CHAPTER 05

ROTATIONAL AND CIRCULAR MOTION

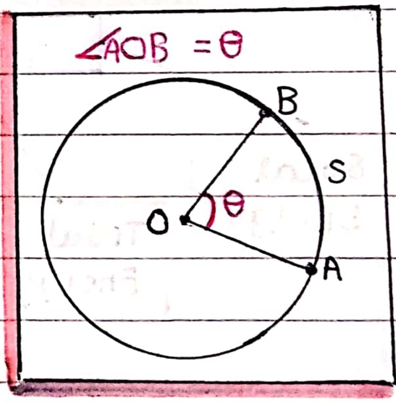
EXAMPLES

1. The wheels of a car or bicycle.
2. The earth in its orbit round th sun.
3. The motion of fans.
4. A spinning DVD in a laptop.

ANGULAR DISPLACEMENT

Def: The angle θ through which the object has moved is known as its angular displacement

Mathematically:



$$\theta = \frac{\text{length of arc}}{\text{Radius}}$$

$$\theta = \frac{AB}{r}$$

$$\theta = \frac{S}{r}$$

$$\boxed{S = r\theta}$$

1	SI Unit of angular 'S' is Radian
2	1 rad = 57.3°
3	2π rad = 360° (1 revolution)

ANGULAR VELOCITY

Angular velocity	Average angular velocity	Instantaneous angular velocity
The rate of change of angular displacement	Total angular displacement divided by the total time taken.	Rate of change of angular displacement at any instant of time is instantaneous angular velocity
$\vec{\omega} = \frac{\Delta\vec{\theta}}{\Delta t}$	$\langle\vec{\omega}\rangle = \frac{\Delta\vec{\theta}}{\Delta t}$	$\vec{\omega}_{inst} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{\theta}}{\Delta t}\right)$

SI UNIT	NON-SI UNIT
Radians per second ($rad\ s^{-1}$)	revolution per second ($rev\ s^{-1}$)

ANGULAR ACCELERATION

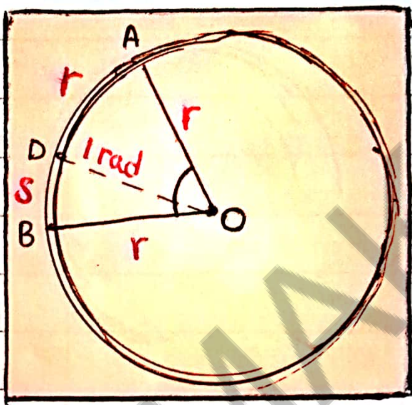
Angular Acceleration	Average angular acceleration	Instantaneous angular acceleration
The Rate of change of angular velocity.	Total change in angular velocity divided by total time is called average angular 'α'	Rate of change of angular velocity at any instant of time is called instantaneous angular acceleration
$\alpha = \frac{\Delta\vec{\omega}}{\Delta t}$ $\alpha = \frac{\Delta\vec{\omega}_f - \vec{\omega}_i}{\Delta t}$	$\langle\vec{\alpha}\rangle = \frac{\Delta\vec{\omega}}{\Delta t}$	$\vec{\alpha}_{ins} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{\omega}}{\Delta t}\right)$

SI UNIT	NON SI UNIT
$rad\ s^{-2}$	$rev\ s^{-2}$

RELATION B/W ANGULAR DISPLACEMENT AND LINER DISPLACEMENT

Arc DB = θ = s

Arc AB = radius = r



$\angle AOB$ = 1 rad = Arc AB

$\angle DOB$ = θ = Arc DB

$$\frac{\text{Arc } DB}{\text{Arc } AB} = \frac{\angle DOB}{\angle AOB}$$

$$\frac{s}{\text{radius}} = \frac{\theta}{1 \text{ rad}}$$

$$\frac{s}{r} = \frac{\theta}{1}$$

$$\boxed{s = r\theta}$$

RELATION B/W LINEAR AND AND ANGULAR VELOCITY

$s = vt$ ——— (i)

$s = r\theta$ ——— (ii)

By comparing eq (i) and eq (ii)

$vt = r\theta$

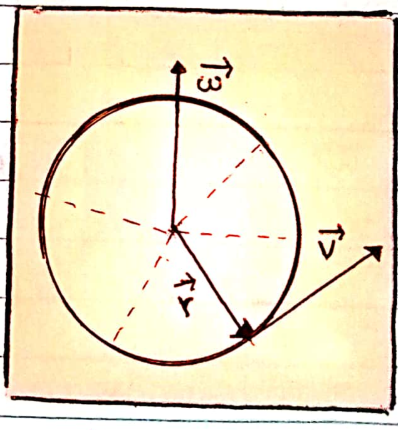
Dividing both sides by 't'

$$\frac{vt}{t} = \frac{r\theta}{t}$$

$$v = \frac{r\theta}{t} \quad (\because \frac{\theta}{t} = \omega)$$

$v = r\omega$

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

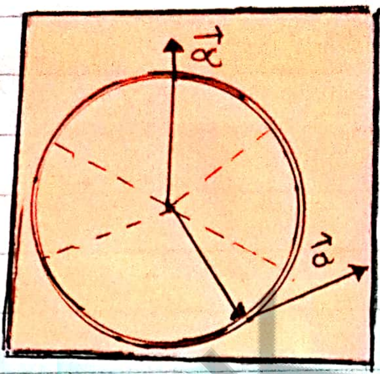


- $\vec{v}$ is in direction of z-axis

RELATION B/W LINER ACCELERATION AND ANGULAR ACCELERATION

$V = r\omega$ (Relation b/w 'V' and 'ω')

