CHAPTER : 04

WORK AND ENERGY

DEFINITION

Work is done when a force acts on a body and body moves in the direction of force

MATHEMATICAL FORM

Wor is : scalar product of force and displacement

W = Force $\cdot$ Displacement

$W = F \cdot d$

$W = Fd\cos\theta$

SI UNIT

$W = F \cdot d$

$W = kgms^{-2}\,m$

$W = Nm$ **OR** Joule 'J'

DIMENSION

$W = Fd$

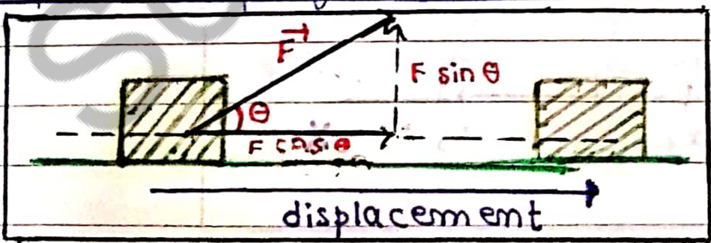
$W = kgms^{-2}\,m$

$W = kg\,m^2 s^{-2}$

$W = [ML^2T^{-2}]$

FORCE AT AN ANGLE

When a force acts at angle, then "work done by the component of force is in the direction of displacement"



- Y-component is ⊥ to 'd' so it does not produce work
- X-component is in the direction of 'd' so it will produce work

Fcosθ = effective component

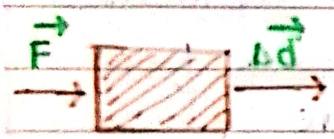
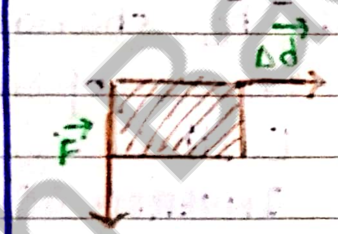
Work = X-component $\cdot$ displacement

$W = \vec{F_x} \cdot \vec{d}$

$W = F\cos\theta\, d$

$W = Fd\cos\theta$

RANGE OF WORK

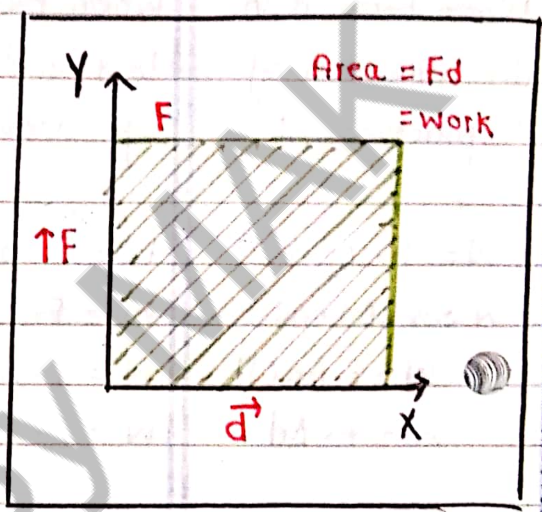
W_{MAX}	W_{MIN}	W_{-IVE}	No WORK
WHEN $\theta = 0°$	WHEN $\theta = 90°$	WHEN $\theta = 180°$	WHEN $d = 0$
OR $F \parallel d$	OR $\vec{F}$ ⊥ $\Delta\vec{d}$	OR $\vec{F} \uparrow \downarrow \vec{d}$	$\vec{F}$ → $\Delta d = 0$ →
Work will be maximum when Force is in the direction of displacement	Work will be minimum Force is perpendicular to displacement.	Work will be -ive when gravitational Force is opposite to displacement	when a force acts on a body but displacement is 0 then work will be 0
$W = Fd\cos 0$ $W = Fd(1)$ $W = F\Delta d$	$W = Fd\cos 90$ $W = Fd(0)$ $W = F\Delta d$	$W = F\Delta d\cos 180°$ $W = F\Delta d(-1)$ $W = -F\Delta d$	$W = F\Delta d\cos\theta$ $W = F(0)\cos 0$ $W = 0$
			 No motion

IF $\theta = 90°$ No work is done
IF $\theta < 90°$ +ive work
IF $\theta > 90°$ -ive work

WORK DONE BY CONSTANT FORCE

When a constant force acts through distance 'd', the event can be plotted on a simple graph.

* X-axis = distance '$\vec{d}$'
* Y-axis = Force '$\vec{F}$'



⇒ As force does not vary, the graph will be horizontal straight line.

⇒ If the constant 'F' and 'd' are in same direction, then work done is 'Fd'

⇒ The Area under F-d curve can be taken to represent the work done by the force.

