WORK

Work is said to be done when a force acts on a body and displaces it in the direction of force.

$$W = \vec{F} \cdot \vec{d}$$

$$W = Fd\cos\theta$$

• main points

→ Effective or active component of force causes work to be done.

→ The only component of the force acting on an object that does work is the component that is parallel to the direction of the displacement.

→ Maximum work is done if $\theta = 0°$ (positive work) or $\theta = 180°$ (negative work)

→ Minimum work is done if $\theta = 90°$.

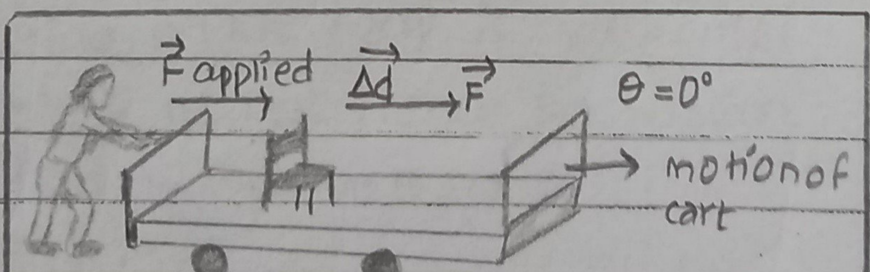
• Cases

(i) Positive work:

→ Force acting on a body is in the same direction as the motion of the body

→ $W = F \cdot \Delta d = F\Delta d\cos\theta = F\Delta d$

where $\theta = 0$



→ Positive work increases kinetic energy of body.

(ii) Negative work:

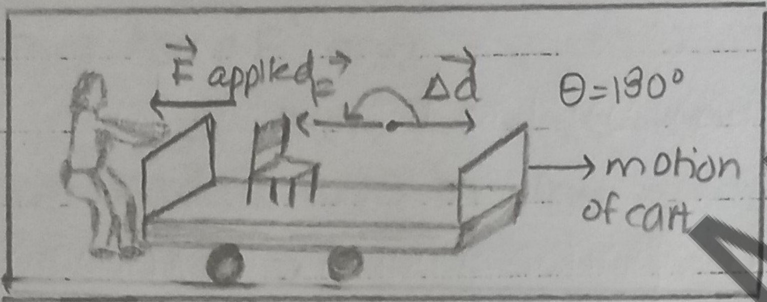
→ Force acting on a body is in the opposite direction as the motion of the body

→ $W = F \cdot \Delta d = F d \cos 180^\circ = -F \Delta d$

$\theta = 180^\circ$

$\cos\theta = -1$

→ Negative work decreases K.E of body.



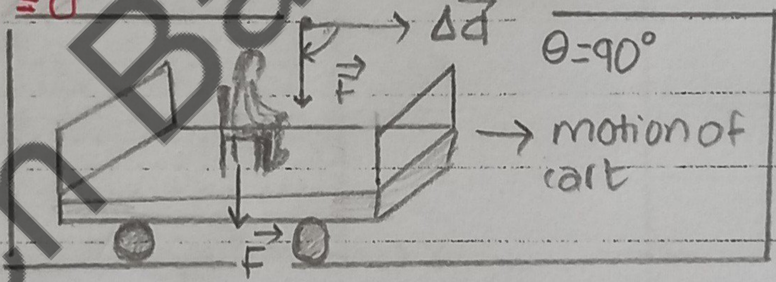
(iii) No work — Horizontal motion, vertical force:

→ Force acting on the body is perpendicular to the direction of motion.

→ $W = F \cdot \Delta d = F \Delta d \cos 90^\circ = 0$

$\theta = 90^\circ$

$\cos 90^\circ = 0$



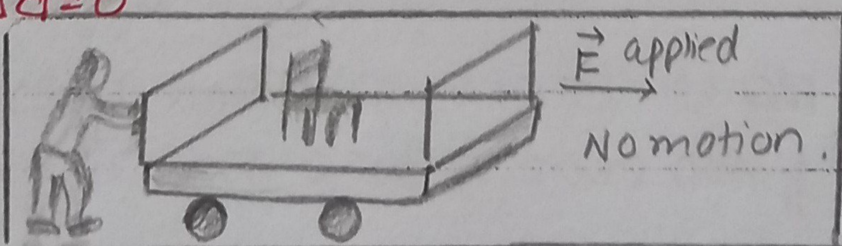
(iv) No work — zero motion, horizontal force

→ Fore acting on the body is horizontal

→ The body is at rest hence no motion.

