(Forces And Motion)

Rest	Motion
Definition	
A body is at rest w.r.t an observer if does not change its position w.r.t observer	A body is in motion w.r.t an observer if it changes its position w.r.t an observer.
Examples	
1. 2 passangers are at rest w.r.t each other in a moving bus.	1. Passangers in a moving . are in motion w.r.t outsider.
2. A book lying on the table	2. A flying bird.

Note: The state of rest and motion of the body is relative to each other. A body can be both at rest and in motion at the same time.

Example: • 2 passengers in a moving bus are at rest w.r.t each other but in motion w.r.t outside observer.

Similarly:

- A person sitting on a chair is at rest (speed = 0) but it is in motion (speed = 30 kms^{-1}) w.r.t sun.

DISPLACEMENT

DEFINITION

"Shortest directed distance b/w 2 positions is called displacement"

EXPLANATION:

NATURE

Displacement is a vector quantity.

- The magnitude of Δd vector is the shortest distance b/w the initial and final positions
- It is directed from initial to final position

NOTATION

Donated by $\Delta\vec{x}$, $\Delta\vec{r}$, $\Delta\vec{s}$, $\Delta\vec{l}$, $\Delta\vec{d}$

SI UNIT

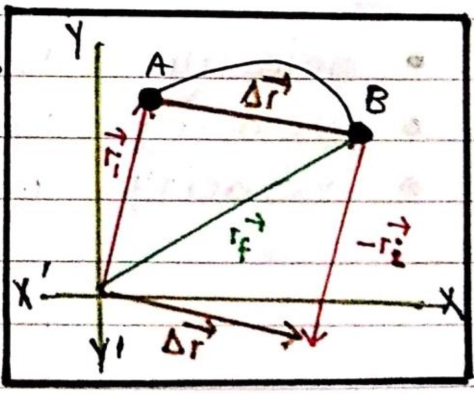
SI unit of $\Delta\vec{d}$ is 'm'

DIMENSION

[L]

EXAMPLE

→ Consider the motion of object moves from initial position A to final position B.



- These positions are drawn from arbitrary coordinate origin 'O'

$\vec{r_i}$ = position A from origin O

$\vec{r_f}$ = position B from origin O

$\Delta\vec{r}$ = vector drawn from the initial position A to the final position B

$$\Delta\vec{r} = \vec{r_f} - \vec{r_i}$$

VELOCITY ($\vec{v}$)

DEFINITION

"Measure of displacement covered ($\Delta\vec{s}$) with passage of time (Δt) is called velocity."

MATHEMATICAL FORM

$$\vec{v} = \frac{\text{displacement}}{\text{time}}$$

$$\vec{v} = \frac{\vec{S_f} - \vec{S_i}}{t_f - t_i}$$

$$\vec{v} = \frac{\Delta\vec{S}}{\Delta t}$$

SI UNIT

SI Unit of velocity is ms^{-1}.

DIMENSION

$\vec{v} = ms^{-1} \implies \vec{v} = [LT^{-1}]$

NATURE

Velocity is vector quantity having same direction as displacement vector.

- Velocity is +ive if displacement is +ive.
- velocity is -ive if displacement is -ive.
- Velocity is 0 if displacement is 0.

TYPES OF VELOCITY

AVERAGE VELOCITY $\langle \vec{v} \rangle$

"Total displacement divided by total time."

$$\langle \vec{v} \rangle = \frac{\vec{s}}{t}$$

INSTANTANEOUS VELOCITY $\vec{v}_{inst}$

"Velocity at a particular instant of time"

→ change in $(\Delta\vec{s})$ in short interval of time (Δt) which approaches to zero.

$$\vec{v}_{ins} = \lim_{\Delta t \to 0} = \frac{\Delta \vec{s}}{t}$$

UNIFORM VELOCITY

"If body covers equal displacements in equal interval of time, a body is said to be moving with uniform velocity.

At uniform velocity:

$$\langle \vec{v} \rangle = \vec{v}_{inst}$$

VARIABLE VELOCITY

If a body does not cover equal displacement in equal intervals of time then body is said to be moving with variable $\vec{v}$.