⇒ In this case, 'F' is not in the direction of 'd', the graph is plotted b/w $F\cos\theta$ and d

(WORK DONE BY VARIABLE FORCE)

"For whose magnitude or direction changes is called variable force"

EXAMPLES:

1. Gravitation force changes as inverse square of distance from Earth's centre.

2. Force exerted by a spring increases with the amount of stretch so force does not remain constant.

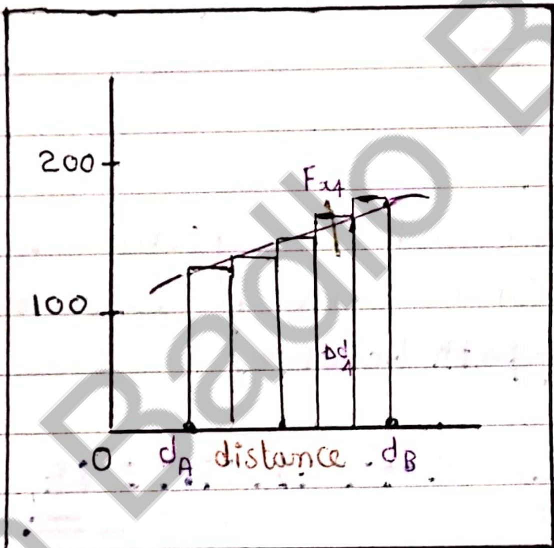
⇒ Work done by variable force is determined graphically.

DISTANCE

* Δd is plotted on x-axis
* Δd is divided into small fragments $\Delta d_1, \Delta d_2, \ldots \Delta d_n$
* Smaller the segments, more accurate will be the work done

FORCE

* F_x is plotted on y-axis
* Forces corresponding to Δd segments $F_{x_1}, F_{x_2}, \ldots F_{x_n}$
* For each segment of distance, Force of F_x is shown by dashed line



$W_1 = F_{x_1} \Delta d_1 = (F_{x_1} \cos\theta_1)\, \Delta d_1$

$W_2 = F_{x_2} \Delta d_2 = (F_{x_2} \cos\theta_2)\, \Delta d_2$

$W_3 = F_{x_3} \Delta d_3 = (F_{x_3} \cos\theta_3)\, \Delta d_3$

$W_4 = F_{x_4} \Delta d_4 = (F_{x_4} \cos\theta_4)\, \Delta d_4$

$W_n = F_{x_n} \Delta d_n = (F_{x_n} \cos\theta_n)\, \Delta d_n$

Total W $W = W_1 + W_2 + W_3 + W_4 + \ldots W_n$

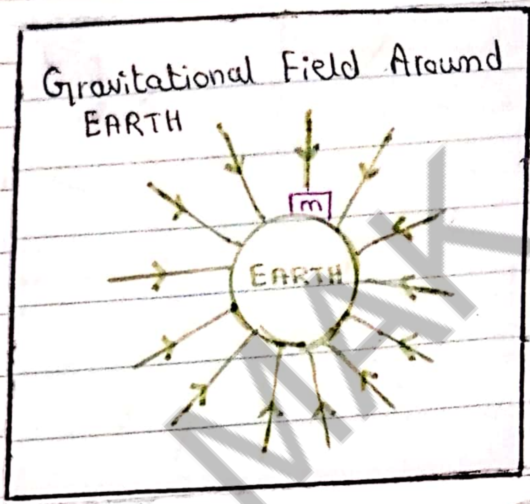
$W = (F_{x_1} \cos\theta_1)\Delta d_1 + (F_{x_2} \cos\theta_2)\Delta d_2 + (F_{x_3} \cos\theta_3) + \ldots (F_{x_n} \cos\theta)\Delta d_n$

$$\boxed{W = \lim_{\Delta d \to 0} \sum_{i=i}^{i=n} (F_{x_i} \cos\theta_i)\, \Delta d_i}$$

(Work Done In A Gravitational Field)

Gravitational Field

The space around the earth with in which it exerts a force of attraction on ther bodies is known as gravitational field.



