SLO Based Quick Revision Notes of Forces & Motion

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REST	MOTION
definition	
→ A body is said to be at rest if it does not change its position with respect to its surroundings.	→ A body is said to be in motion if it changes its position with respect to its surroundings
example	
→ For example, a passenger sitting in a moving bus is at rest according to an observer inside the bus	→ To an observer outside the moving bus, the passengers and the objects inside the bus are in motion.

∴ **NOTE:** The specification of the observer is important while interferring about the state of rest or motion of the body

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DISPLACEMENT:

→ **definition:**

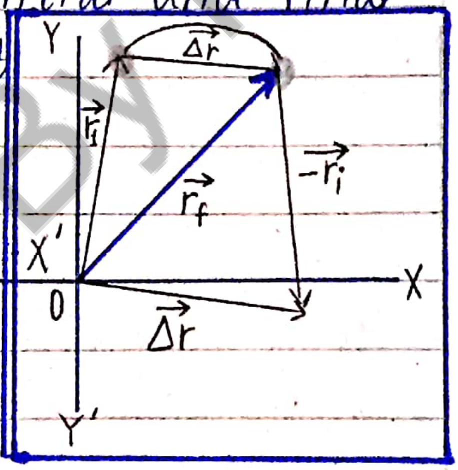
→ Displacement is the shortest directed distance between two ~~points~~ positions

→ **Unit:** meter(m) **(Vector Quantity)**

→ **Denoted by:** $\vec{\Delta x}$, $\vec{\Delta r}$, $\vec{\Delta s}$, $\vec{\Delta l}$ or $\vec{\Delta d}$

→ **Magnitude:** The magnitude of the displacement vector is the shortest distance between the initial and final positions of the object

→ **Formula:** $\boxed{\vec{\Delta r} = \vec{r_f} - \vec{r_i}}$



→ The displacement Δr of ~~x~~ the object is the vector drawn from the initial position A to final position B.

Velocity:

→ **definition:**

→ Measure of displacement covered $(\vec{\Delta s})$ with passage of time (Δt) is called velocity.

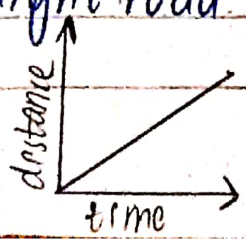
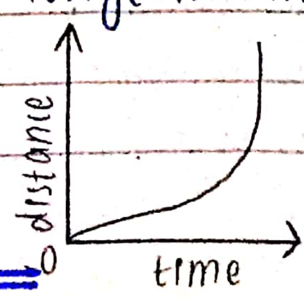
→ **Unit:** metre per second (m/s) **(Vector quantity)**

→ **Denoted by:** $\vec{V}$

→ **Formula:** velocity $(\vec{V})$ $= \dfrac{\text{displacement}}{\text{elapsed time}} = \dfrac{\vec{S_f} - \vec{S_i}}{t_f - t_i}$

$$\boxed{\vec{V} = \frac{\Delta \vec{S}}{\Delta t}}$$

Average Velocity $\langle \vec{V} \rangle$	Instantaneous Velocity $\vec{V}_{inst}$
definition	
→ Average velocity is the net (total) displacement ($\vec{S}$) divided by total time (t)	→ Velocity at particular instant of time is called instantaneous velocity.
formula	
$\langle \vec{V} \rangle = \dfrac{\text{Total displacement}}{\text{Total time}} = \dfrac{\vec{S}}{t}$	$\vec{V} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta \vec{S}}{\Delta t}$

Uniform Velocity	Variable Velocity
~~If a I~~	
definition	
→ If a body covers equal displacement in equal intervals of time.	→ If a body covers unequal displacement in equal intervals of time.
condition	
→ Both the speed and direction remain constant	→ The variable velocity is due to change in speed or direction or both
example	
→ A car moving at a constant speed of 30 kmh^{-1} in a straight road	→ A fan rotating (change in direction)
	

Acceleration: (Time rate of change in velocity)

→ **definition:**

→ The measure of change in velocity ($\Delta\vec{v}$) with the passage of time (Δt) is called acceleration.

→ **Unit:** meter per second squared (m/s^2) (Vector quantity)

→ **Formula:** $\vec{a} = \frac{\text{change in velocity}}{\text{elapsed time}} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$

$$\boxed{\vec{a} = \frac{\Delta\vec{v}}{\Delta t}}$$

→ Measure of how rapidly the velocity is changing.