In Non-uniform velocity:

$\langle \vec{v} \rangle$ and $\vec{v}_{ins}$ are different.

ACCELERATION

DEFINITION

'The measure of change in $\vec{v}$ with the passage of time is called acceleration'

MATHEMATICAL FORM

$$\vec{a} = \frac{\text{change in velocity}}{\text{time}}$$

$$\vec{a} = \frac{\vec{v_f} - \vec{v_i}}{t_f - t_i}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

NATURE

Acceleration is a vector quantity having same direction as change in velocity.

SI UNIT

SI unit of $\vec{a}$ is 'ms^{-2}'

DIMENSION

$[LT^{-2}]$

$\vec{a}$ is a measure of how rapidly the velocity is changing.

TYPES OF ACCELERATION

AVERAGE $\vec{a}$

"The total $\vec{v}$ divided by time"

$$\langle \vec{a} \rangle = \frac{\vec{v}}{t}$$

INSTANTANEOUS $\vec{a}$

"$\vec{a}$ at a particular instant of time"

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

UNIFORM $\vec{a}$

- When $\vec{v}$ changes by equal amount in equal intervals of time.
- In uniform $\vec{a}$:

$$\langle \vec{a} \rangle = \vec{a}_{inst}$$

VARIABLE $\vec{a}$

When $\vec{v}$ changes by unequal amount in equal intervals of time

+tive $\vec{a}$

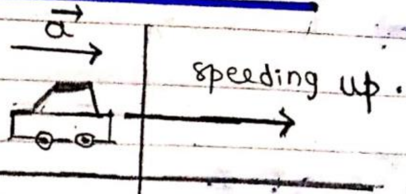
When $\vec{v}$ of a body is increasing the resulting $\vec{a}$ is called +tive $\vec{a}$

NEGATIVE $\vec{a}$

- When an object is slowing down, there is -ive $\vec{a}$.
- -ive $\vec{a}$ is also called deceleration or retardation
- In deceleration, the magnitude of $\vec{v}$ is decreasing.

Direction Of Acceleration

Case-1



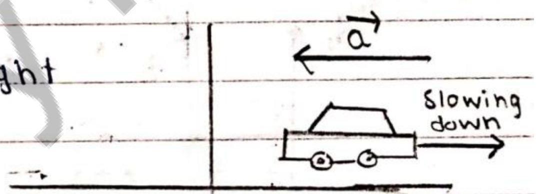
→ When car is moving toward right (+ive x-axis)

→ velocity is increasing.

→ has accelerated motion.

+ive $\vec{a}$ toward right

Case-2



→ When car is moving toward right (+ive x-axis)

→ velocity is decreasing

→ has retarted motion.

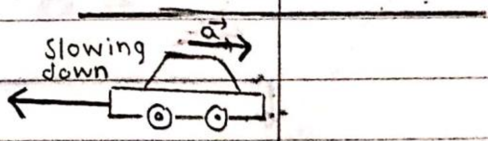
-ive $\vec{a}$ toward left

Case-3

→ When car is moving toward left (-ive x-axis)

→ velocity is decreasing

→ has retarted motion



+ive acceleration

Case-4

→ When car is moving toward left (-ive x-axis)

→ velocity is increasing

→ has accelerated motion

-ive $\vec{a}$

{EQUATIONS OF MOTION}

Motion In Horizontal Line

Consider a body moving in horizontal line As body is moving in straight line so direction of motion does not change.

FIRST EQUATION

$$V_f = V_i + at$$

SECOND EQUATION

$$S = V_i t + \frac{1}{2} at^2$$

THIRD EQUATION

$$2aS = V_f^2 - V_i^2$$

Motion In Vertical Line

In the absence of air resistance, all objects free fall near the surface of Earth with acceleration due to gravity.