Gravitationa Field Strenth

The Gravitation Field per unit mass on a body is called gravitational field strength [F/1 Kg]

More force acting $\Rightarrow$ more gravitation field strength

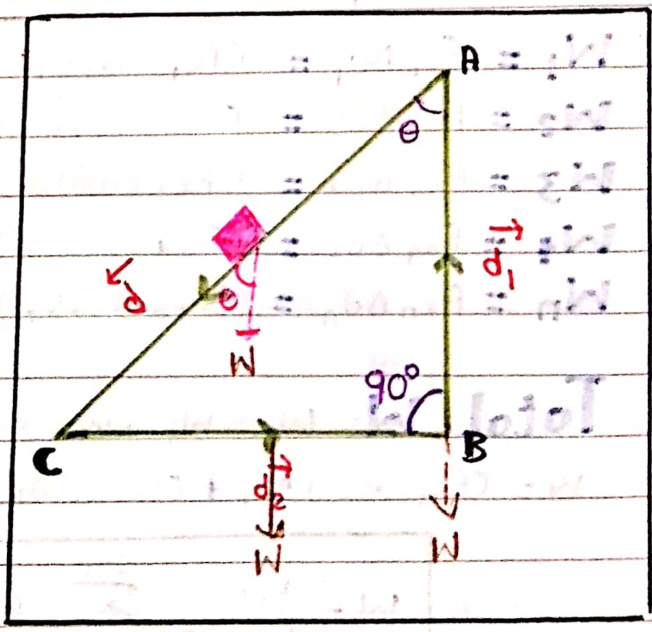
$\Rightarrow$ Work done in a closed path field is:

1. Conservative: Total work is zero
2. Independent of path followed

Work Done Is Conservative

A body of mass 'm' and weight 'w' moves from:

1. C to B then
2. B to A then
3. A to C



WORK DONE b/w C and B.

$\Delta W_{C\to B} = \vec{W}\cdot\vec{d_2} = wd_2 \cos\theta = wd_2 \cos 90^\circ = 0$

WORK DONE b/w B and A

$\Delta W_{B\to A} = \vec{W}\cdot\vec{d_1} = wd_1 \cos\theta = wd_1 \cos 180^\circ = -wd_1$

WORK DONE b/w A and C

$\Delta W_{A\to C} = \vec{W}\cdot\vec{d} = wd\cos\theta \longrightarrow (i)$

FOR $\cos\theta$

$\cos\theta = \dfrac{\text{base}}{\text{hypotenuse}}$

$\cos\theta = \dfrac{d_1}{d}$

$d\cos\theta = d_1$

Putting value of $d\cos\theta$ in eq (i)

$\Delta W_{A\to C} = \vec{W}\cdot\vec{d} = W(d\cos\theta) = wd_1$

TOTAL WORK DONE

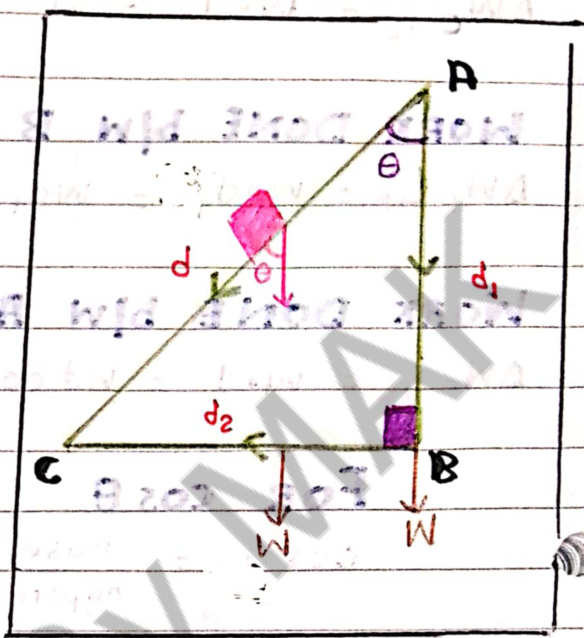
$\Delta W_T = \Delta W_{C\to B} + \Delta W_{B\to A} + \Delta W_{A\to C}$

$\Delta W_T = 0 + (-wd_1) + (wd_1)$

$\Delta W_T = 0$

Conclusion: Since the total work done in a closed path in G.F is zero, therefore G.F is conservative field

WORK DONE IS INDEPENDENT OF PATH FOLLOWED



Path 1 : A to C
Path 2 : A to B then B to C

WORK DONE IN PATH 1.

$\Delta W_T = W \cdot d = Wd \cos\theta$
$(\because d\cos\theta = d_1)$

$\Delta W_T = Wd_1$ —— (ii)

WORK DONE IN PATH 2

$\Delta W_T = W_{A \to B} + W_{B \to C}$

$\Delta W_T = (Wd_1 \cos 0^\circ) + (Wd_2 \cos 90^\circ)$

$\Delta W_T = Wd_1 + 0$
$\Delta W_T = Wd_1$ —— (iii)

Conclusion: By comparing eq (ii) and eq (iii), it is clear that work done in both paths is same, therefore work done in G.F is independent of path followed

(Power)

DEFINITION

Power is the measure of the rate at which work is being done. [Scalar Quantity]

MATHEMATICAL FORM

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

$$P = \frac{W}{t} \quad (\because W = F \cdot d)$$

$$P = \frac{F \cdot d}{t}$$

$$P = F \cdot \frac{d}{t} \quad (\because v = \frac{d}{t})$$

$$P = \vec{F} \cdot \vec{v}$$

[Dot Product of Force and velocity is called power]

EXPLANATION

* No time factor is inovled in the difinition of work.
* In the definition of work, it is not clear, weather the same amount is done in 1s or in 1 hour.
* The agency which performs the specific amount of work in lesser time has more power than the other agency.

