Physics

SLO-Based Quick Notes

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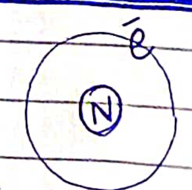
{1st year}

ROTATIONAL & CIRCULAR MOTION

circular motion	rotational motion
definition	
The object in a circular motion only moves in a circle	The object rotates around an axis in rotational motion
axis of rotation	
The axis of rotation remains fixed	The axis of rotation can change
example	
Artificial satellites; ∴ orbit the Earth at a fixed altitude.	The Earth; ∴ rotates on its own axis

angular motion:

- **definition**: The motion of a body about a fixed axis
- **cause**: When forces act on extended objects
- **equal to**: It is equal to the angle passed over at the point/axis by a line drawn to the body
- **examples**: satellite orbiting around the Earth, motion of fans, stone on the end of string.



electron revolving around nucleus.

	definition	formula	SI unit
angular displacement	It is the angle subtended by an arc with centre of circle	$\theta = \frac{S}{r}$	degrees, revolutions or radians
angular velocity	The rate of change of angular displacement.	$\langle\vec{\omega} = \frac{\Delta\vec{\theta}}{\Delta t}$	$rad\ s^{-1}$, $degree\ s^{-1}$, $rev\ s^{-1}$
angular acceleration	The rate of change of angular velocity	$\vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t}$	$rad\ s^{-2}$

Instantaneous angular Velocity	**Instantaneous angular Acceleration**
definition	
Angular velocity of a body at any instant.	Angular acceleration of a body at any instant
formula	
$\vec{\omega}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t}$	$\vec{\alpha}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t}$

Angular velocity (Average)	**Angular acceleration (Average)**
definition	
Total change in angular displacement divided by total time.	Total change in angular velocity divided by total time.
formula	
$\langle\vec{\omega}\rangle = \frac{\Delta\vec{\theta}}{\Delta t}$	$\langle\vec{a}\rangle = \frac{\Delta\vec{\omega}}{\Delta t}$

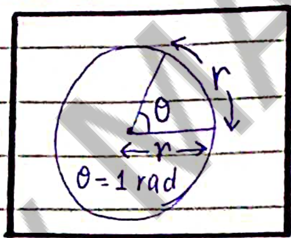
angular quantities Vs Linear quantities

In order to understand the relationship b/w linear and angular quantities, first understand the concept of **radian**.

QUICK RECAP:

A radian is the angle that subtends an arc length equal to the radius of the circle

$\therefore$ 2π radians = 360°



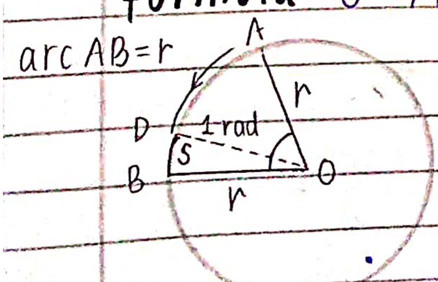
Linear Quantity		Angular Quantity	Relationship
Displacement	s	Angular displacement θ	$s = r\theta$
Speed	v	Angular speed ω	$v = r\omega$
Acceleration	a	Angular acceleration α	$\alpha = r\alpha$
Mass	m	Moment of inertia I	

The correspondence b/w linear and angular quantities gives us corresponding angular kinematic equations

- $V_f = V_i + at$ | $\omega_f = \omega_i + \alpha t$
- $S = V_i t + \frac{1}{2} a t^2$ | $\theta = \omega_i t + \frac{1}{2} \alpha t^2$

relation b/w angular and Linear displacement

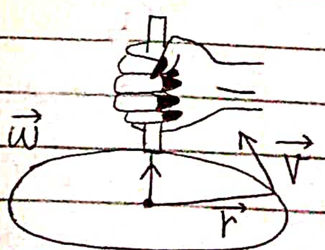
formula: $S = r\theta$



direction:
Linear: up/down/side to side
angular: right hand rule

relation b/w angular and Linear velocity

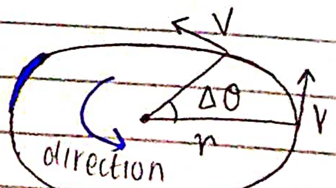
formula: $\vec{v} = \vec{\omega} \times \vec{r}$



direction:
Linear: tangent to path
angular: clockwise/anti-clockwise

relation b/w angular and Linear acceleration

formula: $\vec{a} = \vec{\alpha} \times \vec{r}$



direction:
Linear: tangent to path
angular: right hand rule.