VERTICALLY DOWNWARD MOTION

1. $V_f = V_i + gt$
2. $h = V_i t + \frac{1}{2} g t^2$
3. $2gh = V_f^2 - V_i^2$

VERTICALLY UPWARD MOTION

1. $V_f = V_i - gt$
2. $h = V_i - \frac{1}{2} g t^2$
3. $2gh = V_f^2 - V_i^2$

NEWTON'S LAWS OF MOTION

FIRST LAW OF MOTION

STATEMENT

"A body at rest will remain at rest and a body moving with uniform velocity will continue to do so unless acted upon by external force."

EXPLANATION

CASE-I

When object is at rest it will remain at rest unless an external force act on it

EXAMPLE

A book laying on the table will remain at rest until an external force act on it.

CASE-II

When an object is moving, it will keep moving unless an external force act on it.

EXAMPLE

A ball rolling on the ground will keep rolling until an external force acts on it

MATHEMATICALLY

If $F_{net} = 0$

then $\Delta V = 0\ m/s$

$a = 0\ m/s^2$

Newton's 1st law of motion is also known as "law of inertia

mass $\propto$ inertia

(SECOND LAW OF MOTION)

STATEMENT

A net force applied on the body produces acceleration. The acceleration is directly proportional to the magnitude of net force and inversely proportional to the mass of the object.

MATHEMATICALLY

$a \propto F$ ——(i)

$a \propto \frac{1}{m}$ ——(ii)

Combining eq (i) and (ii)

$$a \propto \frac{F}{m}$$

$$a = k\frac{F}{m}$$

$$a = (1)\frac{F}{m}$$

$$a = \frac{F}{m}$$

$$F = ma$$

Forces are unbalanced
↓
There is an acceleration
↙ ↘
$a \propto F$ $a \propto \frac{1}{m}$

⟹ The $\vec{a}$ produced is in the same direction as that of the net force

~(THIRD LAW OF MOTION)~

STATEMENT

"Every action has an equal and opposite reaction".

OR

"When 2 objects interact, the forces they exert on each other are equal and opposite."

EXPLANATION

→ Action is a force exerted by first body on second body.

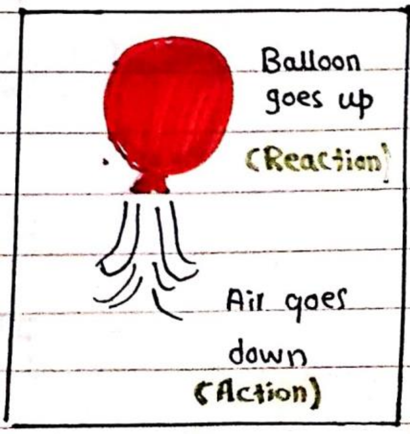
→ Reaction is a force exerted by the second body on to the first body.

→ Action and reaction do not act on the same body

$$F_{action} = -F_{action}$$

EXAMPLE

When ball is set free, the air inside rushes out and balloon moves forward



(LINEAR MOMENTUM)

"Linear $\vec{P}$ is the product of mass and linear velocity."

MATHEMATICALLY

$$\vec{P} = m\vec{V}$$

SI UNIT

$$\vec{P} = m\vec{V}$$

$$\vec{P} = kg\ ms^{-1}$$

OR $\vec{P} = Ns$

NATURE

- vector quantity
- points in same direction as velocity.

MOMENTUM IN TERMS OF NEWTON'S SECOND LAW

According to Newton's Second Law.

$$\vec{F} = m\vec{a} \quad \text{—— (i)}$$

As acceleration is:

$$\vec{a} = \frac{v_f - v_i}{\Delta t} \quad \text{—— (ii)}$$

Putting value of 'a' from eq (ii) in (i).

$$\vec{F} = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right)$$

$$\vec{F} = \frac{mv_f - mv_i}{\Delta t}$$

$$\vec{F} = \frac{P_f - P_i}{\Delta t}$$

$$\vec{F} = \frac{\Delta \vec{P}}{\Delta t}$$

"The rate of change of linear momentum is equal to the force acting on the body"

IMPULSE AND CHANGE OF MOMENTUM

IMPULSE ($\vec{J}$)

"The product of force '$\vec{F}$' and time interval during which force acts is called impulse"

MATHEMATICAL FORM

$$\vec{J} = \vec{F} \times \Delta t$$

NATURE

- Vector quantity
- Same direction as the average force

SI UNIT

N s

EXPLANATION

Describe the effect of how large force is acting and for how long force acts.