$v_f = r\omega_f$, $v_i = r\omega_i$



$$a = \frac{v_f - v_i}{\Delta t}$$

$$a = \frac{r\omega_f - r\omega_i}{\Delta t}$$

- $\vec{a}$ is in the direction of z-axis

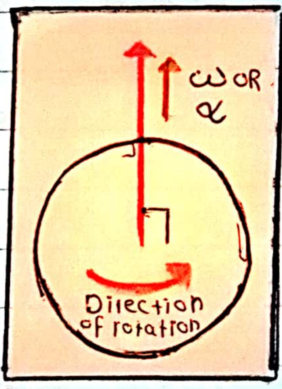
$$a = \frac{r(\omega_f - \omega_i)}{\Delta t}$$

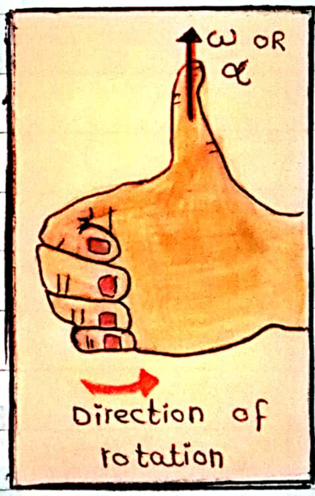
$$a = r\,\frac{\omega_f - \omega_i}{\Delta t} \qquad \left(\because \frac{\omega_f - \omega_i}{\Delta t} = \alpha\right)$$

$$\boxed{a = r\alpha}$$

RIGHT HAND RULE

Right hand rule can be used to find the direction of angular velocity and angular acceleration



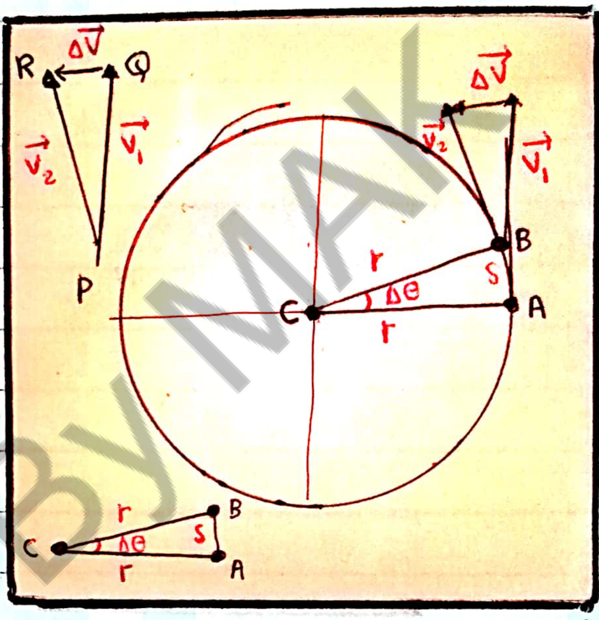


"Rotate the fingers of the Right hand in the direction of motion of rotating body, then thumb will point the direction of angular velocity and angular acceleration"

CENTRIPETAL ACCELERATION

Def: The change in velocity of body produces acceleration directing towards the centre of circle. Such acceleration is known as centripetal acceleration.

Explanation: consider a body of mass 'm' moving in a circle of radius 'r' with uniform velocity '$\vec{v}$'



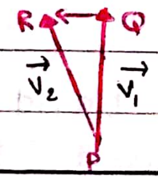
At Point A: at time $t_1 = \vec{V_1}$

At Point B: at time $t_2 = \vec{V_2}$

- ΔV is the difference in velocities $\Delta V = \vec{V_2} - \vec{V_1}$
- Speed is same but differ in the direction

In ΔPQR

- $PQ = \vec{V_1}$
- $PR = \vec{V_2}$



$$\frac{\text{Arc } AB}{AC} = \frac{QR}{PQ}$$

$$\frac{S}{r} = \frac{\Delta V}{V} \quad \text{—— (1)}$$

→ when θ is very small OR $\Delta t = t_2 - t_1$ is very small then point A is very close to B, we can write:

$$S = vt$$

Putting $s = vt$ in eq (1)

$$\frac{vt}{r} = \frac{\Delta V}{V}$$

By Cross Multiplication

$$(vt)(V) = (\Delta V)(r)$$

$$v^2 t = \Delta v \, r$$

$$\frac{v^2}{r} = \frac{\Delta v}{t}$$

$$\frac{\Delta v}{t} = \frac{v^2}{r} \quad \left(\because a = \frac{\Delta v}{t}\right)$$

$$a = \frac{v^2}{r}$$

$$\boxed{a_c = \frac{v^2}{r}}$$

$$\frac{s}{r} = \frac{\Delta v}{v}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r} \quad [\Delta t \text{ is very small}]$$

