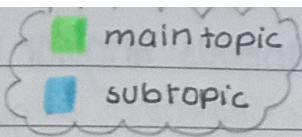


# ■ ANGULAR MOTION

The motion imparted by a body when it spins or rotates about a fixed point is termed as angular motion.



## ■ ANGULAR DISPLACEMENT:-

Definition: When a body rotates about a fixed axis, the angular displacement is the angle swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly.

→ Mathematical form:

$$\theta \text{ (in radian)} = \frac{\text{Arc length}}{\text{radius}}$$

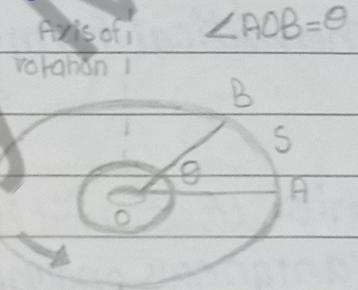
$$= \frac{\text{Arc } AB}{r} = \frac{s}{r}$$

→ SI unit:

SI unit of angular displacement is radian.

$$1 \text{ rad} = 57.3^\circ$$

→ Diagram:



counter-clockwise rotation of the disc

## ■ ANGULAR VELOCITY:-

Definition: The rate of change of angular displacement of a body is called angular velocity.

→ Mathematical form:

$$\langle \vec{\omega} \rangle = \frac{\Delta \vec{\theta}}{\Delta t}$$

→ SI unit:

Its SI unit is  $\text{rad s}^{-1}$

→ Other units:

- degrees $^{\circ}$
- revs $^{-1}$
- revmin $^{-1}$

## → Instantaneous angular velocity:

The instantaneous angular velocity ' $\vec{\omega}$ ' is the limit of the ratio  $\frac{\Delta\theta}{\Delta t}$  as ' $\Delta t$ ' approaches to zero

## → Mathematical form:

$$\vec{\omega}_{\text{inst}} = \lim_{\Delta t \rightarrow 0} \left( \frac{\Delta\theta}{\Delta t} \right)$$

## ■ Angular Acceleration:-

Definition: The time rate of change of angular velocity of a body is called angular acceleration.

## → Mathematical form:

$$\langle \vec{\alpha} \rangle = \frac{\Delta \vec{\omega}}{\Delta t} \quad \text{or}$$

$$\vec{\alpha} = \frac{\vec{\omega}_f - \vec{\omega}_i}{\Delta t}$$

→ SI unit: Its SI unit is  $\text{rad s}^{-2}$

## → Instantaneous angular acceleration:

The rate of change of angular velocity at any instant of time will be instantaneous acceleration.

## → Mathematical form:

$$\vec{\alpha}_{\text{inst}} = \lim_{\Delta t \rightarrow 0} \left( \frac{\Delta \vec{\omega}}{\Delta t} \right)$$

## ■ MCQ points:

- If a body moves w/ uniform angular motion its average angular velocity will be equal to its instantaneous angular velocity.
- The direction of angular acceleration is along the axis of rotation.
- Angular speed for annual rotation of Earth in radians per day is  $2\pi \text{ rad/day}$ .
- The dimension of angular velocity is  $[T^{-1}]$ .

## - Relation b/w ANGULAR & LINEAR Quantities

### - B/w ANGULAR AND LINEAR VELOCITIES:

→ Consider a particle that is moving in a circle of radius 'r' with centre at O. Let particle move from point A to B in circle such that (if we take  $\theta$  in radians)  $S = r\theta \dots (1)$

→ Similarly in linear motion, when a body moves w/ uniform 'v' in time 't'  $S = vt \dots (2)$

→ Comparing above equations we get

$$vt = r\theta \quad (v = r\frac{\theta}{t}) \quad (\frac{\theta}{t} = \omega)$$

$$v = r\omega$$

$$\rightarrow v = r\omega$$

$$\rightarrow \text{in vector form } \vec{v} = \vec{r} \vec{\omega}$$

$$\rightarrow \text{in magnitude form } v = r\omega \sin\theta \quad (\text{for circular motion } \theta = 90^\circ \text{ i.e. } r \perp v)$$

$$v = r\omega \sin 90$$

$$v = r\omega$$

Diagram:

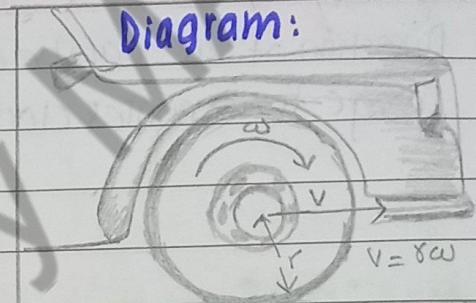
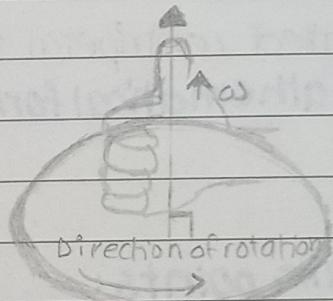