→ large force can act for small interval of time
→ small force can act for long interval of time

↑F	t↓	J↓

↓F	t↑	J↓

RELATION OF '$\vec{J}$' WITH 'ΔP'

$$\vec{J} = \vec{F}_{avr} \times \Delta t$$

$$\vec{J} = \frac{\Delta \vec{P}}{\Delta t} \times \Delta t$$

$$\vec{J} = \Delta \vec{P}$$

CONSERATION OF MOMENTUM

STATEMENT

"In an isolated system, the final momentum must be equal to the initial momentum"

MATHEMATICAL EXPLANATION

Force = Rate of change of momentum

$$F = \frac{\Delta \vec{P}}{\Delta t}$$

$$F = \frac{\vec{P_f} - \vec{P_i}}{\Delta t}$$

In isolated system, $F=0$

$$0 = \frac{\vec{P_f} - \vec{P_i}}{\Delta t}$$

$$0 \times \Delta t = \vec{P_f} - \vec{P_i}$$

$$0 = \vec{P_f} - \vec{P_i}$$

$$\vec{P_f} = \vec{P_i}$$

An isolated system is a collection of particles that can interact with each other but whose interaction with environment outside the collection have a negligible effect on their motion

MOMENTUM AND EXPLOSIVE FORCES

"An explosion is a sudden, intense release of energy that often produces a loud noise, high temperature and flying pieces and generates a pressure wave"

→ In isolated system, total momentum during explosion is conserved

$$P_i = P_f$$

EXAMPLE

FIRING OF PISTOL

- Consider an isolated system of pistol and bullet.

m_p = mass of pistol

m_b = mass of

Before firing → Total momentum = 0

- After firing : * bullet moves with velocity 'v_b' in one direction
  * pistol recoils with 'v_p' in opposite direction

After firing → Total momentum = 0

$P_b + P_p = 0$

$m_b v_b + m_p + v_p = 0$

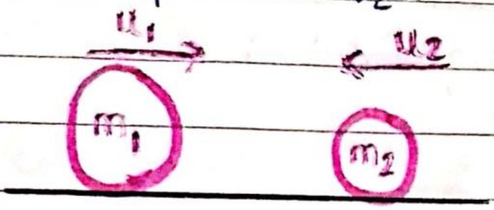
$m_b v_b = - m_p + v_p$

$P_b = - P_p$

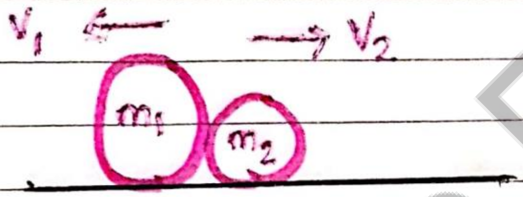
$$v_p = -\frac{m_b v_b}{m_p}$$

PERFECTLY ELASTIC COLLISION IN ONE DIMENSION

Consider 2 bodies of masses m_1 and m_2 moving with velocities u_1 and u_2. When the 2 bodies collide, they move with velocities v_1 and v_2.



$(P_1)_i = m_1u_1$ = momentum of 1st ball before collision
$(P_2)_f = m_2u_2$ = momentum of 2nd ball before collision



⇒ Due to Reactional force, they will move in opposite direction.