→ $W = F \Delta d = F \Delta d \cos\theta = 0$

$\Delta d = 0$



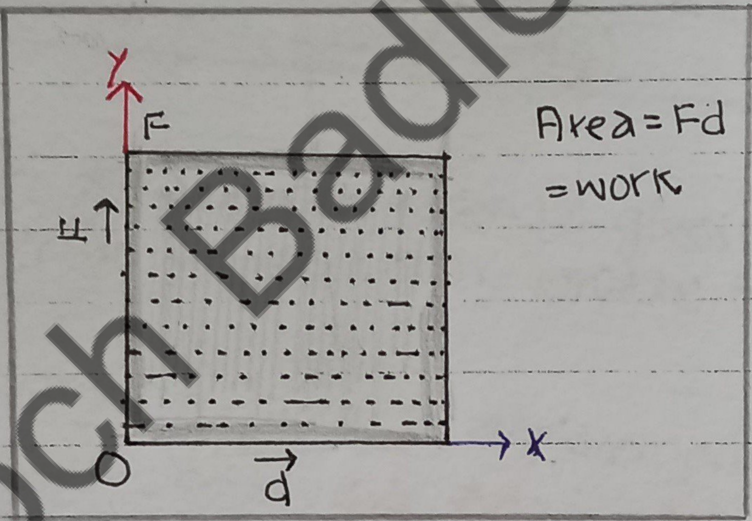
WORK DONE BY A CONSTANT FORCE:

→ When a constant force acts through a distance 'd' the event can be plotted on a graph.

→ If the constant force 'F' and the displacement 'd' are in the same direction then the work done Fd can be shown by the shaded area in the graph given below.

→ "Work is said to be 1 joule if a force of 1 newton displaces a body by 1 metre in the direction of force."

graph



WORK DONE BY A VARIABLE FORCE:

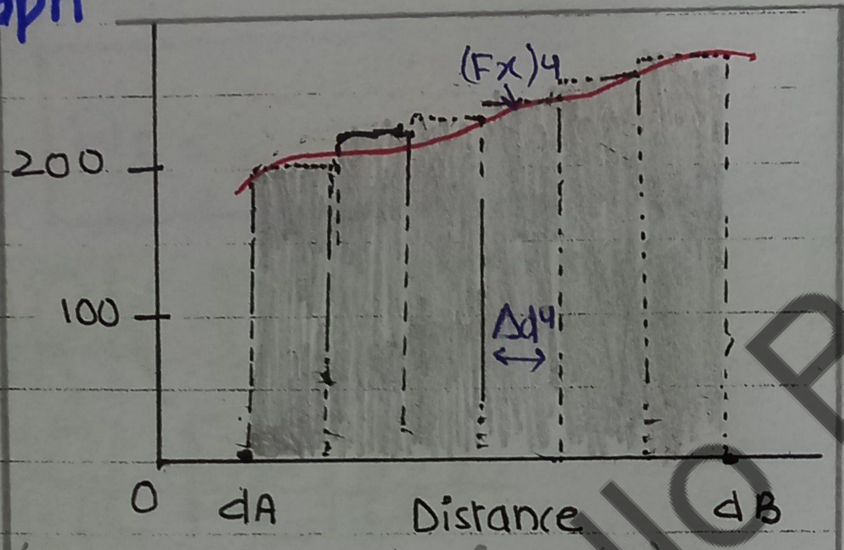
→ In many cases ~~work~~ force doesn't remain constant

For example (i) when a rocket moves away from Earth work is done against the varying force of gravity

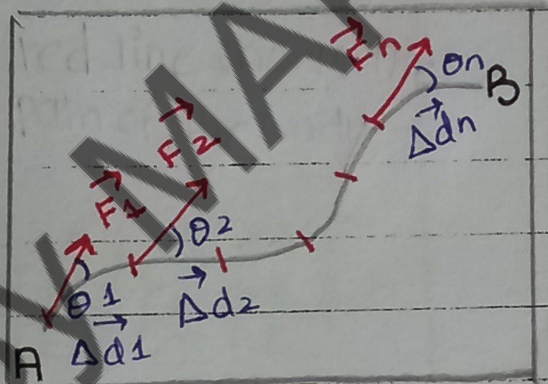
(ii) Force exerted by a spring increases with the amount of stretch so the ~~pe~~ force doesn't remain constant.