Illustration of angular and linear velocities



### - B/w Accelerations:-

→ We know  $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$   
where,

$$\vec{v}_f = \vec{\omega}_f \times \vec{r} \quad \text{and} \quad v_f = \vec{\omega}_f \cdot \vec{r}$$

→ putting  $\vec{a} = \left( \frac{\vec{\omega}_f \times \vec{r} - \vec{\omega}_i \times \vec{r}}{t} \right)$

$$\vec{a} = \left( \frac{\vec{\omega}_f - \vec{\omega}_i}{t} \right) \times \vec{r}$$

where

$$\frac{\vec{\omega}_f - \vec{\omega}_i}{t} = \vec{\alpha}$$

→ Right hand rule:

grasp me imaginary axis in the air such that your fingers curl in the direction of rotation. Thumb will consequently point in the direction of ' $\vec{\alpha}$ '.

→ putting,  $\vec{a} = \vec{\omega} \times \vec{r}$  or  $a = \omega r \sin\theta \hat{n}$

## equations:-

### Equations for linear Motion

$$v_f = v_i + at$$

$$s = v_i t + \frac{1}{2} a t^2$$

$$2as = v_f^2 - v_i^2$$

### Equations for angular motion

$$\omega_f = \omega_i + \alpha t$$

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$2\alpha\theta = \omega_f^2 - \omega_i^2$$

## mcq points:-

- Direction of ' $\omega$ ' can be determined by the <sup>rotation</sup> pointing of thumb (i.e., out of the page for counter clockwise and down on the page for clockwise rotation.)
- 

## -(Centrifugal Force & Acceleration)

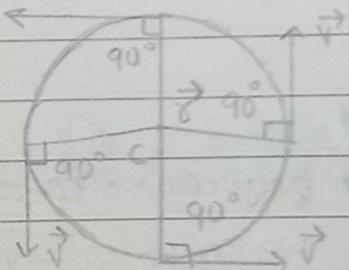
### Centripetal Force

Definition: A force which compels a body to move in a circular path is called centripetal force.

### Mathematical form:

$$F_c = \frac{mv^2}{r}$$

### Diagrams:



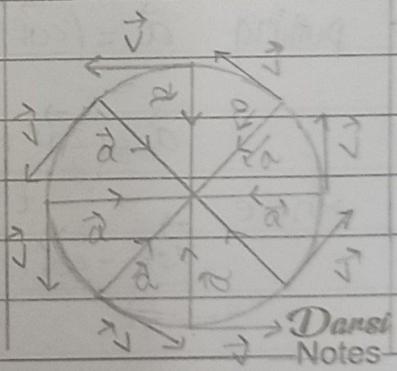
At each instant the direction of velocity changes

### Main points:

- The force always acts inwards as the velocity of the object is directed tangent to the circle.
- The centripetal force is always directed perpendicular to the direction that the object is being displaced.

### Centripetal Acceleration:

Definition: The change in velocity of a body produces acceleration directing towards the centre of the circle, such acceleration is known as centripetal acceleration.



## → Mathematical form:

$$a_c = \frac{v^2}{r}$$

## Mathematical Proof:

Suppose a body of mass 'm' moving in a circle of radius 'r' with uniform speed 'v'. as shown.

- At point A at time  $t_1$ , velocity is  $\vec{v}_1$
- At point B at  $t_2$ , velocity is  $\vec{v}_2$ .
- Let's draw a triangle PQR, such that PQ is equal to and parallel to  $\vec{v}_1$ . PR is equal and parallel to  $\vec{v}_2$
- $\Delta \vec{v}$  is change in velocity corresponding to  $\Delta t$ .

→ When  $\Delta t$  is small, the change  $\Delta v$  is also small. Arc AB approximately becomes equal to chord AB.  
 $(\overline{AB} = \bar{AB})$ .