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = a_{inst}$$

$a_c = \frac{v^2}{r}$ IN VECTOR FORM

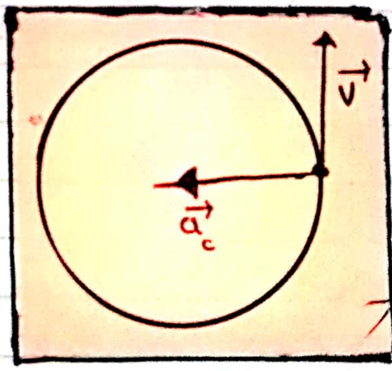
$$a_c = \left(\frac{v^2}{r}\right) \hat{r}$$

$$a_c = \frac{v^2}{r} \left(\frac{\vec{r}}{r}\right) \quad \left[\because \hat{r} = \frac{\vec{r}}{r}\right]$$

$$\boxed{a_c = \left(\frac{v^2}{r^2}\right) \vec{r}}$$

Acceleration and radius are oppositely directed so:

$$\boxed{a_c = -\left(\frac{v^2}{r^2}\right) \vec{r}}$$



'a_c' IN ANGULAR FORM

$[\because v = r\omega]$

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(r\omega)^2}{r}$$

$$a_c = \frac{r^2 \omega^2}{r}$$

$$a_c = r\omega^2$$

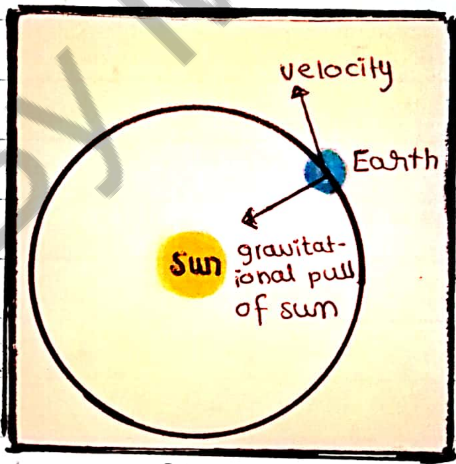
$$\boxed{a_c = -r\omega^2}$$

CENTRIPETAL FORCE

Def: The force which compel the body to move in a circle is called centripetal force.

Explanation: If an object is moving at a steady speed in a circle, we have a body whose velocity is not constant therfore there must be resultant (unbalanced) force acting on it.

- **Example:** As the Earth orbits the sun, it has a constantly changing velocity. Newton's first law suggests that there must be an unbalanced force acting on it. That force is the gravitational pull of sun. If the force disappeard, the earth would travel off in a straight line.



MATHEMATICALLY

From Newton's 2nd Law

$$F_c = m a_c \qquad (\because a_c = \frac{v^2}{r})$$

$$F_c = m\left(\frac{v^2}{r}\right)$$

$$\boxed{F_c = \frac{mv^2}{r}}$$

$F_c = \frac{mv^2}{r}$ IN VECTOR FORM

$$F_c = \left(\frac{mv^2}{r}\right)\hat{r} \qquad \left(\hat{r} = \frac{\vec{r}}{r}\right)$$

$$F_c = \frac{mv^2}{r} \cdot \frac{\vec{r}}{r}$$

$$\boxed{F_c = \left(\frac{mv^2}{r^2}\right)\vec{r}}$$

F_c and radius are oppositely directed
so :-

$$\boxed{F_c = -\left(\frac{mv^2}{r^2}\right)\vec{r}}$$

F_c IN ANGULAR FORM

$(\because v = r\omega)$

$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{m(r\omega)^2}{r}$$

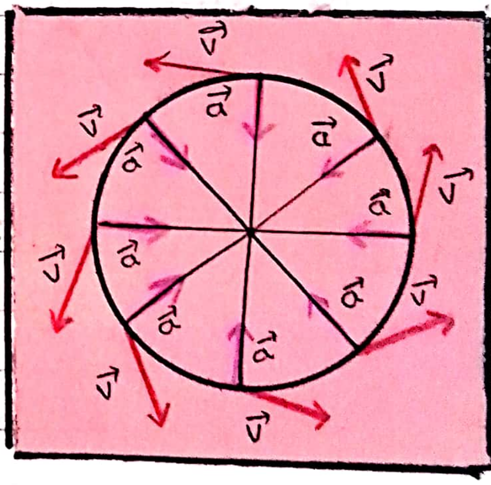
$$F_c = \frac{m r^2 \omega^2}{r}$$

$$F_c = m r \omega^2$$

$$F_c = (m\omega^2) r$$

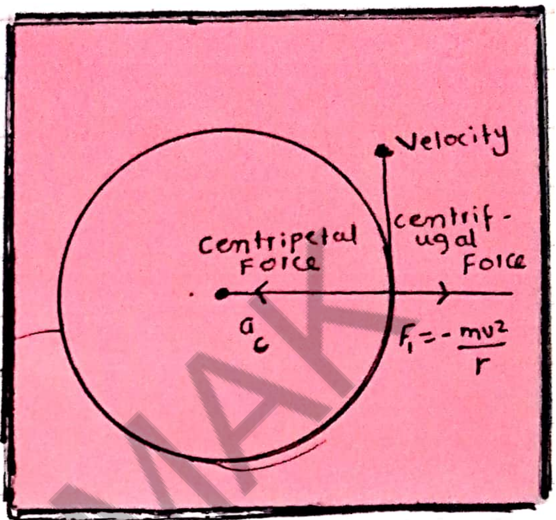
$$F_c = (m\omega^2)\vec{r}$$

$$\boxed{F_c = -(m\omega^2)\vec{r}}$$



- F_c always directed toward centre
- always produces a_c
- Always $\perp$ to direction of $\vec{v}$
- Line of Action of force changes

CENTRIFUGAL FORCE



- Centrifugal force is the tendency of an object moving in a circle to travel away from the centre of circle.
- The magnitude of the centrifugal force is same as that of centripetal force.
- It is a reactional force.