→ In order to plot a graph, the distance is divided into many small segments so that the force 'F' acting through these displacements 'Δd' can be obtained to be almost uniform. In this case $\Delta W = F_x \Delta d$.

graph



(red line shows path of body)



▲ Work done by a varying force

→ The area of each rectangle gives the work done by a specific segment of the variable force through displacement Δd.

→ The small divisions of displacement are

$\vec{\Delta d_1}, \vec{\Delta d_2}, \vec{\Delta d_3}\ \vec{\Delta d_4} \ldots \vec{\Delta d_n}$

the corresponding forces are

$\vec{F}_{x1}, \vec{F}_{x2}, \vec{F}_{x3}, \vec{F}_{x4} \ldots \vec{F}_{xn}$

→ Total work done will be

$$W = \Delta W_1 + \Delta W_2 + \Delta W_3 \ldots + \Delta W_n$$

$$W = (F_{x1}\cos\theta_1)\Delta d_1 + (F_{x2}\cos\theta_2)\Delta d_2 + \ldots + (F_{xn}\cos\theta_n)\Delta d_n$$

$$\boxed{W = \sum_{i=1}^{i=n} (F_{xi}\cos\theta_i)\Delta d_i}$$

→ The smaller Δd is taken, more accurate the value of work can be obtained.

→ "Work done at a specific instant of the curve where time approaches to zero is termed as instantaneous work"

$$W_T = \lim_{\Delta t \to 0} \sum_{i=1}^{i=n} (F_{xi} \cos\theta_i)\Delta d_i$$

WORK DONE IN GRAVITATIONAL FEILD:

• **main points:**

→ The space around the Earth with in which it exerts a force is known as gravitational field.

→ The gravitational force per unit mass on a body is known as gravitational field strength.

→ If we carry a body in a closed path in such field that the total work done will be zero, such a field is known as conservative.

• **conservative force**

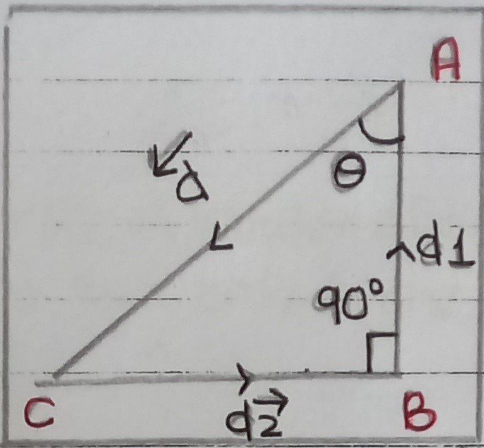
- Conservative forces do not depend on path.

• **non conservative force**

- Forces dependent on path are called non-conservative forces.

• **mathematical explaination of work done in a closed path in gravitational field:**

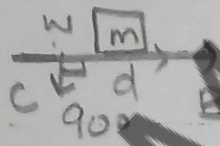
Lets consider closed path 'ABCA'.
A body of mass 'm' is carried from C to B & the from B to A.