DIMENSION

$$P = \vec{F} \cdot \vec{V}$$

$$P = ma \cdot \frac{d}{t}$$

$$P = kg\,ms^{-2} \cdot \frac{m}{s}$$

$$P = kg\,ms^{-2}\,ms^{-1}$$

$$P = kg\,m^2\,s^{-3}$$

$$P = [ML^2T^{-3}]$$

TYPES OF POWER

AVERAGE POWER

The total work done ΔW by a body in total time Δt is called average power

$$\langle P \rangle = \frac{\Delta W}{\Delta t}$$

INSTANTANEOUS POWER

The Rate of doing work in any instant of time is called instantaneous Power

$$P_{inst} = \lim_{\Delta t \to 0} \left(\frac{\Delta W}{\Delta t} \right)$$

SI UNIT

Watt OR Js^{-1}

$$1 \text{ watt power} = \frac{1 \text{ Joule}}{1 \text{ second}} \Rightarrow 1W = \frac{1J}{1s} \Rightarrow 1W = Js^{-1}$$

ONE KILOWATT HOUR

"The workdone in one hour by an agent whose power is one kilowatt"

$$1Kwh = 1000\,W \times 3600\,s$$
$$1Kwh = 3600000\,J$$
$$1Kwh = 3.6 \times 10^6\,J$$
$$1Kwh = 3.6\,MJ$$

- The Power of TV set is 120 W [120 J / 1s]
- The Power of pocket calculator is $7.5 \times 10^{-4}\,W$
- 1hp = 746 W [Unit of electrical Energy]

ENERGY

The capacity of a body to do work OR The agent which causes some change in the state of system is called "Energy".

	KINETIC ENERGY	POTENTIAL ENERGY
DEFINITION	Energy in the body due to its motion is called K.E	Energy in the body due to its position is called P.E
FORMULA	$K.E = \frac{1}{2} mv^2$	$P.E = mgh$
FACTORS	The determing factors for K.E are velocity and mass	The determing factors for P.E are height and mass
TRANSFERABILITY	can be transferred from one moving object to another.	Potential energy can not be transferred.
EXAMPLES	• charged particle in E.F • Moving car	• Water present at the top of the hill.

RELATION b/w K-E AND MOMENTUM

$K.E = \frac{1}{2} mv^2$

Multiply and divide R.H.S by 'm'

$K.E = \frac{m^2v^2}{2m}$

$K.E = \frac{p^2}{2m} \Rightarrow P = \sqrt{2mE}$

ELASTIC P.E

The Energy stored in compressed spring is called E.P.E

$E.P.E = 1/2\ Kx^2$

K = spring constant

x = extension

(ABSOLUTE P.E)

STATEMENT

"The Amount of work done in moving a body at a certain point in gravitational field to a point of zero potential such that the body is never accelerated is called Absolute Potential Energy"

- Near the surface of earth, the gravitational force does not change **so** $E = mgh$ is true.

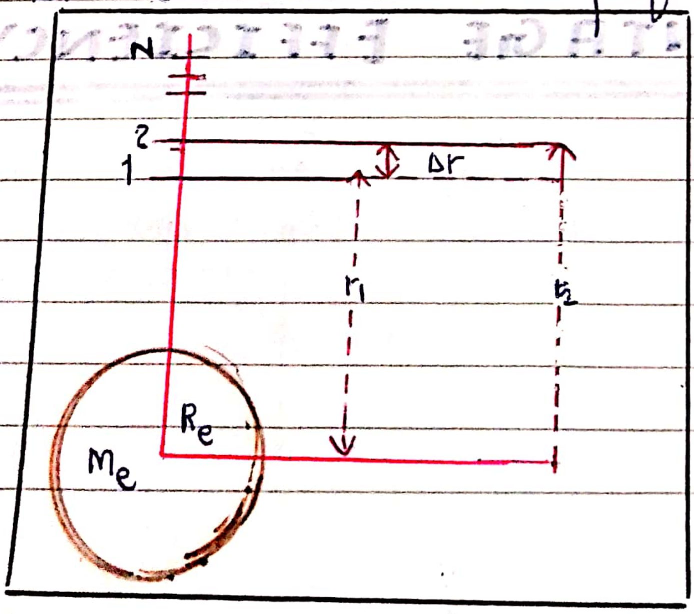
BUT

- If a body is displaced through a large distance more than one Earth Radius then, $E = mgh$ is not valid. **Thats Why** we need absolute value.