Average Acceleration	Instantaneous Acceleration
definition	
→ Average acceleration is the net (total) velocity ($\vec{v}$) divided by total time.	→ Acceleration at particular instant of time is known as instantaneous acceleration
formula	
→ $\langle\vec{a}\rangle = \frac{\vec{v}}{t}$	$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$

→ **UNIFORM & VARIABLE ACCELERATION:**

→ A body is said to be have uniform acceler-ation if its velocity changes by equal amount in equal intervals of time.

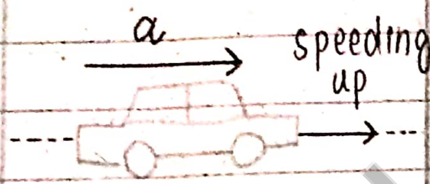
→ RETARDATION:

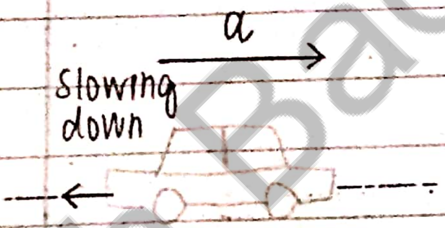
→ There is retardation when:

- When the object is slowing down
- Whenever the magnitude of velocity is decreasing
- When velocity and acceleration point in opposite directions.

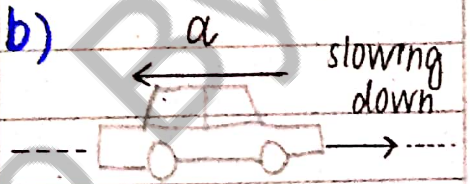
Positive Acceleration	Negative Acceleration
a)	b)

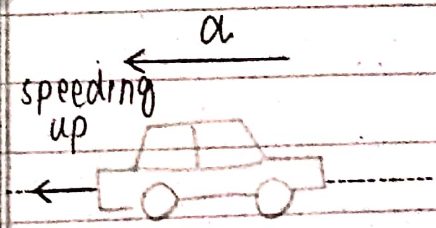
a)





b)





→ Conservation of Momentum:

→ For an isolated system there is no net force acting F = 0, therefore Newton 2nd law in terms of momentum.

$$0 = \frac{\Delta \vec{P}}{\Delta t} \quad \text{or} \quad 0 = \frac{\vec{P}_f - \vec{P}_i}{\Delta t} \; ; \; 0 = \vec{P}_f - \vec{P}_i$$

therefore, $\boxed{\vec{P}_f = \vec{P}_i}$

Graphical Analysis of Motion

→ Graph is an effective way of showing relationship between physical quantities by coordinate systems

Displacement Time graph	Velocity Time graph

Slope gives

→ The slope gives velocity

→ The slope gives info about acceleration

Negative slope

→ A negative slope means motion is in the negative direction.

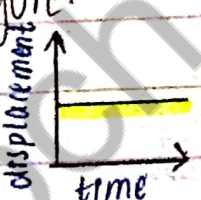
→ If slope is negative, acceleration will be negative

Positive slope

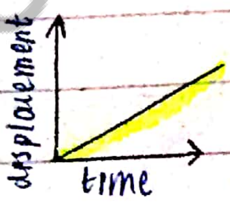
→ A positive slope means motion is in the positive direction

→ If slope is positive, acceleration will be positive

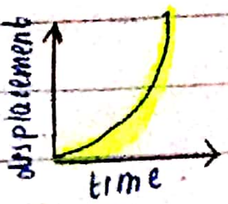
figure:



zero velocity



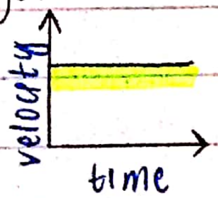
uniform velocity

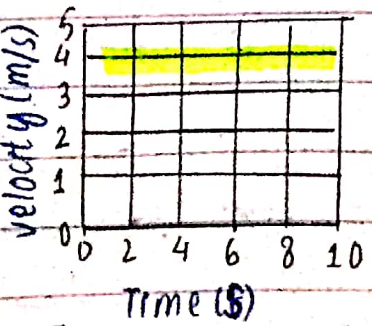


variable velocity

figure:

∴ shallow slope represents slow change in velocity





∴ The area under velocity represents the displacement

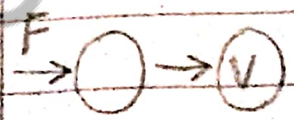
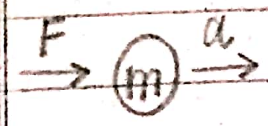
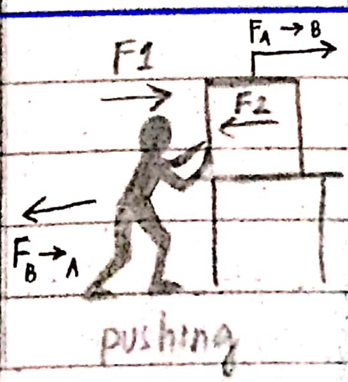
→ Equations for Uniformly Accelerated Motion:

1st :	$v_f = v_i + at$
2nd :	$S = v_i t + \frac{1}{2}at^2$
3rd :	$2aS = v_f^2 - v_i^2$

where,
S : displacement
v_i : initial velocity
v_f : final velocity
a : acceleration
t : time

∴ The equations for uniformly accelerated motion can also be applied to free fall motion of the objects by replacing 'a' by 'g'.

→ Newton's Law's of Motion:

	1st law	2nd law	3rd law
def:	A body at rest will remain at rest and a body in motion will remain in motion unless it is acted upon by an external force	The force acting on an object is equal to the mass of that object times its acceleration.	For every action, there is an equal and opposite reaction.
Mathematically:	$\vec{a} = 0$	$\vec{F} = m\vec{a}$	$\vec{F}_{A\to B} = -\vec{F}_{B\to A}$
figure:			

→ LINEAR MOMENTUM:

→ **definition:**

→ The linear momentum $\vec{P}$ of an object is the product of the object's mass m and velocity $\vec{v}$.

→ **unit:** $Kgms^{-1}$ or Ns

→ **formula:** $\vec{P} = m\vec{v}$

→ **Quantity:** vector quantity

→ Newton's 2nd law & Linear Momentum:

→ By Newton's 2nd law, $\vec{F} = m\vec{a}$ — ①

By definition of acceleration $a = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$ — ②

Putting ~~① and~~ ② in ①,

$$\vec{F} = m\left[\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right] = \left[\frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}\right] = \left[\frac{\vec{P}_f - \vec{P}_i}{\Delta t}\right]$$

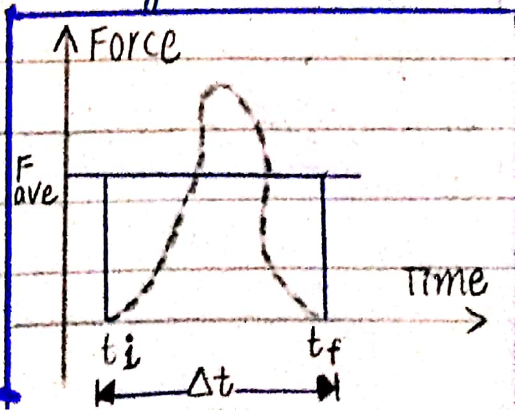
therefore, $\boxed{\vec{F} = \dfrac{\Delta \vec{P}}{\Delta t}}$

→ "The time rate of change of linear momentum of a body is equal to the force acting on the body"

→ IMPULSE AND CHANGE OF MOMENTUM:

"The impulse 'J' is the product of the force '$\vec{F}$' and the time interval 'Δt' during which the force acts.

$$\boxed{\vec{J} = \vec{F} \times \Delta t}$$



Since, $\vec{F} = \dfrac{\Delta \vec{P}}{\Delta t}$

Therefore, $\vec{J} = \Delta \vec{P}$

or, $\boxed{\vec{J} = m\vec{v}_f - m\vec{v}_i}$

COLLISIONS:

definition:

→ An event during which particles come close to each other and interact by means of forces is called collision.

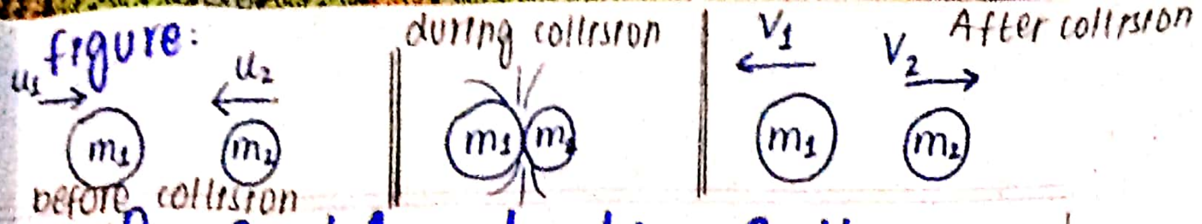
→ **condition:** For collision to occur, the colliding object must not necessarily touch.