(i) work done between C and B

$W_{C\to B} = \vec{W}\cdot\vec{d_2} = Wd_2\cos 90^\circ$

$\Delta W_{C\to B} = 0$

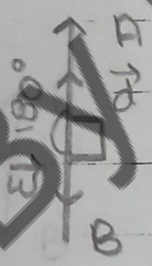


(ii) work done between B and A

$\Delta W_{B\to A} = Wd_1\cos 180^\circ$

$Wd_1(-1)$

$\Delta W_{B\to A} = -Wd_1$



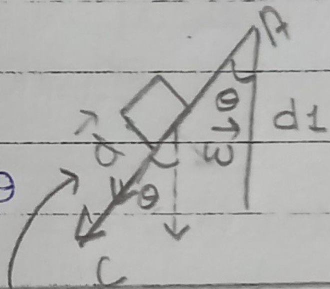
(iii) work done between A and C

$\Delta W_{A\to C} = Wd\cos\theta = W(d\cos\theta)$

From right angle triangle $d_1 = d\cos\theta$

$\Delta W_{A\to C} = W(d_1)$

$\Delta W_{A\to C} = Wd_1$.



note: we will consider the effective component of $\vec{d}$ which is $\vec{dx}$ and $dx = d\cos\theta = d_1$

→ Now finding total work done

$W_T = \Delta W_{C\to B} + \Delta W_{B\to A} + \Delta W_{A\to C}$

$W_T = -d_1 + d_1 + 0$

$\Delta W_T = \text{zero}$.

• mcq points

→ Electric force, elastic spring force and gravitation force are conservative forces.

→ Frictional forces, Air resistance, tension in a string, normal force, propulsion force of a motor, propulsion force of a rocket are non-conservative forces.

POWER

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- Power is defined as the rate of transfer of energy.

or The amount of work done by a body in one second is called power.

$$P = \frac{\vec{F}\cdot\vec{d}}{t} = \vec{F}\cdot\vec{v}$$

• mainpoints:

→ The product of force and velocity gives power.

→ Two persons may have done same work but the time taken by them differs. They can be differentiated in terms of power i.e, person who finished the same work in lesser time has a greater power compared to the other.

→ Unit of power is watt. larger units are KW, MW etc.

$1\,watt = Js^{-1}$

→ "Power is said to be 1watt if 1 Joule of work is done in 1s"

→ "The total work ΔW done by a body in total time 'Δt' is called average power"

$$P_{ave} = \frac{\text{total work done } (\Delta W)}{\text{total time taken } (\Delta T)}$$

→ "The rate of doing work in any instant of time is called istantaneous power."

$$P_{inst} = \lim_{\Delta t \to 0}\left(\frac{\Delta W}{\Delta T}\right)$$

→ The commercial unit of work (electrical energy is Kilowatt hour

→ "One kilowatt-hour is the work done in one hour by an agent whose power is one kilowatt."

$1 kwh = 1000 watts \times 3600 s = 3.6 \times 10^6 J = 3.6 MJ$.

→ 1hp = 746 watts.

• **mcq points**

→ The power of TV set is 120 watts

→ The power of pocket calculator is 7.5×10^{-4} watts.

ENERGY

■ The capacity of a body to do work

or Energy is the agent which causes some change in the state of system.

energy and work are interconvertable according to work energy principle.

KINETIC ENERGY

■ Energy in body due to its motion.

$$K.E = \frac{1}{2} mv^2$$

POTENTIAL ENERGY

■ Energy in a body due to its position (with respect to the surface of the Earth)

$$P.E = mgh$$

EFFICIENCY

- Mechanical efficiency is the ratio of work output to work input. It's expressed in persentage.

$$\text{Efficiency } \eta = \frac{\text{output work}}{\text{input work}}$$

• output:

If a machine moves a load W through a distance h then useful work done by machine is called output; **output = $F_{out} \times D_{out}$**

• input:

If an effort Fin acts through a distance Din then the work done on the machine is called input; **input = $F_{in} \times D_{in}$**

• ideal system:

An ideal system is the one which has the same output as input i.e, output = input. In real life, such a system doesn't exist.

ENERGY LOSSES:

→ No practical system can be ideal due to energy lossess.

→ Energy is lost in doing work against resistive forces like friction.

→ Energy is lost in various forms i.e, heat energy, sound energy, light energy etc.

→ This loss in energy effects the input output ratio and hence the effeciency of a system is reduced.

• **mcq points**

→ LED bulbs are more preferred than light bulbs because there're more efficient and consume less energy.

→ AC/DC fans cut down your consumption to 65% and are designed to run on 12V. and consume around 26-35W.

→ ordinary fans consume about 35-40watt and are less efficien than AC/DC fans.