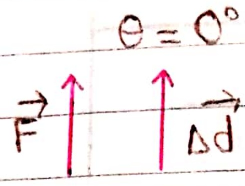
(DERIVATION)

- To calculate the workdone in moving a body from earth surface to a very far off distance, gravitational force does not remain same.

- Distance is divided into small distances of magnitude Δr so that value of force remains constant:



$$\Delta W_{1\to 2} = \vec{F}\cdot\vec{\Delta d}$$
$$= F\Delta d\cos\theta$$
$$= F\Delta d\cos 0$$
$$= F\Delta d$$



$$\Delta W_{1\to 2} = F_{ave}\cdot\Delta d$$
$$= F_{ave}\cdot\Delta r$$
$$= F_{ave}(r_2 - r_1) \quad \text{—— (i)}$$

$$F_{ave} = \frac{G m_1 m_2}{r^2_{ave}} \quad \text{—— (ii)}$$

FOR r^2_{ave}

$$r_{ave} = \frac{r_1 + r_2}{2}$$

$$\boxed{\begin{aligned} \Delta r &= r_2 - r_1 \\ \Delta r + r_1 &= r_2 \\ r_2 &= \Delta r + r_1 \end{aligned}}$$

$$r_{ave} = \frac{r_1 + \Delta r + r_1}{2}$$

$$r_{ave} = \frac{2r_1 + \Delta r}{2}$$

$$(r_{ave})^2 = \left(\frac{2r_1 + \Delta r}{2}\right)^2$$

$$r^2_{ave} = \left[\frac{(2r_1)^2 + (\Delta r)^2 + 2(2r_1)(\Delta r)}{4}\right]$$

$\Delta r^2 \approx 0 \Rightarrow r_2 - r_1 \approx 0 \Rightarrow \Delta r^2 < r^2$ [So ignored]

$$r^2_{ave} = \left[\frac{4r_1^2 + 4r_1\Delta r}{4}\right]$$

$$r^2_{ave} = r_1^2 + r_1\Delta r$$

$$r^2_{ave} = r_1^2 + r_1(r_2 - r_1)$$

$$r^2_{ave} = r_1^2 + r_1 r_2 - r_1^2$$

$$\boxed{r^2_{ave} = r_1 r_2} \quad \text{—— (iii)}$$

Putting the value of 'r^2_{ave}' in eq (ii)

$$\boxed{F_{ave} = \frac{G m_1 m_2}{r_1 r_2}} \quad \text{———} \quad (iv)$$

$$\Delta W_{1\to 2} = F_{ave} \cdot \Delta r$$

$$\Delta W_{1\to 2} = \left(\frac{G m_1 m_2}{r_1 r_2}\right) \cdot (r_2 - r_1)$$

$$\Delta W_{1\to 2} = G M_e m \left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

$$\Delta W_{1\to 2} = G M_e m \left(\frac{r_2}{r_1 r_2} - \frac{r_1}{r_1 r_2}\right)$$

$$\Delta W_{1\to 2} = G M_e m \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

SIMILARLY:

$$\Delta W_{2\to 3} = G M_e m \left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$\Delta W_{N-1\to N} = G M_e m \left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

$$W_T = \Delta W_{1\to 2} + \Delta W_{2\to 3} + \cdots \Delta W_{N-2\to N-1} + \Delta W_{N-1\to N}$$

$$W_T = G M_e m \left[\frac{1}{r_1} - \frac{1}{r_2}\right] + G M_e m \left[\frac{1}{r_2} - \frac{1}{r_3}\right] + \cdots G M_e m \left[\frac{1}{r_{N-2}} + \frac{1}{r_{N-1}}\right] + G M_e m \left[\frac{1}{r_{N-1}} + \frac{1}{r_N}\right]$$

$$W_T = G M_e m \left[\frac{1}{r_1} - \cancel{\frac{1}{r_2}} + \cancel{\frac{1}{r_2}} - \frac{1}{r_3} + \cdots + \frac{1}{r_{N-2}} - \cancel{\frac{1}{r_{N-1}}} + \cancel{\frac{1}{r_{N-1}}} - \frac{1}{r_N}\right]$$

$$\boxed{W_{T_{1\to N}} = G M_e m \left[\frac{1}{r_1} - \frac{1}{r_N}\right]}$$

If N lies on infinity $[r_N = \infty]$

$$W_{1\to\infty} = G M_e m \left[\frac{1}{r_1} - \frac{1}{\infty}\right]$$

$$W_{1\to\infty} = Gm_em\left[\frac{1}{r_1} - \frac{1}{0}\right]$$

$$W_{1\to\infty} = Gm_em\left[\frac{1}{r_1} - 0\right]$$

$$W_{1\to\infty} = Gm_em\left[\frac{1}{r_1}\right]$$

$$W_{1\to\infty} = \frac{Gm_em}{r_1}$$

If we displace the body from earth surface with Radius Re to infinity then:

$$\Delta W = \frac{Gm_em}{R_e}$$

A.P.E = work done in moving body from earth surface to infinity.