CENTRIFUGE

Such machines which are based on centrifugal force is called centrifuge. e.g washing machine.

MATHEMATICALLY

$$F = \frac{mv^2}{r}$$

$$F_c = \frac{-mv^2}{r}\hat{r}$$

$$F_c = \frac{-mv^2}{r}\frac{\vec{r}}{r}$$

$$F_c = \frac{-mv^2}{r^2}\vec{r}$$

$$\vec{F} = -F_c$$

$$\vec{F} = -\left(\frac{-mv^2}{r^2}\vec{r}\right)$$

$$\vec{F} = \frac{mv^2}{r^2}\vec{r}$$

BANKING OF ROAD

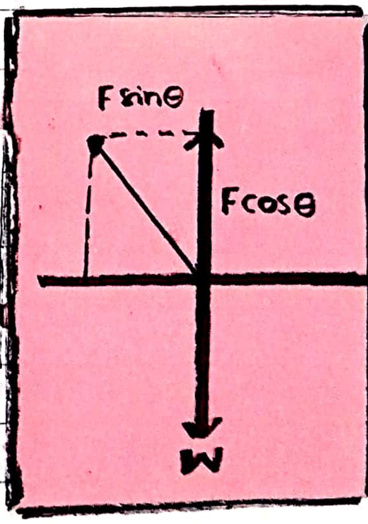
Definition: The phenomenon in which outer edges are raised for the curved roads above the inner edge to provide necessary centripetal force to the vehicles.

EXPLANATION

When the road is banked, the horizontal component of the normal reaction provides the necessary centripetal force required for circular motion of vehicle. To provide the necessary centripetal force at curved road, banking of road is necessary.

→ Centripetal force arises from the interaction b/w ~~road~~ and tyre.

MATHEMATICALLY



$F\cos\theta = mg$ ---- (1)

$F\sin\theta = \frac{mv^2}{r}$ ---- (2)

Dividing eq (2) by (1)

$$\frac{F\sin\theta}{F\cos\theta} = \frac{mv^2/r}{mg}$$

$$\tan\theta = \frac{v^2 m}{rmg}$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$v^2 = rg\tan\theta$$

$$v = \sqrt{rg\tan\theta}$$

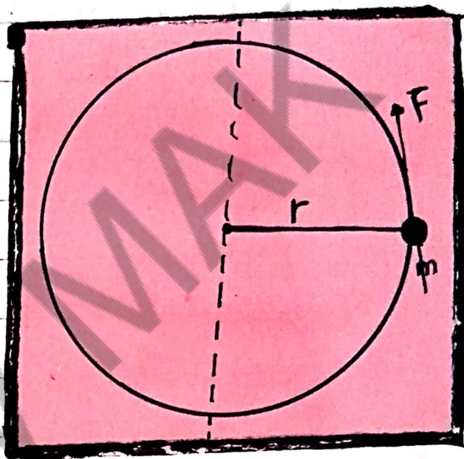
$$v \propto \theta$$

$$\theta \propto \frac{1}{r}$$

Moment Of Inertia

Definition The property due to which a body opposes any change in state of rest or rotatory motion.

Explanation



consider a particle of mass 'm'. The distance of mass from axis of rotation is radius 'r'. When we apply force perpendicular to radius, acceleration will produce.

Mathematically

$F = ma$ —— (i)

$F = m(r\alpha)$ $(\because a = r\alpha)$

$F = mr\alpha$

As turning effect is produced by torque :-

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} = \vec{r}\,(mr\alpha)$

$\vec{\tau} = mr^2\alpha$

$\boxed{\vec{\tau} = I\alpha}$ $(\because I = mr^2)$

Unit

$I = mr^2$

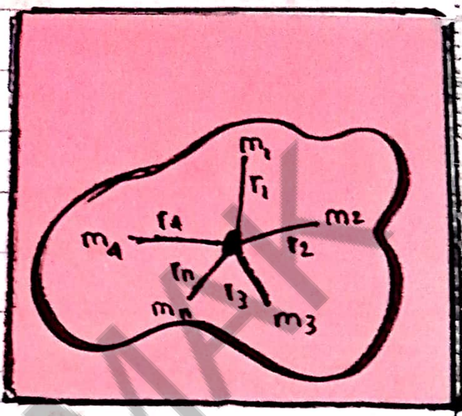
$I = kg\,m^2$

Dependance

- $I \propto m$
- $I \propto r^2$
- $I \propto \frac{1}{\alpha}$

Moment of Inertia for Non Symmetrical Rigid Body

Consider a rigid body which is divided into small pieces of masses $m_1, m_2, m_3 \ldots m_n$ having radii $r_1, r_2, r_3 \ldots r_n$.