→ centripetal force & centripetal acceleration

→ Centripetal force	Centripetal acceleration

definition

- A force which compels a body to move in a circular path is called centripetal force.
- The change in velocity of body produces acceleration directing towards centre of circle

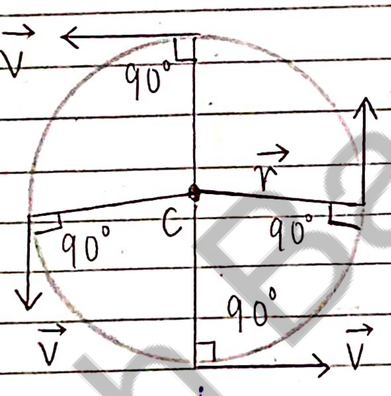
formula

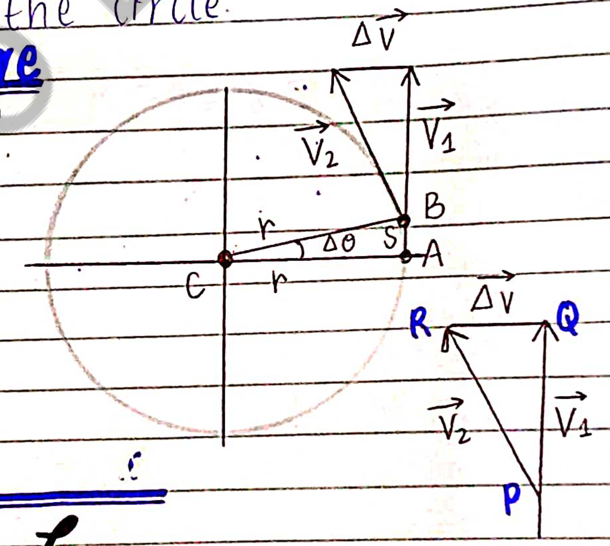
- $F_c = \dfrac{mv^2}{r}$
- $a_c = \dfrac{v^2}{r}$

direction

- It is always directed ($\perp$) perpendicular to direction of object's displacement
- Its direction is always towards the centre of the circle.

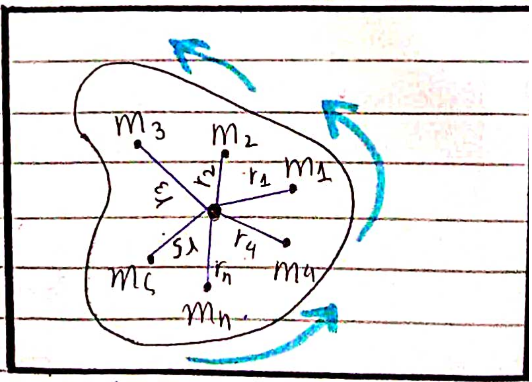
figure

- 

- 

→ Rotational K.E

- **definition:** The energy possessed by a body due to its rotation about an axis is called rotational K.E.



- **formula:**

$$\boxed{K.E_{rot} = \frac{1}{2} I\omega^2}$$

SI unit: Joules (J)

dimentions: $[ML^2T^{-2}]$

Quantity: scalar quantity.

Increases: Increases with increase in rotating speed

~~Energ~~ **e.g:** ball rolling down a ramp.

Centrifugal Force
aka reaction force

- **definition:** It is the force which tries to move the body away from the centre of the circle.
- **Also known as:** It is reaction of centripetal force
- **Formula:** centrifugal force $\boxed{= \frac{mv^2}{r}}$
- **Examples:**
  - Whirling a ball at the end of the string.