→ geometrically,  $\frac{\text{Arc AB}}{AC} = \frac{QR}{PQ}$

(if  $\Delta t \rightarrow 0$ , Arc AB = side length AB = S)

$$\text{or, } \frac{S}{\delta} = \frac{\Delta V}{V}$$

→ We know that  $S = v \Delta t$  then,  $\frac{v \Delta t}{r} = \frac{\Delta V}{V}$

$$\Rightarrow \frac{\Delta V}{\Delta t} = \frac{v^2}{r}$$

→ since  $\lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t} = a_{\text{inst}}$

$$a_{\text{inst}} = v^2$$

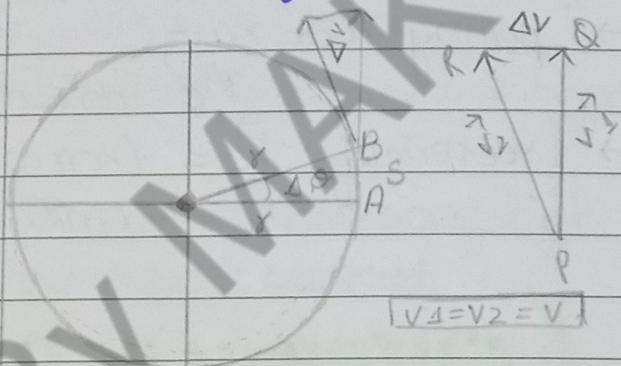
→ thus  $a_c = \frac{v^2}{r}$ . Vectorsially,  $\vec{a}_c = \left(\frac{v^2}{r}\right) \hat{r}$

$$\hat{r} = \frac{\vec{r}}{r}$$

$$\vec{a}_c = -\left(\frac{v^2}{r}\right) \frac{\vec{r}}{r}$$

→ negative sign indicates direction.  $\vec{a}_c = -\left(\frac{v^2}{r}\right) \vec{r}$

## Diagram:



mass 'm' moving in circular path

→ putting  $v = \omega r$ ,  $\vec{a}_c = -\omega^2 \vec{r}$

→ Using Newton's second law of motion,

$$F_c = mac$$

→ putting  $F_c = \frac{mv^2}{r}$

→ in vector form  $\vec{F}_c = \left(\frac{mv^2}{r}\right) \hat{r} \Rightarrow \vec{F}_c = \left(\frac{mr\omega^2}{r}\right) \vec{r}$

→ putting  $a_c = r\omega^2$ ,  
 $F_c = mr\omega^2$ .

→ vectorially,  $\vec{F}_c = -(mr\omega^2) \hat{r}$

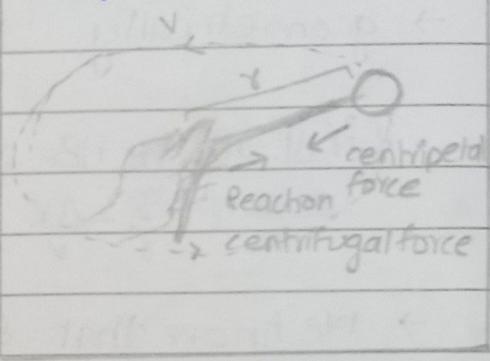
$$\vec{F}_c = -(mr\omega^2) \frac{\vec{r}}{r} \Rightarrow \vec{F}_c = - (mr\omega^2) \vec{r}$$

## • ENTRIFUGAL FORCE

centrifugal force:

Definition: According to Newton's 3rd law of motion, reaction of centripetal force is centrifugal force.

Diagram:



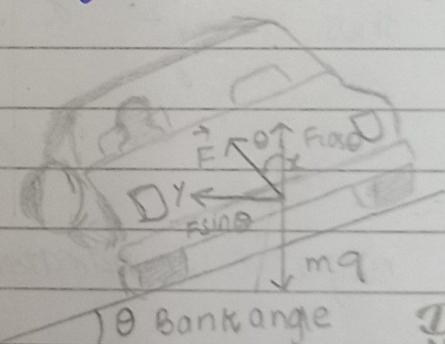
### Main points

→ As centripetal and centrifugal force cancel each other out, the body is enabled to move in a circle. If either of them increases due to an external factor, the body will either fall in the centre of travel along a tangent to the circle.

### Mathematical form:

$$\text{centrifugal force} = \frac{mv^2}{r}$$

Diagram:



## • BANKING OF ROAD

Definition: If outer edge of the road is higher than inner edge of road that turns then it's called banking of road.

## → main points:

- The direction of car is considered to be a circular one when it takes a turn.
- To provide the necessary centripetal force, air's force exerting on the body and friction force between the road and the tyres is required.
- If the friction of air and road are not sufficient to provide the necessary centripetal force, due to exceed in centrifugal force the car will skid on the road.
- To prevent the car from depending upon unreliable sources of friction such as fair and road friction, the road is banked.