$$I_1 = m_1 r_1^2$$
$$I_2 = m_2 r_2^2$$
$$I_n = m_n r_n^2$$

Total moment of inertia can be calculated as:

$$I_T = I_1 + I_2 + \ldots I_n$$
$$I_T = m_1 r_1^2 + m_2 r_2^2 + \ldots m_n r_n^2$$

$$\boxed{I_T = \sum_{i=1}^{n} m_i r_i^2}$$

Angular Momentum

Definition Angular momentum is the cross product of moment arm r & linear momentum P

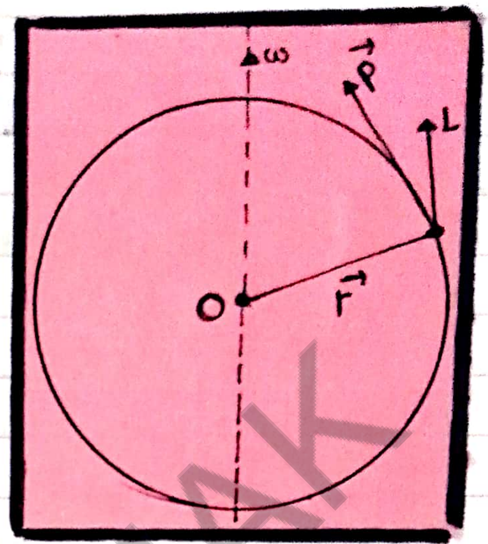
Unit Kgm^2/s , Js

represented by L

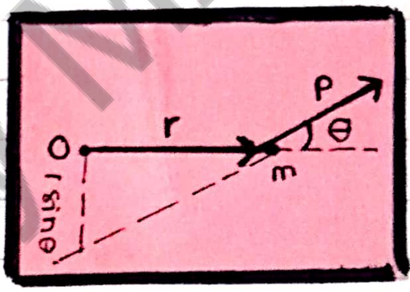
Nature vector quantity.

Mathematical Explanation.

consider a particle of mass m moving in a circle of radius r with velocity and momentum P relative to the origin O



$$\vec{L} = \vec{r} \times \vec{P}$$
$$\vec{L} = rp \sin 90^\circ$$
$$\vec{L} = rp$$
$$\vec{L} = rmv \qquad (\because P = mv)$$

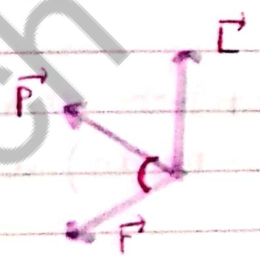


In Angular Form

$$L = rmv$$
$$L = rm(r\omega) \qquad (v = r\omega)$$
$$L = mr^2\omega$$
$$(\because I = mr^2)$$

$$\boxed{L = I\omega}$$

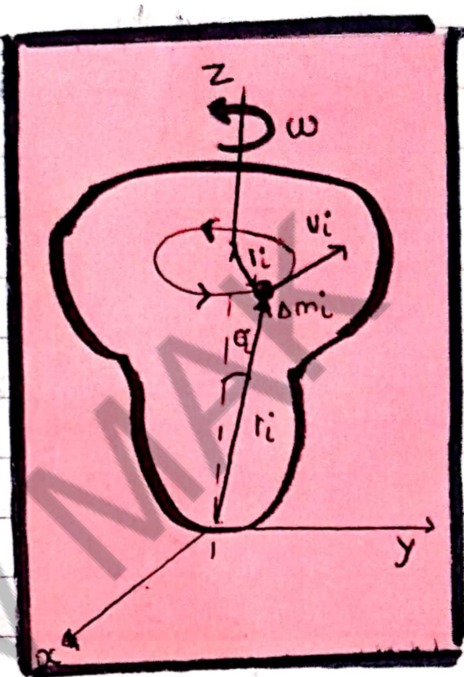
$$I \propto \frac{1}{\omega}$$



→ $\vec{L}$ is perpendicular to r and p.

ANGULAR MOMENTUM FOR SYMMYTRICAL RIGID BODY

Consider a symmetrical rigid body which consists of masses $m_1, m_2, \ldots m_n$. Their respective distances are $r_1, r_2 \ldots r_n$



Suppose the rotation is taken anticlockwise.