$$\text{Absolute P.E} = \frac{Gm_em}{R_e}$$

As work is done against the gravity so its value is taken -ive

$$\boxed{\text{Absolute P.E} = -\frac{Gm_em}{R_e}}$$

GRAVITATIONAL P.E PER UNIT MASS

→ The P-E per unit mass at that point which is at distance 'r' from the center of earth

$$V_{(r)} = \frac{-\frac{Gm_em}{R_e}}{m}$$

$$V_{(r)} = -\frac{Gm_e}{R_e}$$

~(Escape Velocity)~

Definition

Initial velocity of projectile which allows projectile at earth surface to escape from gravitational field of earth is called escape velocity.

Derivation

G.F of earth extends up to infinity. The projectile must have initial K.E that can move up to infinite distance in space

$$K.E = \text{Absolute } P.E$$

$$\frac{1}{2}mv^2 = \frac{GM_em}{R_e}$$

$$\frac{1}{2}mv^2_{esc} = \frac{GM_em}{R_e}$$

$$\frac{1}{2}v^2_{esc} = \frac{GM_e}{R_e}$$

$$v^2_{esc} = \frac{2GM_e}{R_e}$$

$$\sqrt{v^2_{esc}} = \sqrt{\frac{2GM_e}{R_e}}$$

$$\boxed{V_{esc} = \sqrt{\frac{2GM_e}{R_e}}} \quad \text{——— (1)}$$

According to Newton's Law of Gravitation

$$F = \frac{GM_em}{R_e^2} \quad \text{——— (2)}$$

According to Newton's 2nd Law of motion

$$F = mg \quad \text{——— (3)} \quad (a = g)$$

Comparing eq (2) And eq (3)

$$\frac{GM_em}{R_e^2} = mg$$

$$\frac{GM_e}{R_e^2} = g$$

$$GM_e = gR_e^2$$

Putting the value of GM_e in eq (1)

$$V_{esc} = \sqrt{\frac{2GM_e}{R_e}}$$

$$V_{esc} = \sqrt{\frac{2gR_e^2}{R_e}}$$

$$\boxed{V_{esc} = \sqrt{2gR_e}}$$

ESCAPE VELOCITY ON EARTH

$$V_{esc} = \sqrt{\frac{2GM_e}{R_e}}$$

$R_e = 6.4 \times 10^6 \, m$

$M_e = 6 \times 10^{24} \, kg$

$G = 6.67 \times 10^{-11} \, Nm^2 \, kg^{-2}$

$$V_{esc} = \sqrt{\frac{2(6.67 \times 10^{-11})(6 \times 10^{24})}{6.4 \times 10^6}}$$

$$V_{esc} = \sqrt{\frac{80.04 \times 10^{13}}{6.4 \times 10^6}}$$

$$V_{esc} = \sqrt{12.50 \times 10^7}$$

$$V_{esc} = \sqrt{12.50 \times 10^1 \times 10^6}$$

$$V_{esc} = \sqrt{125 \times 10^6}$$

$V_{esc} = 11.18 \times 10^3 \, m/s$

$V_{esc} = 11.2 \times 10^3 \, m/s$

$V_{esc} = 11.2 \, km/s$.

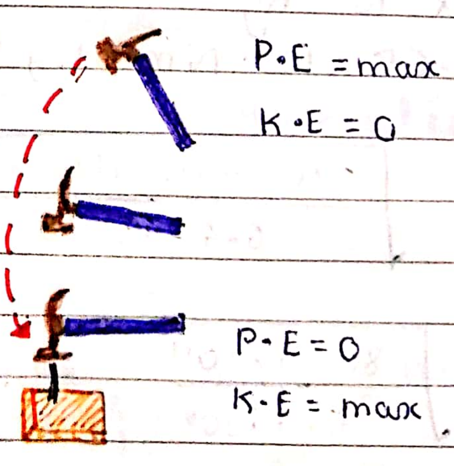
WORK ENERGY THEOREM IN RESISTIVE MEDIUM

→ There are 2 basic types of Energy:
1. Kinetic Energy
2. Potential Energy.