→ Banking of roads:

- **definition:**

→ If outer edge of the road is higher than inner edge of the road at turns then it is called banking of road

- **reason:**

→ The banking of a road is done to provide centripetal force

- **how the force arises:**

→ This (centripetal force) ~~occu~~ arises from the interaction of the car with the ground and air

- **advantage:**

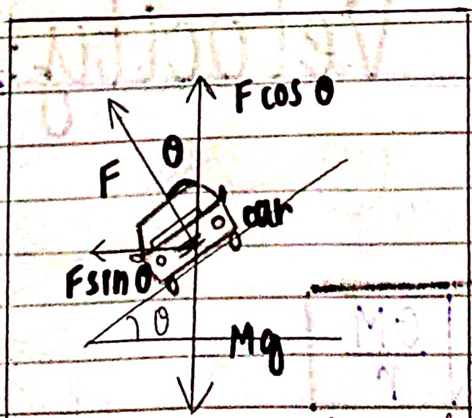
→ Banking helps to avoid skidding.

→ Helps to prevent overturning or toppling.

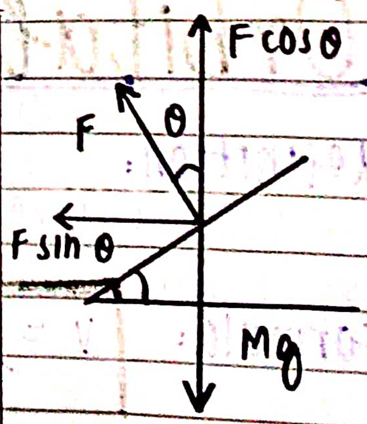
- **formula:**

$v = \sqrt{rg \tan\theta}$

- **figure:**



Banking of road



Torque & Moment of inertia

→ Moment of inertia:

→ **definition:**
The property due to which a body opposes any change in its state of rest or of angular motion.

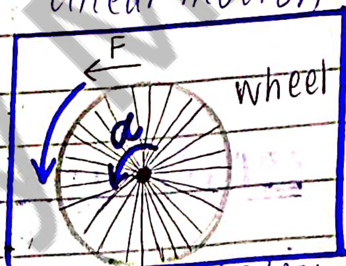
→ **denoted by:** I **formula:** $I = mr^2$

→ **SI unit:** $kg\,m^2$

→ **Dimension:** $[ML^2]$

→ **significance:**
→ It plays the same role in angular motion as inertia in linear motion

→ **example:**
→ Applying force on a bike wheel.



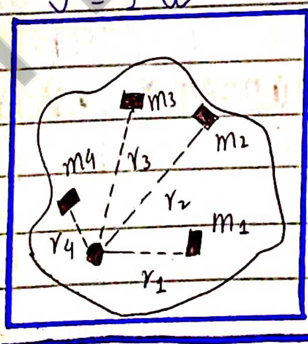
→ **dependence:**
- mass 'm' of body
- square of perpendicular distance 'r'
- distribution of mass from position of axis of rotation

→ Torque:

→ **definition:**
→ The product of moment of inertia 'I' and angular acceleration 'α' of the body gives the magnitude of the torque acting on it.

→ **formula:** $\tau = I\alpha$

→ **figure:**

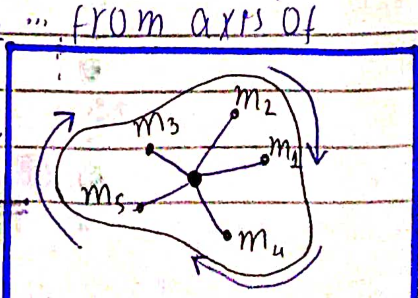


MOMENT OF INERTIA OF A RIGID BODY:

→ **consideration:**
Consider a rigid body made up of 'n' small pieces of masses $m_1, m_2, \ldots$ at distance $r_1, r_2 \ldots$ from axis of rotation of O.

→ **Formula:** $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \ldots$

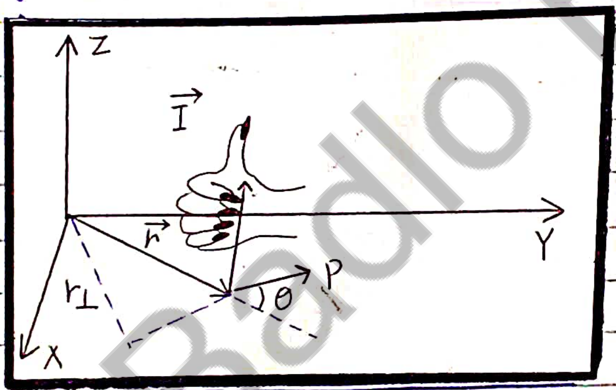
$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$$



