(A_1^2 - A_2^2)}}}$$ $(\because h_1 - h_2 = h)$

Soch Badlo By MAK

CS CamScanner

FILTER PUMP



Filter pump is a device used to produce partial vacuum in vessel attached to it.

USE

Pumps are used to transfer liquids from low pressure zones to high pressure zones.

CONSTRUCTION

Filter pump consists of a tube with jet attached to it, in which water flows from the tube toward the jet.

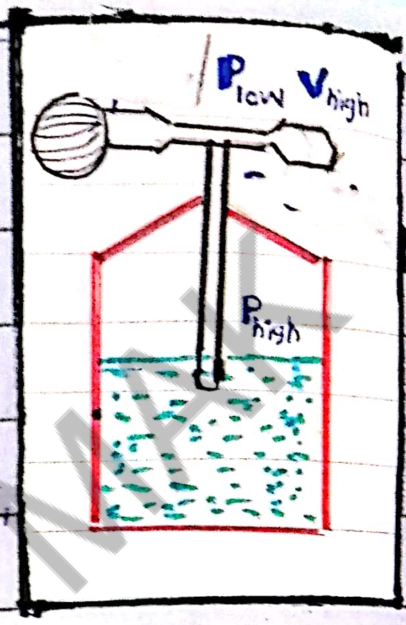
WORKING

1. When water reaches from jet section its **speed increases**, as result **pressure decreases**
2. The drop in pressure allows the air to flow in from the side tube to which the vessel is connected, thus air and water are forced together at the bottom of filter pump.
3. In this way, partial vacuum is created in the vessel attached to it.

Soch Badlo By MAK

CS CamScanner

ATOMIZERS



→ Atomizer is a device used for emitting water, perfume or other liquids as a fine spray

USE

Such atomizers can be seen in perfume bottles, engine carburetor, water filter pumps and paint sprays.

CONSTRUCTION

1. bulb 2. jet 3. nozzle 4. tube immersed in liquid

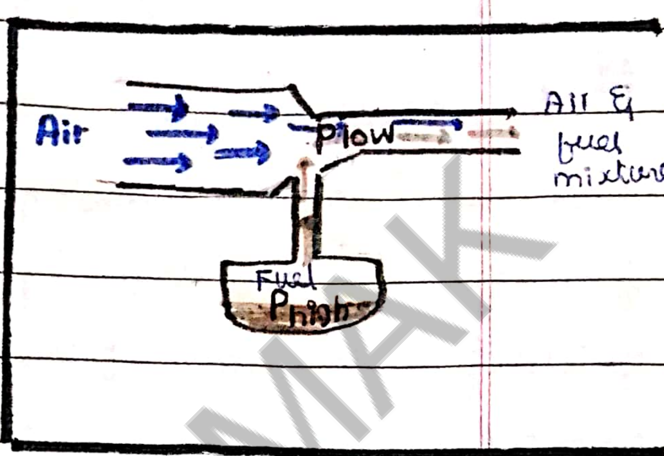
WORKING

1. A stream of air passing over one end of an open tube, the other end of which is immersed in liquid reduces the pressure above the tube.
2. The reduction in pressure causes the liquid to to rise into the air stream.
3. The liquid is then dispersed into a fine spray of droplets.

Soch Badlo By MAK

CamScanner

ENGINE CARBURETOR



- An engine carburetor is a device that blends air and fuel for an internal combustion engine.
- Newer cars tend to have Electronic fuel Injectors (EFI) rather than carb-urators but carburators are common in older cars.

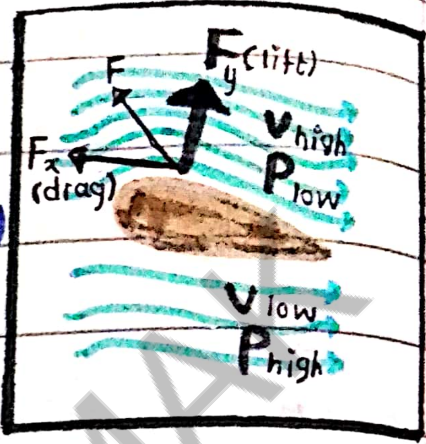
WORKING

1. Part of carburator is a tube with constriction. The pressure on the petrol in the fuel supply is the same as the pressure in the thicker part of the tube.
2. Air flowing through the narrow section of the tube, which is attached to fuel supply is at low pressure, so fuel is forced into the air flow.
3. By regulating the flow of air in the tube, the amount of fuel is mixed into the air can be changed.

Soch Badlo By MAK

CS CamScanner

AEROFOIL



" The devices which are ~~us~~shaped so that the relative motion b/w it and the fluid produces a force perpendicular to the flow are called aerofoils. "

EXPLANATION

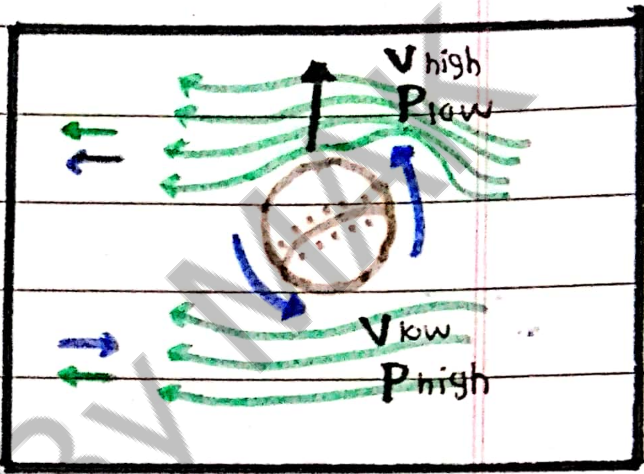
1. The shape of aerofoil is made such that the fluid speed at the top surface is greater than the bottom (closer stream lines) due to which the pressure is less
(closeness of streamlines ∝ velocity) $[v \propto \frac{1}{P}]$
2. An aero-foil-shaped body moved through a fluid produces an **aerodynamic force.**
3. The component of this force perpendicular to the direction of motion is called **lift.** (F_y)
4. The component parallel to the direction of motion is called **drag**

Aerofoils are found in aeroplane wings, helicopters, sailboats, propellers, fans, compressors and turbines.

Soch Badlo By MAX

CS CamScanner

HOW SWING IS PRODUCED IN A FAST MOVING BALL ?



When a ball is thrown, it spins as well as moves forward. Due to this, the speed of air on one side of the ball increases and pressure decreases. On the other side of the ball, the speed of air is low whereas pressure is high. Due to pressure difference, a swing is produced in the ball. This gives an extra curvature to the ball.

Soch Badlo By MAK

CS CamScanner