EXPLANATION

IN THE ABSENCE OF AIR FRICTION

* Consider a hammer which is raised to a certain height
* Hammer acquires gravitational P.E
* Its Gravitational P.E can be used to drive nail into the wood.
* If a hammer of weight 'w' is released, it will fall under the force of gravity F and will do work on the nail by driving it into the wood block
* work done = w x h
* Under the action of gravitational force, a hammer
  - loses its P.E
  - acquiring K.E while falling downward
* Before hitting the ground, $(P.E)_{hammer}$ = minimum
  $(K.E)_{hammer}$ = maximum.



MATHEMATICALLY

loss in P.E = Gain in K.E

$$mgh = \frac{1}{2}mv^2$$

> Conclusion: P.E of body decreases when an equal increase in K.E occurs

IN THE PRESENCE OF AIR FRICTION

- Frictional Forces reduces the mechanical energy (but not the total energy), they are called "**dissipative forces**".

- Friction causes the loss of energy.
- A part of P.E is used in work done against the friction. (fh) and remaining P.E is converted into K.E.

WHEN BODY MOVES DOWN

Loss in P.E = workdone against friction + gain in K.E

$$mgh = fh + \frac{1}{2}mv^2$$

WHEN BODY MOVES UP

Loss in K.E = work done against friction + gain in P.E

$$\frac{1}{2}mv^2 = fh + mgh$$

Conservation Of Energy

Statement

Energy can neither be created nor destroyed but can be transformed from one form into other.

⇒ Energy remains constant

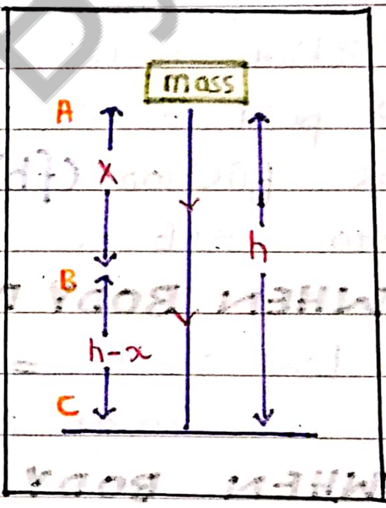
Mathematically

Total Energy = K.E + P.E = constant

Interconversion of K.E and P.E

Explanation

Consider a brick whose mass is 'm'. and it is at point A at height 'h' on the ground. Then its brick start falling and after coming at distance 'x', it is at poin B and has a height of h-x. We have point C when the brick hits the ground.



K.E = 0
P.E = mgh

K.E = mgx
P.E = mg(h-x)

K.E = mgh
P.E = 0

Point A [Object Start Falling]

$(T.E)_A = (K.E)_A + (P.E)_A$

$(T.E)_A = \frac{1}{2} m v_A^2 + mgh_A$

At A : object is at Rest ($v_A = 0$)

At A : $h_A = h$

$(T.E)_A = \frac{1}{2} m (0)^2 + mg(h)$

$(T.E)_A = mgh$

POINT B [WHEN OBJECT IS B/W A and C]

$(T.E)_B = (K.E)_B + (P.E)_B$

$(T.E)_B = \frac{1}{2} m v_B^2 + mgh_B \qquad h_B = h - x$

$2aS = v_f^2 - v_i^2$

$2gx = v_B^2 - v_A^2$

$2gx = v_B^2 - 0^2$

$2gx = v_B^2$

$(T.E)_B = \frac{1}{2} m (2gx) + mg(h-x)$

$(T.E)_B = mgx + mgh - mgx$

$(T.E)_B = mgh$

POINT C [BEFORE HITTING THE GROUND]