$P_1' = m_1v_1$ = momentum of 1st ball after collision
$P_2' = m_2v_2$ = momentum of 2nd ball after collision

According to law of conservation of momentum

$$P_i = P_f$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 - m_1v_1 = m_2u_2 - m_2v_2$$

$$m_1(u_1 - v_1) = m_2(u_2 - v_2) \quad \text{——— (A)}$$

According to law of conservation of Energy

$$(K.E)_i = (K.E)_f$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\frac{1}{2}(m_1 u_1^2 + m_2 u_2^2) = \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2)$$
$$m_1 u_1^2 - m_1 v_1^2 = m_2 v_2^2 - m_2 u_2^2$$
$$m_1 (u_1^2 - v_1^2) = m_2 (v_2^2 - u_2^2)$$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$$m_1 (u_1 + v_1)(u_1 - v_1) = m_2 (v_2 + u_2)(v_2 - u_2) \quad \text{—— (B)}$$

Dividing eq B by A

$$\frac{m_1 (u_1 + v_1)(u_1 - v_1)}{m_1 (u_1 - v_1)} = \frac{m_2 (v_2 + u_2)(v_2 - u_2)}{m_2 (v_2 - u_2)}$$

$$u_1 + v_1 = v_2 + u_2$$
$$u_1 - u_2 = v_2 - v_1$$
$$u_1 - u_2 = -(v_1 - v_2)$$

Relative Speed of Approach = Relative Speed of separation

$$v_1 = u_2 + v_2 - u_1 \quad \text{—— (C)}$$
$$v_2 = u_1 + v_1 - u_2 \quad \text{—— (D)}$$

Velocity of 1st ball after collision (v_1)

$\star_1$ $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

Putting the value of v_2

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 (u_1 + v_1 - u_2)$$
$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 u_1 + m_2 v_1 - m_2 u_2$$
$$m_1 u_1 + m_2 u_2 - m_2 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_1$$
$$m_1 v_1 + m_2 v_1 = m_1 u_1 + m_2 u_2 - m_2 u_1 + m_2 u_2$$

$$v_1(m_1+m_2) = u_1(m_1-m_2) + 2m_2u_2$$

$$\boxed{v_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1 + \left(\frac{2m_2}{m_1+m_2}\right)u_2}$$

Velocity of 2nd ball after collision (v_2)

$*_1$ $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Putting the value of v_1

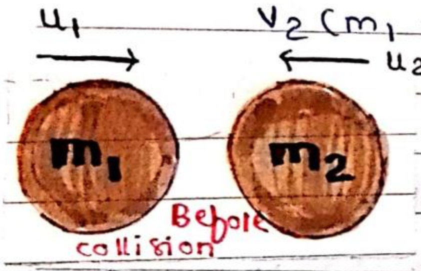
$$m_1u_1 + m_2u_2 = m_1(u_2 + v_2 - u_1) + m_2v_2$$

$$m_1u_1 + m_2u_2 = m_1u_2 + m_1v_2 - m_1u_1 + m_2v_2$$

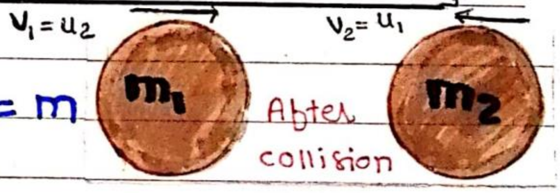
$$m_1u_1 + m_2u_2 - m_1u_2 + m_1u_1 = m_1v_2 + m_2v_2$$

$$m_1v_2 + m_2v_2 = m_1u_1 + m_2u_2 - m_1u_2 + m_1u_1$$

$$v_2(m_1+m_2) = 2m_1u_1 - (m_1-m_2)u_2$$



$$\boxed{v_2 = \left(\frac{2m_1}{m_1+m_2}\right)u_1 - \left(\frac{m_1-m_2}{m_1+m_2}\right)u_2}$$