Angular Momentum

- **definition:**
Quantity of angular motion in a body is called angular momentum

- **cross-product:** $\boxed{\vec{L} = \vec{r} \times \vec{P}}$
- **magnitude:** $L = rp\sin\theta$
$L = rp\sin(90°)$
$L = rP$ as $P = mv$ so, $\boxed{L = rmv}$
but $v = r\omega$, so $L = m(r\omega)r = mr^2\omega$
$\boxed{L = I\omega}$
- **Quantity:** Vector
- **SI unit:** kgm^2s^{-1} or Js
- **Dimensions:** $[ML^2T^{-1}]$
- **Figure:**



- $\vec{r}$ and $\vec{p}$ = x-y plane
- $\vec{I} \perp$ xy plane

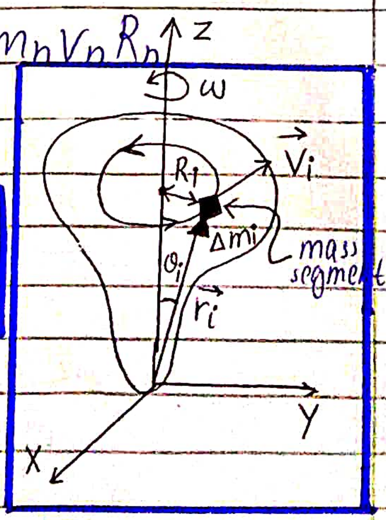
Angular momentum of a rigid body:

- **Introduction:**
The sum of angular momentum of all particles gives the total momentum of the rigid body

As, $L = L_1 + L_2 + \dots + L_n$
since, $L = m_1V_1R_1 + m_2V_2R_2 + \dots + m_nV_nR_n$
so, $L = \sum_{i=1}^{n} m_i v_i R_i$

putting $v_i = R_i\omega$, $\boxed{L = \sum_{i=1}^{n} m_i R_i^2 \omega}$

As, $\sum_{i=1}^{n} m_i R_i^2 = I$; So $\boxed{L = I\omega}$



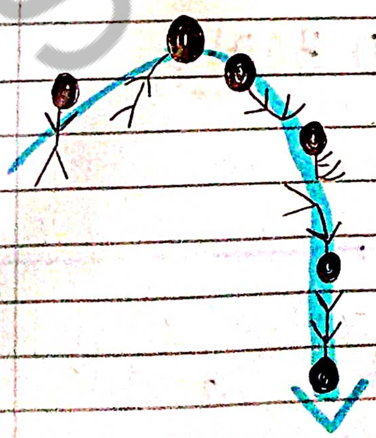
Conservation of angular momentum:-

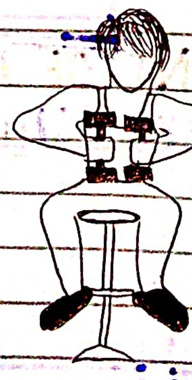
- **definition:**
  In the absence of any external torque ~~force~~, the angular momentum of a system remains constant.
- **derivation:** If $\tau = 0$

$$\text{then } \frac{\Delta L}{\Delta t} = 0$$
$$\Delta L = 0$$
$$L_2 - L_1 = 0$$
$$L_2 = L_1$$
$$L = \text{constant}$$

- **conservation:**
  For isolated system in which no force and no torque acts on system, therefore, angular momentum is conserved.
- **Applications:**

DIVERS	GYMNASTS	MAN HOLDING DUMB-BELLS
when:	**when:**	**when:**
• When legs and arms of diver are drawn in closed tuck position.	• By tucking his knees, he brings his mass closer to the axis of rotation	• When he pulls the dumb-bells by stretching his arms
• **what happens?**	• **what happens?**	• **what happens?**
$I_1 \omega_1 = I_2 \omega_2$ $I_2 < I_1$ $\omega_2 > \omega_1$	$I\omega = \text{constant}$	$I_1 \omega_i = I_2 \omega_f$ $I_2 < I_1$ $\omega_f > \omega_i$