$(T.E)_C = (K.E)_C + (P.E)_C$

$(T.E)_C = \frac{1}{2} m v_C + mgh_C \qquad h_C = 0$

$2aS = v_f^2 - v_i^2$

$2gh = v_C^2 - v_A^2$

$2gh = v_C^2 - 0^2$

$v_C^2 = 2gh$

$(T.E)_C = \frac{1}{2} m (2gh) + mg(0)$

$(T.E)_C = mgh$

CONCLUSION

1. Ignoring Air Friction

loss in P.E = gain in K.E

$mgh_1 - mgh_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

$mg(h_1 - h_2) = \frac{1}{2} m (v_2^2 - v_1^2)$

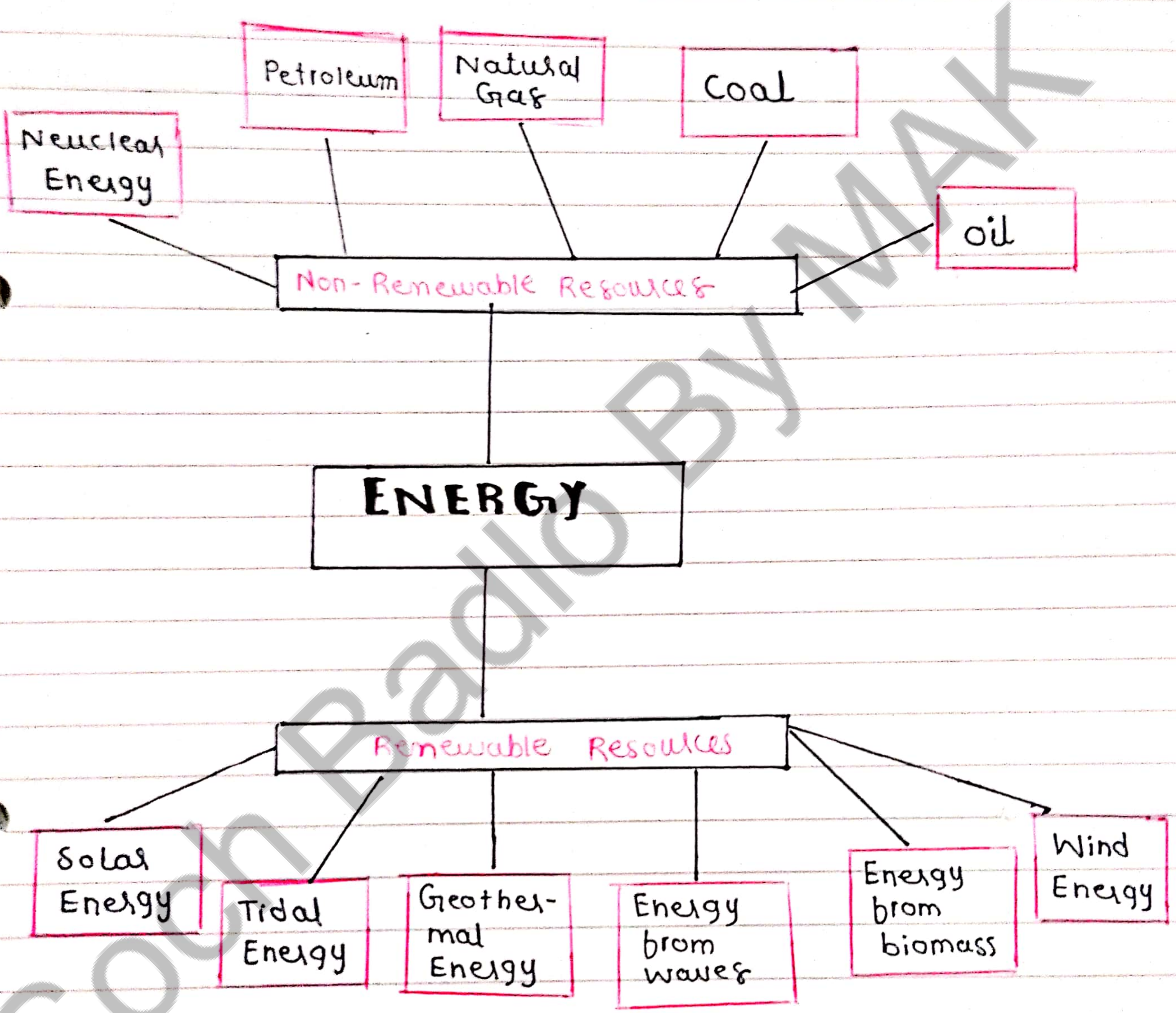
Ideal Case

2. In the presence of friction

loss in P.E = gain in K.E + 'W' against 'f'

$mgh = \frac{1}{2} m v^2 + fh$

Non Renewable Resources	Renewable Resources
Definition	
Non-Renewable resources can be described as convential sources of energy which have been used since a long time	An energy source that cannot be depleted and are able to supply a continous source of clean energy.
Depletion	
deplete over time	can't be depleted overtime
Sources	
Natural gas, oil, coal, petroleum.	Geothermal energy, Tidal energy. Energy from biomass
Environmental Impact	
has higher carbon emission	has low carbon emission
Cost	
high cost	low cost
Usage	
cannot be used again and again	can be used again and again
Pollution	
Not pollution free	Pollution free
Limit	
Limited	Unlimited


Neuclear Energy
Petroleum
Natural Gas
Coal
oil
Non-Renewable Resources
ENERGY
Renewable Resources
Solar Energy
Tidal Energy
Geother-mal Energy
Energy from waves
Energy from biomass
Wind Energy