CASE-I If $m_1 = m_2 = m$

$$v_1 = \frac{2m_2u_2}{m_1+m_2} + \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1$$

$$v_1 = \frac{2mu_2}{m+m} + \left(\frac{m-m}{m+m}\right)u_1$$

$$v_1 = \frac{2mu_2}{2m} + 0$$

$$v_1 = u_2 \text{ ms}^{-1}$$

➡ Velocity of 1st ball after collision is equal to velocity of 2nd ball before collision [interchange]

$$v_2 = \frac{2m_1u_1}{m_1+m_2} + \left(\frac{m_2-m_1}{m_1+m_2}\right)u_2$$

$$v_2 = \frac{2mu_1}{m+m} + \left(\frac{m-m}{m+m}\right)u_2$$

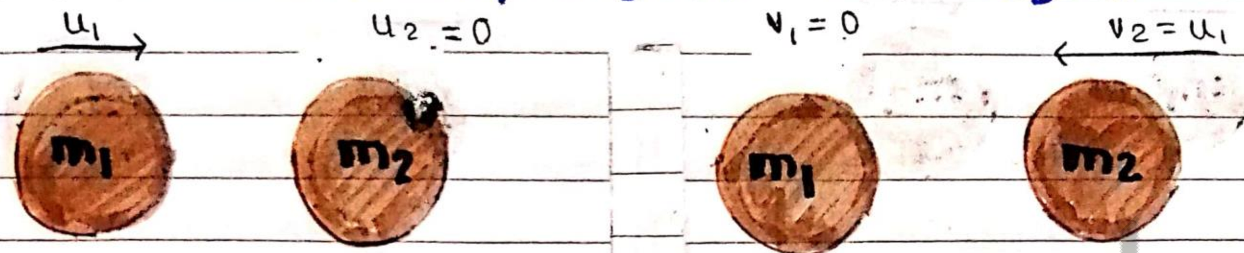
$$\frac{2m}{2m}(u_1) + 0$$

$$v_2 = \frac{2mu_1}{2m} + 0$$

$$v_2 = u_1 \text{ ms}^{-1}$$

➡ Velocity of 2nd ball after collision is equal to the velocity of first ball before collision

CASE-II If $m_1 = m_2 = m$ AND $u_2 = 0$



Before Collision   After collision

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

$$v_1 = \left(\frac{m - m}{m + m}\right)u_1 + \frac{2m(0)}{m + m}$$

$$v_1 = 0 + \frac{0}{2m}$$

$$v_1 = 0 \ ms^{-1}$$

➡ Velocity of 1st ball after collision become 0 when strike with stationary object.

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$$

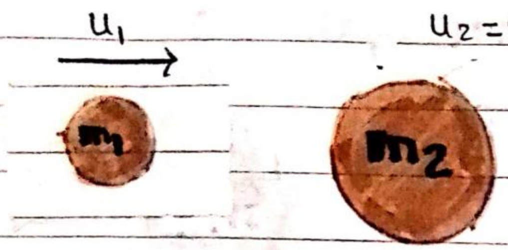
$$v_2 = \left(\frac{m - m}{m + m}\right)0 + \frac{2m(u_1)}{m + m}$$

$$v_2 = 0 + u_1$$

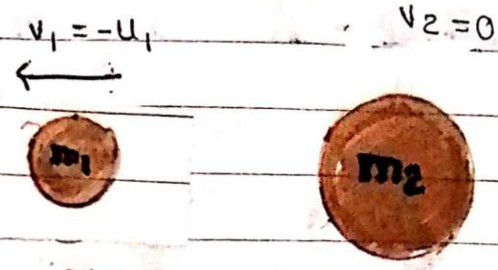
$$v_2 = u_1 \ ms^{-1}$$

➡ velocity of 2nd ball after collision is equal to the velocity of 1st ball before collision

CASE-III WHEN $m_1 << m_2$, $u_2 = 0$, $m_1 = 0$



Before Collision



After collision

$$V_1 = \frac{2m_2u_2}{m_1+m_2} + \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1$$

$$V_1 = \frac{2m_2(0)}{0+m_2} + \left(\frac{0-m_2}{0+m_2}\right)u_1$$

$$V_1 = 0 - u_1$$

$$V_1 = -u_1 \ ms^{-1}$$

→ velocity of 1st ball after collision is same as before collision only its direction reverses.