ABSOLUTE POTENTIAL ENERGY:

■ Potential energy possessed by a body when the factor of gravity 'g' is not constant is called absolute potential energy.

$$\text{absolute potential energy} = \frac{GmMe}{Re}.$$

main points:

→ when there is no change in 'g', mgh is valid.

→ If there's a change in 'g' we expell factor of 'g' and calculte. Absolute P.E.

→ Change in 'g' occurs above 1 Re. (radius of Earth)

→ In order to calculate absolute P.E of a body, we divide the distance 'd' covered by it in small intervals so that the change occuring in gravity is equal in each interval.

→ We will calculate work done in all patches and add them.
for instance, $\Delta W_{1-2} = \vec{F}_{ave} \cdot \Delta d \cos\theta$.
where ΔW_{1-2} is work done by the body when it moved from point 1 to point 2.

- Average force is taken because it's between two points
- θ will be zero, $\cos\theta = \cos 0 = 1$
- 'F' isn't provided by us, it's $\frac{GmMe}{r^2}$

• derivation:

First calculating work done between point '1' and '2'.

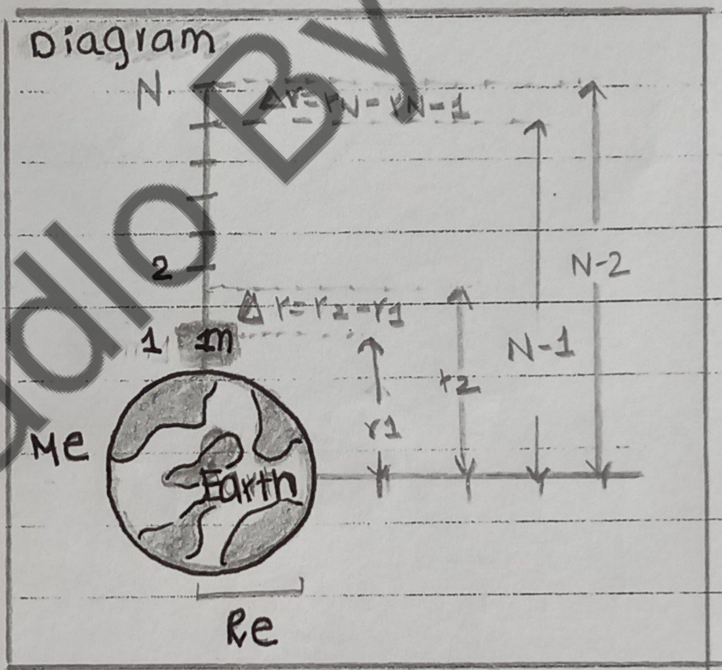
$\Delta W_{1\to2} = F_{av}\Delta r$

$\Delta W_{1\to2} = F_{av}(r_2 - r_1)$ (i)

$F_{average} = \frac{GmMe}{r^2}$

(i) becomes

$\Delta W_{1\to2} = \frac{GmMe}{r_{av}^2}(r_2 - r_1)$ ... (ii)



where $r_{av} = \frac{r_1 + r_2}{2}$

note:
important derivation of '$r_1 r_2$', not written in book.

$r_{ave} = \frac{r_1 + r_2}{2}$. $\because \Delta r = r_2 - r_1$

$\Delta r + r_1 = r_2$.

$r_{ave} = \frac{r_1 + \Delta r + r_1}{2}$

(taking square as required by the formula).

$(r_{ave})^2 = (\Delta r + 2r_1 \div 2)^2$

$$= \frac{\Delta r^2 + 4r_1^2 + 4r_1\Delta r}{4} \quad \because (a+b)^2 = a^2+b^2+2ab.$$

here we'll take $\Delta r \approx 0$

$$= \frac{4r_1^2 + 4r_1\Delta r}{4} \Rightarrow \frac{4(r_1^2 + r_1\Delta r)}{4}$$

$$(r_{ave})^2 = r_1^2 + r_1(r_2 - r_1)$$

Δr isn't taken zero when it's being multiplied by another variable

$$r_{ave}^2 = r_1^2 + r_1 r_2 - r_1^2$$

$$r_{ave}^2 = r_1 r_2$$

putting this value of r_{ave}^2 in eq(ii).