$L_1 = m_1 r_1^2 \omega_1$

$L_2 = m_2 r_2^2 \omega_2$

$L_3 = m_3 r_3^2 \omega_3$

$L_n = m_n r_n^2 \omega_n$

Condition

Each and every particle of the body is moving with uniform angular speed

$\omega_1 = \omega_2 = \omega_3 = \cdots \omega_n = \omega$

$L_T = L_1 + L_2 + L_3 + \cdots L_n$

$L_T = m_1 r_1^2 \omega + m_2 r_2 \omega + m_3 r_3 \omega + \cdots m_n r_n \omega$

$L_T = (m_1 r_1^2 + m_2 r_2 + m_3 r_3 + \cdots m_n r_n)\,\omega$

$$L = \sum_{i=1}^{n} (m_i r_i^2)\,\omega$$

$L = I\omega$

CONSERVATION OF ANGULAR MOMENTUM

Definition Total angular momentum of system is constant if the resultant torque acting on the body is zero. if the system is isolated

$$L_T = \text{constant}$$

Mathematically

According to difinition:

"Rate of change of angular momentum is equal to torque"

$$\frac{\Delta L}{\Delta t} = \frac{\Delta rP}{\Delta t}$$

$$\frac{\Delta L}{\Delta t} = r\frac{\Delta P}{\Delta t}$$

$$\frac{\Delta L}{\Delta t} = r(F)$$

$$\boxed{\frac{\Delta L}{\Delta t} = \vec{\tau}}$$

If $\vec{\tau} = 0$

$$\frac{\Delta L}{\Delta t} = 0$$

$$\Delta L = 0$$

$$L_f - L_i = 0$$

$$\boxed{L_f = L_i}$$

APPLICATIONS

1. DIVER'S MOTION

Before diving

When a diver come on the diving board, his arms and legs are extended. Moment of inertia I_1 will be more and angular velocity will be less ω_1

$$I_1 > \omega_1$$

After diving

When he start diving he pulls his arms and legs. His moment of inertia I_2 decreases and angular speed increases.

$$I_2 < \omega_2$$

2. MOTION OF GYMNAST

Gymnast starts dismount at full extension. Before tucking his knees, his angular velocity is less and moment of inertia more. As he tucks his knees, his moment of inertia decreases and angular speed increases. In this way he keeps his angular momentum constant

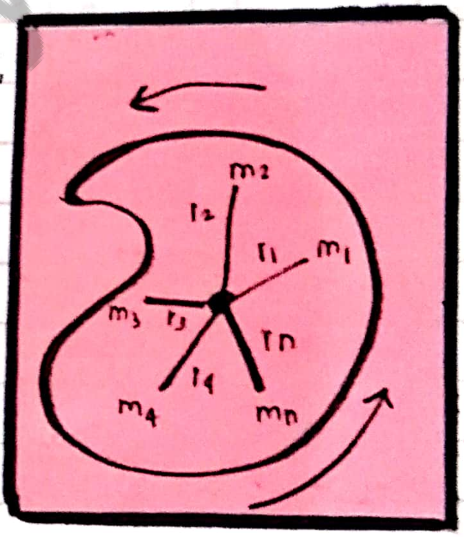
$$I\omega = \text{constant}.$$

3. TURN TABLE

A man standing on turn table and holding heavy weights. Initially, his body is stretched due to which his moment of inertia is more and angular speed less. When his arms are close to chest, his angular speed increases and moment of inertia decrease. In this way, angular momentum remains constant.

K.E OF ROTATION

Consider an irregular shaped object having masses $m_1, m_2 \cdots m_n$. Their respective distances from axis of rotation are $r_1, r_2 \ldots r_n$.



$$(K.E)_1 = \frac{1}{2} m_1 r_1 \omega_1^2$$
$$(K.E)_2 = \frac{1}{2} m_2 r_2 \omega_2^2$$
$$(K.E)_3 = \frac{1}{2} m_3 r_3 \omega_3^2$$
$$(K.E)_n = \frac{1}{2} m_n r_n \omega_n^2$$

$$(K.E)_T = \frac{1}{2} m_1 r_1 \omega_1^2 + \frac{1}{2} m_2 r_2 \omega_2^2 + \cdots \frac{1}{2} m_n r_n \omega_n^2$$

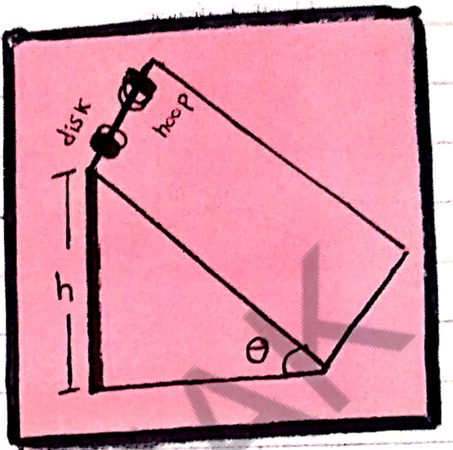
$$\omega_1 = \omega_2 = \cdots \omega_n = \omega$$

$$(K.E)_{rot} = \frac{1}{2} m_1 r_1 \omega + \frac{1}{2} m_2 r_2 \omega + \cdots \frac{1}{2} m_n r_n \omega$$
$$(K.E)_{rot} = \frac{1}{2} (m_1 r_1 + m_2 r_2 + \cdots m_n r_n) \omega$$

$$(K.E)_{rot} = \frac{1}{2} \left(\sum_{i=1}^{n} m_i r_i^2 \right) \Rightarrow \boxed{(K.E)_{rot} = \frac{1}{2} I\omega}$$

Rolling Of Disk And Hoop Down Inclined The Plane

consider a disk and hoop which are placed on inclined plane at height h. When we allow them to move, they will have translational K·E as well as rotational K·E at a time.