→ **example:** collision between a proton and nucleus of an atom

TYPES:

Elastic Collision ($KE_{initial} = KE_{final}$)	**Inelastic Collision** ($KE_{initial} \neq KE_{final}$)
Momentum	
• Total momentum is conserved.	• Total momentum is conserved.
Kinetic energy	
• Total kinetic energy is conserved.	• Total kinetic energy is NOT conserved.
Forces Conservation	
• Forces involved are conservative forces	• Forces involved are NON-conservative forces
Energy Dissipation	
• Mechanical energy is NOT dissipated.	• Mechanical energy is dissipated into heat, light, sound etc.
Example	
• Collisions between billiard balls	• Mud ball thrown on a rigid wall

figure:



→ Perfectly elastic Collision in One Dimensions:

→ definition:

→ The elastic collision in which the two objects move along the same line before and after collision.

→ energy conservation:

→ Kinetic energy of objects is conserved.

→ examples:

→ Atomic behaviour in Rutherford scattering

→ CONSIDERATION:

By law of conservation of momentum; $\vec{P_i} = \vec{P_f}$

① — or $m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$

② — $m_1(u_1 - V_1) = m_2(V_2 - u_2)$

Since, $KE_i = KE_f$

$$\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 V_2^2$$

(rearranging)

$$\frac{1}{2}m_1 u_1^2 - \frac{1}{2}m_1 V_1^2 = \frac{1}{2}m_2 V_2^2 - \frac{1}{2}m_2 u_2^2$$

therefore,

$$m_1(u_1^2 - V_1^2) = m_2(V_2^2 - u_2^2) \quad \text{— ③}$$

Dividing eq ③ by eq ②, we get

$$\frac{m_1(u_1^2 - V_1^2)}{m_1(u_1 - V_1)} = \frac{m_2(V_2^2 - u_2^2)}{m_2(V_2 - u_2)}$$

$$\frac{(u_1^2 - V_1^2)}{(u_1 - V_1)} = \frac{(V_2^2 - u_2^2)}{(V_2 - u_2)}$$

As, $a^2 - b^2 = (a+b)\times(a-b)$

Therefore,

$$\frac{(u_1 + V_1)(u_1 - V_1)}{(u_1 - V_1)} = \frac{(V_2 + u_2)(V_2 - u_2)}{(V_2 - u_2)}$$

therefore,

$$\boxed{u_1 - u_2 = -(V_1 - V_2)}$$

$$\boxed{u_{rel} = -V_{rel}}$$

$$V_2 = u_1 + V_1 - u_2 \quad \text{— ⑤}$$

Putting eq ⑤ in eq ①, we get,

$$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2(u_1 + V_1 - u_2)$$

or,

$$m_1 u_1 + m_2 u_2$$

$$m_1 V_1 + m_2 V_1 = -m_2 u_1 + m_2 u_2$$

or,

$$(m_1 + m_2)V_1 = (m_1 - m_2)u_1 + 2m_2 u_2$$

dividing b.s by $m_1 + m_2$,

$$\boxed{V_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \frac{2m_2}{(m_1 + m_2)}u_2}$$

substituting value of V_1 from eq ④ in eq ①,

$$\boxed{V_2 = \frac{2m_1}{(m_1 + m_2)}u_1 - \frac{(m_1 - m_2)}{(m_1 + m_2)}u_2}$$

Momentum & Explosive forces

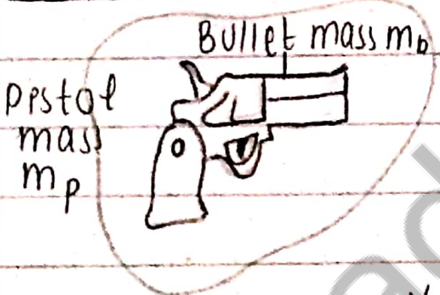
→ Explosion:

→ An explosion is a sudden, intense release of energy that often produces a loud noise, high temperature, and flying pieces, and generates a pressure wave.