## → Mathematical formulae;

### • necessary centripetal force:

resolving  $F$  vertically and horizontally,  $F\cos\theta$  or  $F_x$  balances the weight  $mg$  of vehicle and  $F_y = F\sin\theta$  provides the necessary centripetal force.

$$F\sin\theta = \frac{mv^2}{r}$$

### • bank angle:

We know that,  $F\cos\theta = mg$  and  $F\sin\theta = \frac{mv^2}{r}$   
dividing both,

$$\frac{F\sin\theta}{F\cos\theta} = \frac{mv^2/r}{mg}$$

$$\tan\theta = \frac{v^2}{gr}$$

$$\tan\theta = \frac{v^2}{gr}$$

(Bank angle is only correct for one speed)

$$v^2 = rg\tan\theta$$

$$v = \sqrt{rg\tan\theta}$$

## ■ mca points:-

- Centripetal acceleration is also called radial acceleration
- Centripetal acceleration is not directed along axis of rotation
- If linear velocity and radius are both made half the  $F_c$  becomes half ( $\frac{F_c}{2}$ ).

# ■ TORQUE AND MOMENT OF INERTIA

- Moment of inertia;

- Definition: Moment of inertia is the property due to which body opposes any change in its state of rest or of angular motion.

→ Mathematical form:  $I = m\gamma^2$

→ SI unit:  $\text{kgm}^2$

→ Main points :

→ angular acceleration  $\propto \frac{1}{\gamma}$

→ Force  $\propto$  angular acceleration.

→ Inertia  $\propto 1/\text{angular acceleration}$

→ Dimensions of inertia (moment)  $[\text{ML}^2]$

→ Inertia of a rigid body :

We first divide the rigid body in different mass segments and denote them  $m_1, m_2, \dots, m_n$ .

each mass segment revolves or rotates around the same rigid centre. The circular paths of all the masses hence form concentric circles (having same centre)

all circles having different radii  $r_1, r_2, \dots, r_n$ .

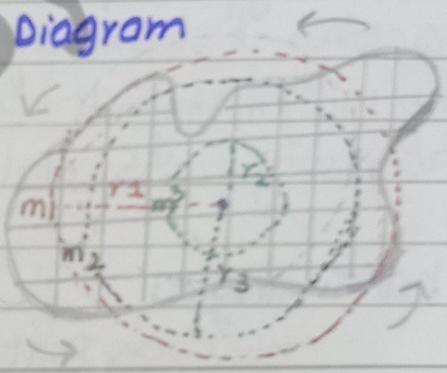
→ Mathematically :

$$I = m_1r_1^2 + m_2r_2^2 + \dots + m_nr_n^2$$

$$\text{or } I = \sum_{i=1}^n m_i r_i^2$$

Shape	moment of inertia.
trapezoid	$I = MR^2$
cylinder	$I = \frac{1}{2}MR^2$
thin walled ring or	$I = \frac{1}{2}MR^2$
Disc or solid cylinder	$I = \frac{1}{2}MR^2$
thick walled disc	$I = \frac{1}{2}M(R_2^2 + R_1^2)$
solid sphere	$I = \frac{2}{5}MR^2$
solid rod	$I = \frac{1}{12}ML^2$
rectangular plate	$I = \frac{1}{12}M(a^2 + b^2)$

Diagram



each particle 'm' is rotating around the same centre in the rigid body with uniform 'ω'.

- Torque of rotating body;

- Definition: The product of moment of inertia 'I' and angular angular 'α' of body gives the magnitude of the torque acting on it.

→ Mathematically: We know that  $a=r\alpha$  and  $F=ma$  thus,  $F=mr\alpha$

$$\text{Torque} = T = Fr$$

$$\text{Torque} = T = mr^2\alpha \quad (\text{putting } I = mr^2) \quad T = I\alpha$$

# Angular Momentum & Torque:-

**Definition:** Angular momentum ' $\vec{L}$ ' of a body is the product of its radius and linear momentum ' $\vec{p}$ '.

→ **Mathematically:** we know  $\vec{L} = \vec{r} \times \vec{p}$

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

magnitude of ' $L$ '

$$L = r p \sin \theta$$

**Important fact:** The factor of radius ' $r$ ' changes a linear quantity into angular.

→ **Main points:**

$\vec{L}$  is perpendicular to the  $xy$  plane in which radius and linear momentum reside.

$r$  is constant hence any change brought in  $\vec{L}$  is due to change in  $p = mv$ . (Linear momentum).

→ **angular momentum of a rigid body:**

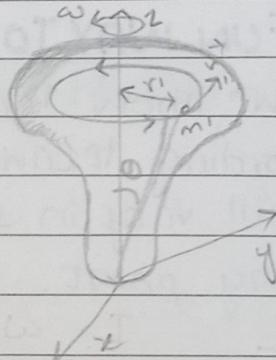
The rigid body is divided into many mass segments. each mass segment has the same angular velocity ' $\omega$ '.