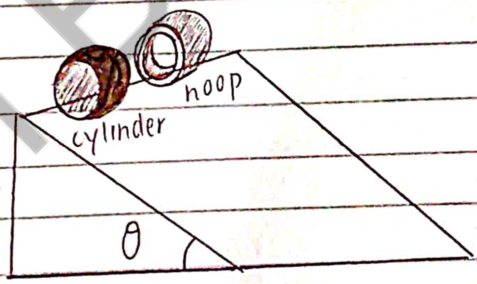


ROLLING OF DISC Vs ROLLING OF HOOP (thin ring)

ROLLING OF DISC	ROLLING OF HOOP (thin ring)
definition	
• A piece of solid cylinder is called disc.	• A piece of thin walled cylinder / hollow sphere.
when it rolls down	
• $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$	• $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
From table	
• $I = \frac{1}{2}mR^2$	• $I = MR^2$
Formula	
• $V = \sqrt{\frac{4}{3}}$	• $V = \sqrt{gh}$
Speed	
• The solid disk will move faster and thus reach the bottom first	• The hoop is slower than disk.

→ **Why speed of Disc > HOOP?**

→ The ratio of the moment of inertia to the mass will be smaller for the disc. So, it will move faster.

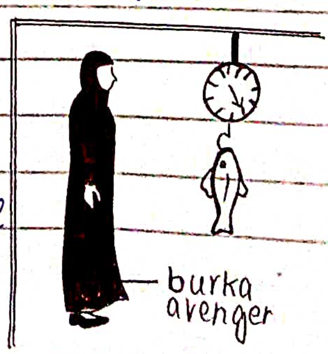


Weightlessness:

"Weightless sensation exists when all contact forces are removed"

example:

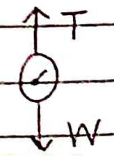
☆ Burka Avenger in an elevator, weighting a fish with a spring balance attached to ceiling to lift.



Real weight	**Apparent weight**
definition	
• It is the gravitational pull of earth on the object.	• Apparent weight is equal to the force required to stop it from falling in the frame of reference.
→ $W = mg$	→ **measurement of weight of body by any device.**

Cases:

Case I

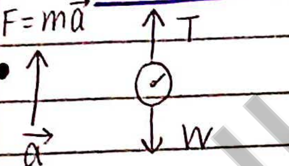


(rest)

- $T = W$
- actual weight
- Lift is at rest / moving with uniform velocity

Case II

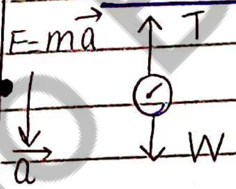
$F = m\vec{a}$



- $\vec{T} > \vec{W}$
  $\vec{T} = \vec{W} + m\vec{a}$
- Apparent weight
- Lift is moving upward with acceleration

Case III

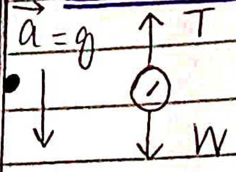
$F = m\vec{a}$



- $\vec{T} < \vec{W}$
  $\vec{T} = \vec{W} - m\vec{a}$
- Apparent weight
- Lift is moving downward with acceleration

Case IV

$\vec{a} = g$



- $T = W - ma$
  $T = mg - mg$
  $T = 0$
- Apparent weight
- Lift is freely falling under gravity.

- **artificial satellite:** man-made objects that orbit around the Earth. e.g GPS

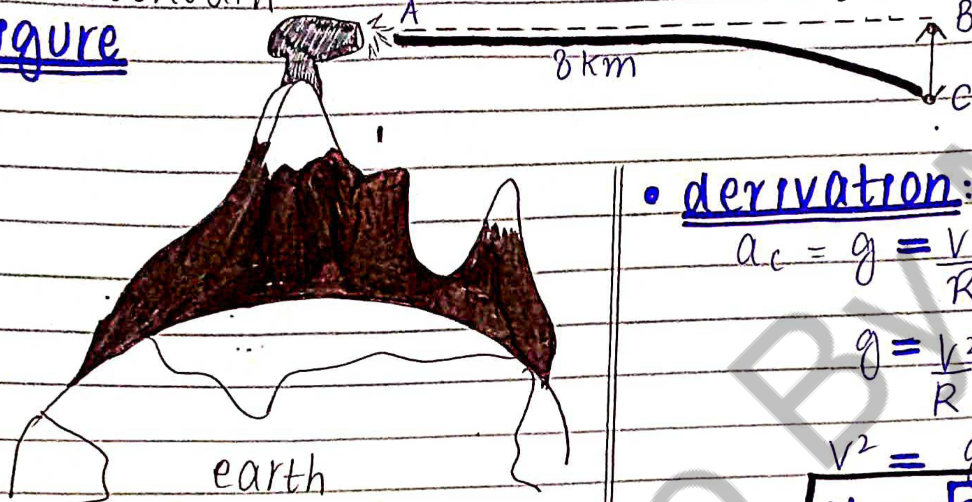
Weightlessness in Satellite (free fall in spaceship)