→ **Mathematically:** $P_i = P_f$

A
Firing of Pistol

$\boxed{P_i = 0}$



$\boxed{P_f = 0}$

→ Pistol recoil when they fire a bullet

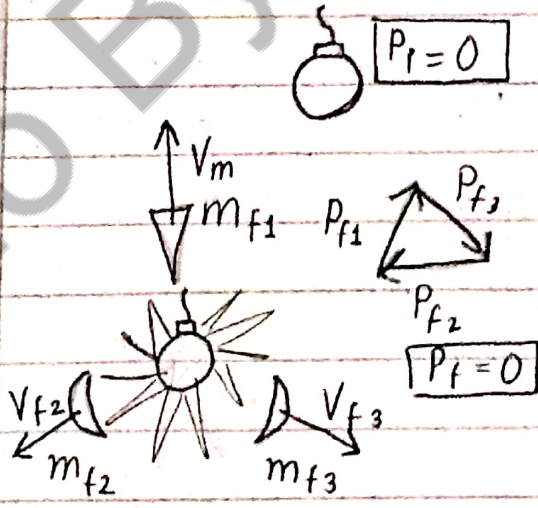
$P_f = 0$

or $m_b V_b + m_p V_p = 0$

therefore,

$$m_b V_b = -m_p V_p$$

B
Explosion of Explosive material



→ Bomb explosion

Projectile Motion

→ definition:

→ Two dimensional motion of a body under the action of gravity and inertia is called projectile motion.

$$\boxed{\text{Projectile motion} = \text{Horizontal Motion} + \text{Vertical Motion}}$$

Horizontal Motion	**Vertical Motion**
$V_{ix} = V_i \cos\theta$	$V_{iy} = V_i \sin\theta$
$a_x = 0$	$a_y = -g$
$V_{fx} = V_f \cos\theta$	$V_y = V_i \sin\theta - gt$
$x = (V_i \cos\theta)t$	$y = (V_i \sin\theta)t - \frac{1}{2}gt^2$

→ Example:

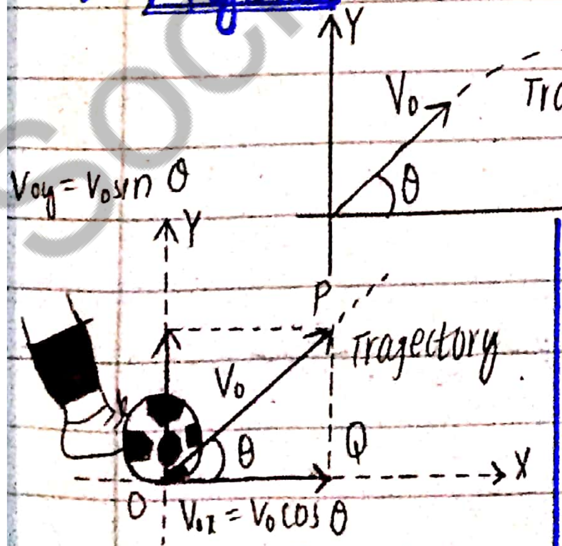
- A ball thrown from some height.
- A football kicked off by a player.

→ Trajectory:

→ The pathway followed by projectile is called trajectory.

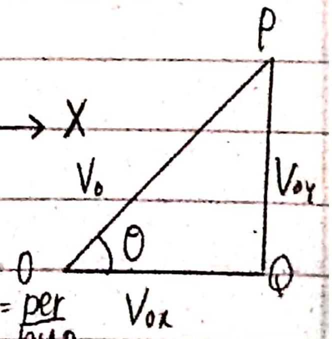
→ Figure:

$a_x = 0 ; a_y = -g$



projectile motion

From the right angle ΔOPQ, we can write that



$\cos\theta = \frac{base}{hyp}$

$base = hyp \cos\theta$

$V_{0x} = V_0 \cos\theta$

$\sin\theta = \frac{per}{hyp}$

$per = hyp \sin\theta$

$V_{0y} = V_0 \sin\theta$

A. Velocity

→ $V_x = V_0 \cos\theta$ →(1) $V_y = V_0 \sin\theta - gt$ →(2)

Magnitude:

$$\boxed{V = \sqrt{(V_0 \cos\theta)^2 + (V_0 \sin\theta - gt)^2}}$$

Direction:

$$\boxed{\phi = \tan^{-1} \frac{V_0 \sin\theta - gt}{V_0 \cos\theta}}$$

B. Maximum Height:

→ Maximum ~~Height~~ Vertical distance reached by projection level is called maximum height of projectile.

$$\boxed{H = \frac{V_0^2 \sin^2\theta}{2g}}$$

C. Time of Flight:

→ Time taken by projectile to go from point of projection to the point of impact is called time of flight of projectile.

$$\boxed{T = \frac{2 V_0 \sin\theta}{g}}$$

D. Range:

→ The horizontal distance from point of projection to point of impact is called range of projectile

$$\boxed{R = \frac{V_0^2}{g} \sin 2\theta}$$

Maximum range Angle: $\theta_{max} = 45^\circ$