For Disk

$(K \cdot E)_T = (K \cdot E)_T + (K \cdot E)_{rot}$

$= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$ — (i)

$I_{disk} = \frac{1}{2} m r^2$

$= \frac{1}{2} I \omega^2$

$= \frac{1}{2} \left[\frac{1}{2} m r^2\right] \omega^2$

$= \frac{1}{4} m r^2 \omega^2$

$= \frac{1}{4} m v^2 \quad (v = r\omega)$

$(K \cdot E_{rot})_{disk} = \frac{1}{4} m v^2$ — (ii)

Putting eq (ii) in eq (i)

$(K \cdot E)_T = \frac{1}{2} m v^2 + \frac{1}{4} m v^2$

$= m v^2 \left[\frac{1}{2} + \frac{1}{4}\right]$

$= \frac{3}{4} m v^2$

According to law of conservation of Energy

Loss in P·E = gain in K·E

$mgh = \frac{3}{4} m v^2$

$\boxed{v = \sqrt{\frac{4gh}{3}}}$

For Hoop

$(K \cdot E)_T = (K \cdot E)_t + (K \cdot E)_{rot}$

$= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$ — (i)

$I_{hoop} = m r^2$

$= \frac{1}{2} I \omega^2$

$= \frac{1}{2} m r^2 \omega^2$

$= \frac{1}{2} m v^2 \quad (v = r\omega)$

$(K \cdot E)_{hoop} = \frac{1}{2} m v^2$ — (ii)

Putting eq(ii) in eq(i)

$(K \cdot E)_T = \frac{1}{2} m v^2 + \frac{1}{2} m v^2$

$(K \cdot E)_T = m v^2$

According to law of conservation of energy

loss in P·E = gain in K·E

$mgh = m v^2$

$\boxed{v = gh}$

Conclusion

$$V_{disk} \qquad V_{hoop}$$

$$\sqrt{\frac{4}{3} gh} > \sqrt{gh}$$

$$\frac{4}{3}\sqrt{gh} > \sqrt{gh}$$

1. Solid disk will move faster than hoop and will reach bottom first
2. Speed is independent of mass in this case.

Real And Apparent Weight

Case-1 lift is at rest	Case-2 lift with upward 'a'	Case-3 lift with downward 'a'	Case-4 lift falling freely
$F_{net} = F_1 + F_2$ $ma = T + (-W)$ $(a = 0)$ $m(0) = T - W$ $0 = T - W$ $W = T$	$F_{net} = F_1 + F_2$ $F_{net} = T - W$ $ma = T - W$ $ma + W = T$ $T = ma + W$	$F_{net} = F_1 + F_2$ $ma = W - T$ $T = W - ma$	$F_{net} = F_1 + F_2$ $ma = W + T$ $mg = mg + T$ $mg - mg = T$ $0 = T$ $T = 0$
$T = W$	$T > W$	$T < W$	$T = 0$
Real and apparent weight are equal.	Apparent weight is greater than actual weight by factor 'ma'	Apparent weight is less than Actual weight by factor 'ma'	Apparent weight is equal to zero

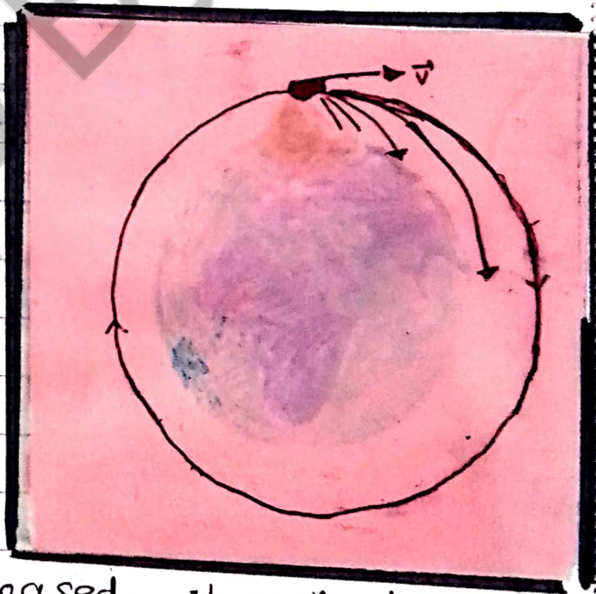
CS CamScanner

Concept Of Weightlessness

- Weightlessness is a state of free fall.
- It exist when all contact forces are removed
- If no external force is acting which is holding us, object will fall with gravitational acceleration
- In isolated system, object is only experiencing gravitational acceleration and will fall freely.

Free Fall In Spaceship

Consider a cannon ball shot parallel to the horizontal surface of the earth from the top of the mountain.



- Since the speed of cannon ball was too small, it will eventually fall to earth.
- When the speed is increased, it will follow another path and curvature will be less.

$speed \propto \frac{1}{curvature}$

- When curvature of path and curvature of earth become equal, it will overshoot circular path and circle round the earth.

When a satellite is moving in a circle of radius R from centre of earth of mass M_e, It has centripetal acceleration

$$a_c = g = \frac{v^2}{R}$$

$$g = \frac{v^2}{R}$$

$$v^2 = gR$$

$$v = \sqrt{gR}$$

$$v = \sqrt{(9.8)(6.4 \times 10^6)}$$

$$v = 7.9 \times 10^3 \text{ m/s}$$

$$v = 7.9 \text{ km/s}$$

Critical Velocity

The minimum velocity required to put a satellite into the orbit which is 7.9 km/s.