→ **Mathematically:**  $\vec{L} = \sum_i m_i R_i v_i$

$$(p = mv), (v = R\omega) \quad = \sum_i m_i R_i^2 \omega$$

$$\vec{L} = \sum_i (m R_i^2) \omega = I \omega$$

**Diagram**



→ **units:**

Its units can be;

$$\text{kg } m^2 \text{ s}^{-1}$$

$$\text{kg } m \times \text{ms}^{-1}$$

$$\text{Nms}$$

dividing and multiplying by s

$$\text{J s (SI)}$$

$$\text{kg } m \times \text{ms}^{-1} \times \frac{s}{s} \Rightarrow \text{kgm}^2 \text{s}^{-2} \text{s}$$

$$\downarrow \text{Nm.s}$$

↳  $\theta$  is the angle b/w the mass segment and axis of rotation.

■ **main points**

→ Value of angular momentum is maximum when  $\theta$  is  $90^\circ$

→ Dimensions of angular momentum are  $[ML^2 T^{-1}]$

## CONSERVATION OF ANGULAR MOMENTUM

Statement The total angular momentum of a system is constant in both magnitude and direction if the resultant external torque acting on the system is zero, that's the system is isolated.

→ **Mathematically:**

change in angular momentum is  $\Delta L = r \Delta p$ , dividing b.s by  $\Delta t$

$$\frac{\Delta L}{\Delta t} = r \frac{\Delta p}{\Delta t} = r F \quad \left( \frac{\Delta p}{\Delta t} = F \right)$$

Thus the rate of change of angular momentum is equal to the torque.

$$\text{in } \frac{\Delta L}{\Delta t} = \vec{\tau}, \quad \frac{\Delta L}{\Delta t} = 0 \Rightarrow L = (\text{constant in time}).$$

→ Thus the absence of any external torque, the angular momentum remains constant. ( $rF=0$ )

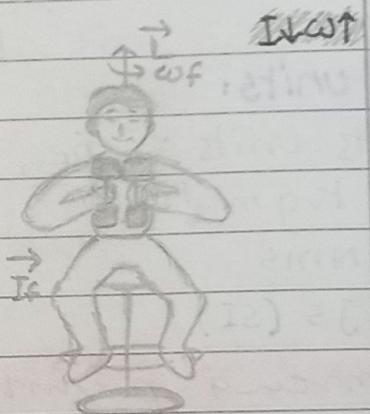
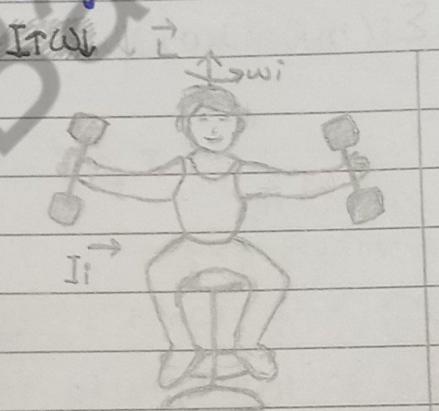
→ **FUN WAY TO LEARN**

We know that  $L = I\omega$

According to Law of conservation of angular momentum, 'I' and ' $\omega$ ' will vary in such a way that value of ' $L$ ' remains same at any point.

Diagram

$L$	$I$	$\omega$
12	6	2
12	12	1
12	1	12
12	2	6
12	3	4
12	4	3



Angular momentum remains constant in both cases

→ **Conclusion:**

As radius decreases, ' $\omega$ ' angular speed increases and viceversa thus ' $I$ ' and ' $\omega$ ' vary in an isolated system in such a way that ' $L$ ' remains same / conserved.

## KINETIC ENERGY OF ROTATION

**Definition** The energy in a body due to its angular motion is called rotational kinetic energy and is given by equation

→ Mathematical form:

$$KE_{(rot)} = \frac{1}{2} I \omega^2 \quad \left[ = \frac{1}{2} m v^2 \Rightarrow \frac{1}{2} m(r\omega)^2 \Rightarrow \frac{1}{2} m^2 \omega^2 \right]$$

→ K.E(rot) of Rigid body:

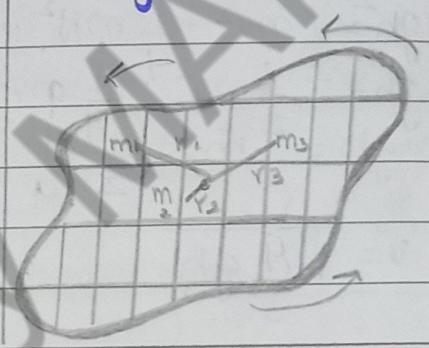
$$K.E.T = K.E_1 (\text{of mass } 1) + K.E_2 + \dots + K.E_n.$$

$$\text{Or} \quad = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dots + \frac{1}{2} m_n v_n^2$$

$$K.E.T = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \dots + \frac{1}{2} m_n r_n^2 \omega^2$$

$$K.E.(rot) = \frac{1}{2} \omega^2 \left( \sum_{i=1}^{i=n} m_i r_i^2 \right)$$

Diagram



## COMPARISON IN LINEAR AND ANGULAR MOTIONS

equations for linear motion

i.  $s = vt$

ii.  $v_f = v_i + at$

iii.  $v_f^2 - v_i^2 = 2as$

iv.  $s = v_i t + \frac{1}{2} a t^2$

v.  $I_{\text{inertia}} = m$

vi. Force =  $ma$

vii.  $\vec{P} = m\vec{v}$

viii.  $K.E(\text{trans}) = \frac{1}{2} m v^2$

equations for angular motion

i.  $\theta = \omega t$

ii.  $\omega_f = \omega_i + \alpha t$

iii.  $\omega_f^2 - \omega_i^2 = 2\alpha\theta$

iv.  $\theta = \omega_i t + \frac{1}{2} \alpha t^2$

v. moment of inertia  $= m r^2 = I$

vi. Torque  $= \tau = I\alpha$

vii.  $\vec{L} = \vec{\sigma} \times \vec{P}$  or  $\vec{L} = I\vec{\omega}$

viii.  $K.E(\text{rot}) = \frac{1}{2} I \omega^2$

# ROLLING OF DISC & HOOP DOWN THE INCLINED PLANE:-

For Disc

$$\rightarrow \text{Loss in P.E} = \text{Gain in K.E}_{\text{trans}} + \text{Gain in K.E}_{\text{rot}}$$

$$I = \frac{1}{2} mR^2$$

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} mR^2 \omega^2 \quad [v = R\omega]$$

$$mgh = \frac{1}{2} \frac{1}{2} mv^2 + \frac{1}{2} mv^2$$

$$mgh = \frac{2}{2} mv^2 \left( \frac{1}{2} + \frac{1}{4} \right)$$

$$v = \sqrt{\frac{4}{3} gh}$$

$$I = mR^2$$

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 \downarrow$$

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} mv^2$$

$$mgh = \frac{2}{2} mv^2 (1 + \frac{1}{4})$$

$$mgh = \frac{2}{2} mv^2$$

$$gh = mv^2/m$$

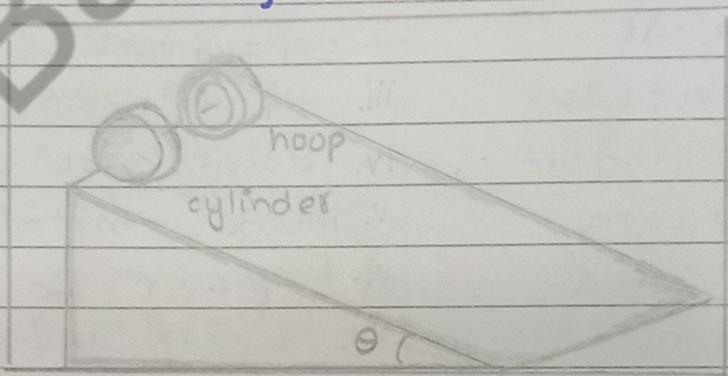
$$gh = v^2/m$$

$$v = \sqrt{gh}$$

$\rightarrow$  conclusion

$\rightarrow$  velocity of disc is 1.15 times the velocity of hoop thus disc will move faster and reach the bottom first.

$\rightarrow$  Diagram



## MCQ points

- The rolling body which has 50% K.E<sub>rot</sub> and 50% K.E<sub>trans</sub> is ring.
- The ratio of a solid sphere's rotational K.E to translational K.E as it rolls on a horizontal plane without slipping is 2:5.

## ▪ REAL AND APPARENT WEIGHT

Intro: A woman weighs a fish with a spring scale attached to the ceiling of a lift.

Real and apparent weight of the fish are explained in accordance with 5 cases.