- **Introduction:**

So, after the law of gravitation, he began thinking about artificial satellites by experimenting:

→ "put a cannon at the top of an extremely high mountain and shoot a canon ball horizontally"

- **figure**



- **critical velocity:**
  - The minimum velocity required to put the satellite into orbit close to Earth.

- **derivation:**

$$a_c = g = \frac{V^2}{R}$$

$$g = \frac{V^2}{R}$$

$$V^2 = gR$$

$$\boxed{V = \sqrt{gR}}$$

$$V = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$\boxed{V = 7.9\ km/s}$$

artificial gravity:

- **definition:**

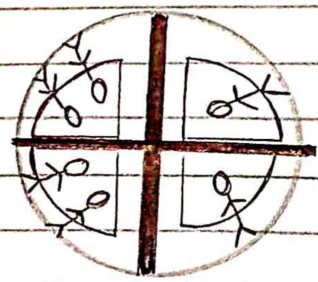
Artificial gravity is the gravity-like effect produced in an orbitting satellite by spinning it about its axis

- **formula:**

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{r}}$$

∴ r is radius of space-craft NOT earth

- **figure**



space-stations built in the form of large wheels with hollow rims

- **explanation:**

Weightlessness creates problems for astronauts by affecting the performance of crew present in space craft. This is done by the help of spin motion of satellites.

The astronauts, then press outer rim and exert force on floor of spaceship. To get artificial gravity = gravity on surface of earth, we've to adjust frequency of spin motion of satellites

→ The Geo-stationary Orbits

→ **definition:**

- Geostationary orbit: The orbit in which the time period of revolution of satellite = time period of rotation of earth about its axis.
- Geostationary satellite: The satellite which completes its one ~~rotation~~ revolution around earth in **24 hours** is called geo-stationary satellite.

→ **applications**

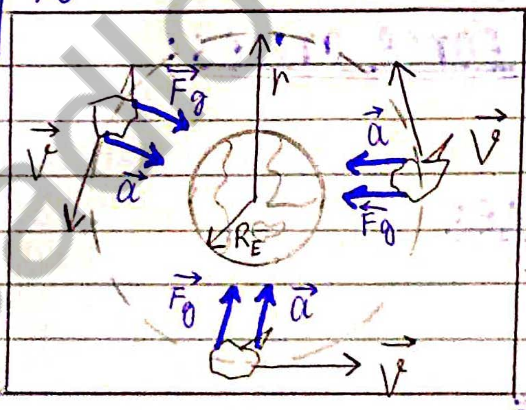
- communication satellite,
- ~~wh~~ weather observation
- purpose of broadcasting.
- military uses

→ **radius of geostationary orbit:**

$$r_0 = \left[\frac{G M_E T^2}{4\pi^2}\right]^{1/3}$$

→ **Figure**

∴ The satellite is in a circular orbit: Its acceleration $\vec{a} \perp \vec{v}$, so its '**v**' is constant

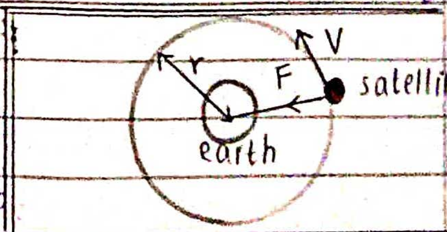


→ Orbital Velocity:

→ **definition:**

The tangential velocity to put satellite in orbit around the Earth.

→ **formula:** $\boxed{V = \sqrt{\frac{GM}{r}}}$



→ **Independent of:** mass of satellite

→ **inversely proportional to:** radius of the orbit of satellite