CONCLUSION

The spaceship is falling toward the centre of earth all the time but the curvature of the earth prevents the spaceship from hitting the ground.

ARTIFICIAL GRAVITY

- Artificial gravity is produced in the satellite by rotating them about their own axis.

- Artificial gravity is provided by centrifugal force.

Mathematically.

$$a_c = \frac{v^2}{r}$$

$$g = \frac{(r\omega)^2}{r} \quad (v = r\omega) \quad (a = g)$$

$$g = \frac{r^2\omega^2}{r}$$

$$g = r\omega^2$$

$$g = r\left(\frac{2\pi}{T}\right)^2 \quad (\because \omega = 2\pi/T)$$

$$g = r\,\frac{4\pi^2}{T^2}$$

$$g = r\,4\pi^2\left(\frac{1}{T^2}\right)$$

$$g = 4\pi^2 r\, f^2$$

$$\frac{g}{4\pi^2 r} = f^2$$

$$\sqrt{\frac{g}{4\pi^2 r}} = \sqrt{f^2}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{r}}}$$

$$S = vt$$

$$S = 2\pi r$$

$$vt = 2\pi r$$

$$T = \frac{2\pi r}{v}$$

$$(v = r\omega)$$

$$T = \frac{2\pi r}{r\omega}$$

$$T = \frac{2\pi}{\omega}$$

ARTIFIAL SATELLITES

→ objects revolving around the earth.

TYPES

1. Natural Satellites e.g moon.
2. Artificial satellites

Navigation Satellite

- GPS is made up of 24 satellites
- It orbit at an altitude of 20000 km above the surface of earth
- The difference in time for signals recieved from four satellites is used to calculate the exact location of GPS.

Communication Satellite

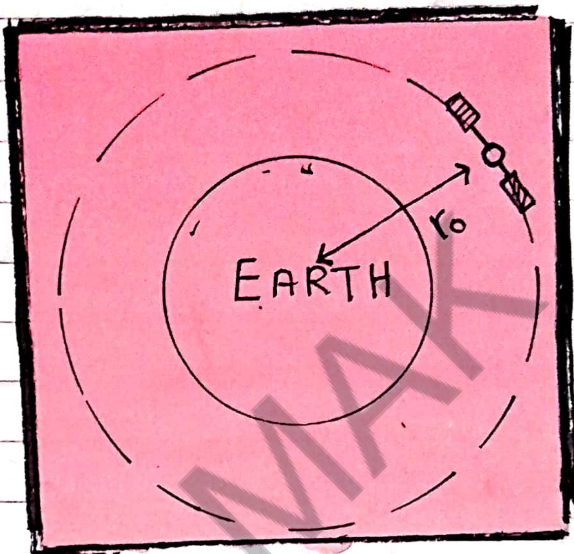
- Used for TV, phone, etc.
- often used in geostationary orbit.
- At the high orbital altitude of 35,800 km, geostationary satellite orbits the earth in the same amount of time it takes the earth to revolve once.
- The area to which it can transmit is called "satellit's foot print"

Weather Satellite

- used to image clouds, measure temperature & rainfall.
- used to help with more accurate weather forecasting.

ORBITAL VELOCITY

Consider a satellite of mass m_s in circular orbit about earth at distance r_o from centre of earth



M_e = mass of earth

m_s = mass of satellite

r_o = radius.

Comparing Newton's law of gravitation and centrifugal force

$$\frac{m_s V^2}{r_o} = \frac{G M_e m_s}{r_o^2}$$

$$V^2 = \frac{G M_e}{r_o}$$

$$\sqrt{V^2} = \sqrt{\frac{G M_e}{r_o}}$$

$$V = \sqrt{\frac{G M_e}{r_o}}$$

$$V = \frac{\text{constant}}{\sqrt{r}}$$

$$V \propto \frac{1}{\sqrt{r}}$$

CONCLUSION

1. A satellite in orbit moves faster when it is close to planet and slower when it is farther away.
2. Orbital velocity depends on mass of the planet and not mass of satellite.

GEO-STATIONARY ORBITS

Definition

The orbit in which the time period of revolution of satellite is exactly equal to the time period of rotation of earth about its axis is called geo-stationary orbit.

Geo-stationary Satellite

- The satellite which is at rest w.r.t earth. Its relative motion w.r.t earth is zero.

Mathematical Derivation

$$V_o = \sqrt{\frac{GM}{r}} \quad \text{—(i)}$$

As we know,

$$V = \frac{s}{t} = \frac{2\pi r}{T} \quad \text{—(ii)}$$

- Comparing eq (i) and (ii)

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

Taking square

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$\frac{GMT^2}{4\pi^2} = r^3$$

Taking cube Root

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{3}$$

$$\boxed{\begin{array}{l} r = 4.23 \times 10^4 \text{ km} \\ \text{from centre of earth} \end{array}}$$

$$\boxed{\begin{array}{l} \text{This distance comes} \\ \text{out be 36000 km} \\ \text{above equator} \end{array}}$$