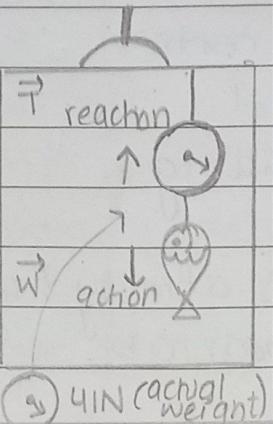
### CASE-1 | lift is at rest

$$\vec{T} = \vec{W}$$

$$\vec{F} = \vec{W} = 41\text{N}$$

apparent and actual weight are same

$$\vec{a}_{lift} = 0$$



### CASE-2 | lift with upward acceleration

### CASE-2 | lift with upward acceleration

→ tension's increasing as

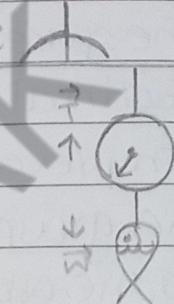
elevator moves up

→  $T$  is increasing because mass is being attracted by Earth more

$$T = W + ma \leftarrow \begin{matrix} \text{acceleration} \\ \text{of elevator} \end{matrix}$$

$$T > W$$

$$\vec{F} = m\vec{a}$$



apparent weight is more than actual weight.

$$\downarrow = 51\text{N}$$

### CASE-3 | lift with downward acceleration

$$\vec{T} = \vec{W} - m\vec{a}$$

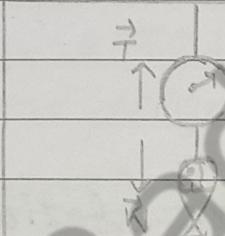
$$\vec{T} < \vec{W}$$

apparent weight is less than actual weight

lift's acceleration

$$\vec{F} = m\vec{a}$$

$$\vec{a} \downarrow$$



$$\uparrow = 32\text{N}$$

### CASE-4 | lift falling freely

$$\vec{T} = \vec{W} - m\vec{a}$$

$$T = mg - mag$$

$$T = 0$$

Apparent weight of freely falling body is 0.

(of earth)

$$\uparrow = 0\text{N}$$

→ Conclusion: When weight is apparent,  $\vec{a}$  is not zero.

## ▪ CONCEPT OF WEIGHTLESSNESS

Definition: Weightlessness in a body is the sensation experienced by the body when no external object is exerting a force by touching it.

## ▪ WEIGHTLESSNESS IN SPACESHIPS

→ Main points

→ Curvature of the Earth drops a vertical height of 5m for every 8000m tangent to the surface.

- If an object is thrown fast enough parallel to Earth that the curvature of its path matches the curvature of Earth it'll orbit the Earth.
- Minimum velocity i.e.,  $8 \text{ km/s}$  is required for an object to orbit the Earth, it's called critical velocity.
- The spaceship is falling towards the centre of Earth at all points but curvature of Earth prevents the space ship from hitting the ground.
- So, the orbiting spaceship/satellite is a freely falling object in space, everything within it will appear to be weightless.
- Derivation of critical velocity:

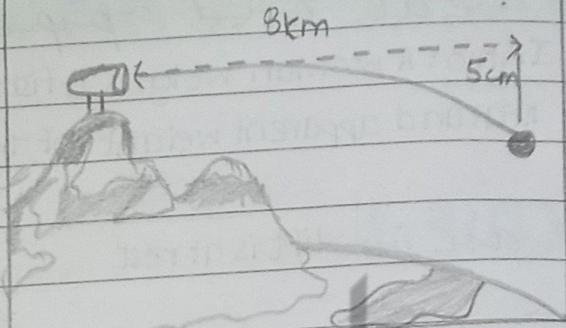
$$a_c = g = \frac{v^2}{R} \Rightarrow v^2 = gR$$

$$v = \sqrt{gR}$$

$$v = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$v = 7.9 \text{ km/s}$$

Diagram



A cannon ball fired with the speed of about  $8 \text{ km/s}$  will orbit the Earth

## ■ Artificial Gravity in Space Stations -

Definition: Artificial gravity is the gravity like effect produced in an orbiting satellite by spinning it by its own axis.

### → Main points:

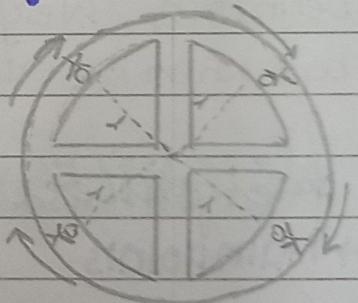
- Satellites are in weightlessness which causes a lot of problems for astronauts. To overcome this difficulty, an artificial gravity is created.
- For this purpose satellites are made to spin about their own axis due to which all objects inside are made to stick to the walls due to centrifugal force. This enables a person to walk just like on Earth.