$$\Delta W_{1\to 2} = \frac{GmM_e}{r_1 r_2}(r_2 - r_1) \quad \cdots (iii)$$

$$\Delta W_{1\to 2} = GmM_e\left(\frac{r_2}{r_1 r_2} - \frac{r_1}{r_1 r_2}\right)$$

$$\Delta W_{1\to 2} = GmM_e\left\{\frac{1}{r_1} - \frac{1}{r_2}\right\}$$

similarly $\Delta W_{2\to 3} = GmM_e\left\{\frac{1}{r_2} - \frac{1}{r_3}\right\}$

and $\Delta W_{N-2\to N-1} = GmM_e\left\{\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right\}$

$$\Delta W_{N-1\to N} = GmM_e\left\{\frac{1}{r_{N-1}} - \frac{1}{r_N}\right\}$$

adding them,

$$\Delta W_{1\to N} = \Delta W_{1\to 2} + \Delta W_{2\to 3} + \Delta W_{3\to 4} + \cdots + \Delta W_{N-2\to N-1}$$

putting corresponding values

$$\Delta W_{1\to N} = GmM_e\left\{\frac{1}{r_1} - \frac{1}{r_2}\right\} + '' \left[\frac{1}{r_2} - \frac{1}{r_3}\right] + '' \left[\frac{1}{r_3} + \frac{1}{r_4}\right]$$

resulting

$$\Delta W_{1\to N} = GmM_e\left\{\frac{1}{r_1} - \frac{1}{r_N}\right\}$$

if 'N' point lies on infinity,
then $r_N = \infty$

$$\Delta W_{1\to\infty} = GmM_e\left\{\frac{1}{r_1} - \frac{1}{r_\infty}\right\}$$

$$= GmM_e\left\{\frac{1}{r_1} - 0\right\}$$

$$\Delta W_{1\to\infty} = \frac{GmM_e}{r_1} \quad \ldots (iv)$$

where we'll replace 'r1' By Earth's radius. eq(iv) becomes

$$\boxed{\Delta W = -\frac{GmM_e}{R_e}}$$

since the ΔW is against gravity, negative sign is used.

- **gravitational potential:**

The potential energy per unit mass at that point which is at distance r from the centre of Earth.

$$V(r) = -\frac{GM_e}{R_e}$$

ESCAPE VELOCITY:

The initial velocity, which a projectile must have at the Earth's surface in order to go out of Earth's gravitational field, is known as escape velocity.

$$v_{esc} = \sqrt{2gR_e}$$

• main points :

→ Rockets require a velocity which would help them reach in space.

→ Earth's gravitational force constantly pulls a body back to Earth. Thus a velocity which would enable the body to escape from Earth's gravitational feild is called escape velocity

→ Its value is $11.2 \times 10^3 ms^{-1}$

• derivation :

Absolute P.E is given by $\frac{GMem}{Re}$

• If a projectile is given kinetic energy equal to $\frac{GMem}{Re}$ it'll reach an infinite distance from the Earth.

hence $\frac{1}{2} mV^2_{esc} = \frac{GMem}{Re}$

$$\frac{1}{2} V^2_{esc} = \frac{GMe}{Re}$$

$$V_{esc} = \sqrt{\frac{2GMe}{Re}}$$

As $\frac{GMem}{Re^2} = mg \Rightarrow GMe = gRe^2$

putting its value

$$V_{esc} = \sqrt{\frac{2gRe^2}{Re}}$$

$$V_{esc} = \sqrt{2gRe}$$

• mcq point:

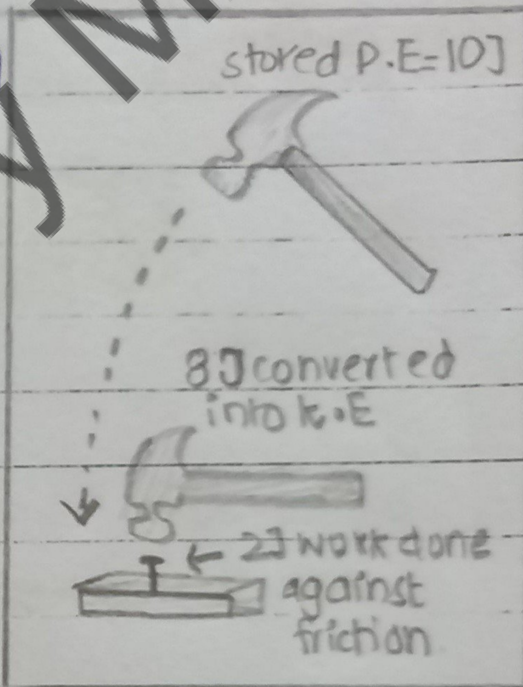
→ escape velocity of moon is $2.360 \times 10^3 ms^{-1}$

WORK ENERGY THEOREM IN RESISTIVE MEDIUM:

• **main points:**

→ Energy can be stored in one of the two basic types kinetic energy and potential energy.

→ Potential energy stored in a body raised to a height 'h' is called gravitational potential energy.



→ In presence of a resistive medium, all P.E is not converted into K.E, rather some energy is utilized in doing work against the resistive medium.

→ gain in K.E = loss in P.E (non-resistive medium)

→ gain in K.E + work done against resistive medium = loss in P.E (resistive medium)

→ $mgh = fh + \frac{1}{2}mv^2$

CONSERVATION OF ENERGY:

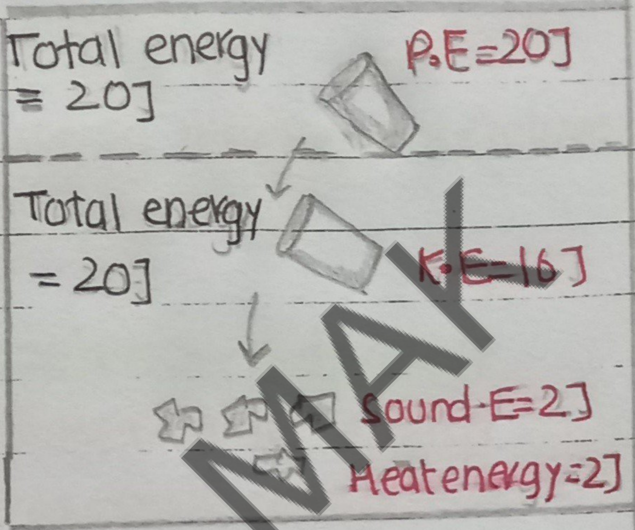
• **main points**

→ Energy can neither be created nor destroyed. It can be transformed from one kind into other, but the total amount of energy remains constant.

Total energy = P.E + K.E = constant

→ Total energy of a system in which energy is being converted from one form into other remains constant.

→ A lot of energy is lost in the forms useless for us. That's why new energy sources have to be developed.



NON-RENEWABLE ENERGY RESOURCES:

■ Energy resources which cannot be renewed or replenished in short duration are called non-renewable energy resources.

• **main points:**

→ These are also called conventional energy resources.

→ It took thousands of years to form these.

→ over 85% of energy used is from conventional supplies.

→ These are: coal, oil, natural gas, nuclear energy.

• **mcq points:**

→ Coal is most abundant form of fossil fuel.

→ Oil is most abundant in middle east countries.

→ Natural gas is gaseous form of fossil fuels.

→ Process of fission takes place inside a nuclear power plant.

RENEWABLE ENERGY RESOURCES:

■ The resources which are being constantly renewed by nature are called renewable resources.

• **main points:**

→ These are also called 'non-conventional' sources of energy

→ Some of these include: energy from biomass, energy from waves, wind energy, tidal energy, solar energy, geothermal energy.

→ Biomass energy is generated from plants and animals.

→ Ocean waves are very powerful and contain a lot of energy, this energy is converted into electricity.

→ The inside of Earth is full of heat which can be converted into different forms of energy.

→ Solar energy is the radiant light and heat from the sun that has been harnessed by humans.

→ Wind energy describes the process by which wind is used to generate electricity

→ Using power of the tides, energy is produced from gravitational pull creating high and low tides. A tidal barrage utilizes potential energy of tides to turn turbines which generates electricity.

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