### → Mathematically: $a_c = \frac{v^2}{r}$

$$g = \frac{r \omega^2}{r}$$

$$g = r \omega^2 \dots (i)$$

Diagram



We know that  $S = 2\pi r$  (one complete revolution)

and  $S = vE$  as well

$$\text{then } vE = 2\pi r \Rightarrow T = \frac{2\pi r}{v} \quad [v = \omega r]$$

$$T = \frac{2\pi r}{\omega r}, \omega = \frac{2\pi}{T}$$

$$\omega = 2\pi \times \frac{1}{T}, \omega = 2\pi f.$$

equation (i)  $\frac{1}{T}$  becomes

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{r}}$$

→ The spaceship rotates with this particular frequency.

## ▪ Artificial satellites:-

→ Main points:-

- A satellite is anything that orbits around another object.
- Moons are natural satellites
- There're currently over 1000 satellites orbiting the Earth.
- The size, altitude and design depend on the purpose of satellite.

## ▪ Types of satellites:-

### Navigation satellites

- The GPS is made up of 24 satellites that orbit the Earth.
- A GPS receiver's location on Earth is interpreted using 4 satellites.

### Communication satellites

- These are used for TV, internet etc transmissions
- They're at high orbital altitude of 35,800 km.
- Such a satellite covers  $120^\circ$  of longitude so the whole Earth can be covered using three of these.

### Weather satellites

- These are used to image clouds, temp, rainfall etc.
- Both geostationary and low orbits are used
- Used to help with more accurate weather forecasts.

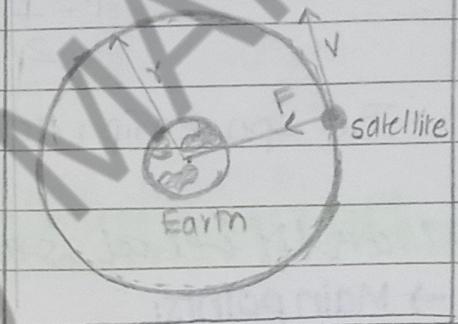
## ■ Orbital Velocity:-

Definition: Orbital velocity is the tangential velocity to put satellite in orbit around the Earth.

→ Mathematically: consider a satellite of mass  $m_s$  moving with orbital velocity  $v$  around the Earth of mass  $M$  and radius  $r$  (orbits radius) then,  $F_c = \frac{m_s v^2}{r}$  ... (1)

This force is provided by  $\text{g}$  by ' $G$ '.

Diagram



therefore,

$$F = \frac{G M m_s}{r^2} \rightarrow (2)$$

$$\frac{G M m_s}{r^2} = \frac{m_s v^2}{r}$$

$$v^2 = \frac{GM}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

[where  $r = R+h$ ]

## → Conclusion:

- Orbital speed of satellite is independent of mass
- If the speed of satellite is less than the orbital speed then it'll not be able to revolve around the Earth.
- $v \propto \frac{1}{\sqrt{r}}$

## ■ Major points:-

- Time period of artificial satellite is given by  $\frac{2\pi r}{v}$
- Relation between escape velocity and orbital velocity is  $v_{esc} = \sqrt{2} v_0$
- Time period of a low flying satellite is 84 minutes.
- Orbital velocity near surface of Earth's given by  $\sqrt{gR}$ .

## Geostationary Orbit

Definition: The orbit in which the time period of revolution of satellite is exactly equal to the time period of Earth is called geo stationary orbit.

Geostationary satellites: The satellite which completes its one revolution around Earth in 24 hours is called geostationary satellite.

→ Expression for orbital radius of geo stationary satellites:

$$\text{we know that, } v_0 = \sqrt{\frac{GM_e}{r_0}}$$

$$v_0 = \frac{s}{t} \rightarrow s = 2\pi r_0 \quad t \text{ or } v_0 = 2\pi r_0$$

hence

$$\left(\frac{2\pi r_0}{T}\right)^2 = \left(\sqrt{\frac{GM_e}{r_0}}\right)^2, \text{ squaring both sides}$$

$$\frac{4\pi^2 r_0^2}{T^2} = \frac{GM_e}{r_0}$$

$$r_0^3 = \frac{GM_e T^2}{4\pi^2}, \text{ taking cubes}$$

$$r_0 = \left[ \frac{GM_e T^2}{4\pi^2} \right]^{\frac{1}{3}}$$

$$r_0 = 4.23 \times 10^4 \text{ km}$$

→ Conclusion :

- This type of satellite is ideal for many communications and weather satellites
- A geostationary orbit has an altitude of 22,240 miles (35,790 km)
- Orbital speed of geostationary satellite is 6,880 mph<sup>2</sup> (11,070 km/h)