$$V_2 = \frac{2m_1u_1}{m_1+m_2} + \left(\frac{m_2-m_1}{m_1+m_2}\right)u_2$$

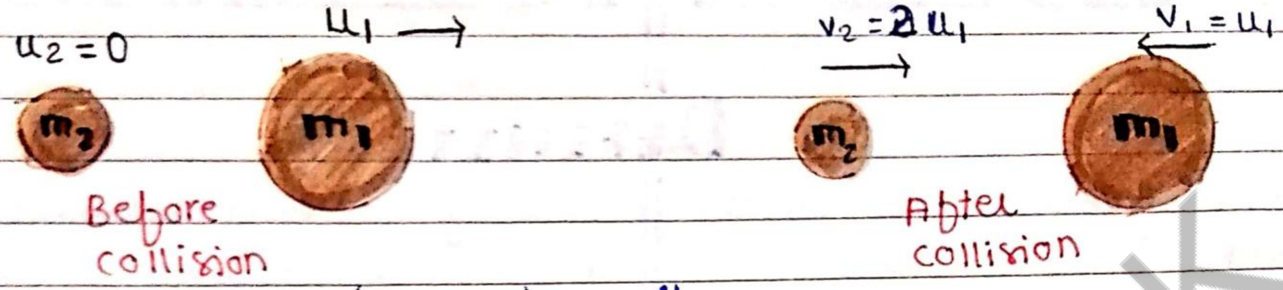
$$V_2 = \frac{2(0)u_1}{0+m_2} + \left(\frac{m_2-0}{0+m_2}\right)0$$

$$V_2 = 0$$

$$V_2 = 0 \ ms^{-1}$$

→ velocity of 2nd ball after collision remain zero.

CASE - IV If $m_1 >> m_2$ $u_2 = 0$ $m_2 = 0$



$$v_1 = \frac{2m_2u_2}{m_1+m_2} + \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1$$

$$v_1 = \frac{2(0)(0)}{m_1+0} + \left(\frac{m_1-0}{m_1+0}\right)u_1$$

$$v_1 = 0 + \frac{m_1}{m_1}(u_1)$$

$$v_1 = u_1 \ ms^{-1}$$

→ Velocity of 1st ball after collision will be same as initial velocity

$$v_2 = \frac{2m_1u_1}{m_1+m_2} + \left(\frac{m_2-m_1}{m_1+m_2}\right)u_2$$

$$v_2 = \frac{2m_1u_1}{m_1+0} + \left(\frac{0-m_1}{m_1+0}\right)0$$

$$v_2 = 2u_1 + 0$$

$$v_2 = 2u_1$$

→ velocity of 2nd ball after collision will be double of initial velocity of 1st ball

PROJECTILE MOTION [2 dimensional motion]

DEFINITION

Such a motion in which a body is thrown at certain angle in such a way that it moves with **constant horizontal velocity** and at the same time, it moves **freely under the action of gravity.** is called projectile motion.

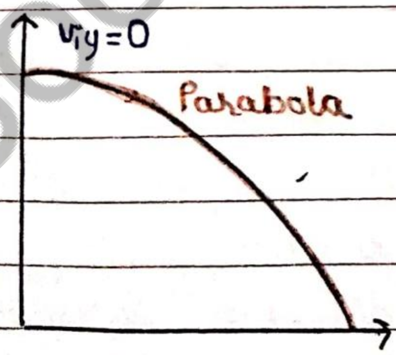
EXAMPLES

1. A football hit into air
2. A missile shot from a gun.
3. A stone thrown down the hill

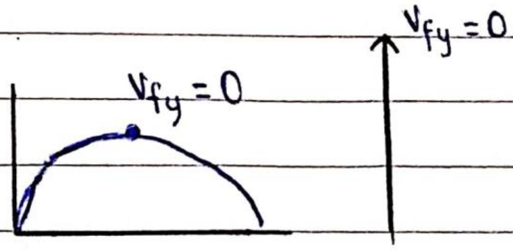
- V_{ix}, V_{fx} remains constant
- V_y changes uniformly
- $V_y = 0$ at highest point

X-axis : Air friction [neglected] ⇒ 0 acceleration ⇒ constant v
Y-axis : gravity ⇒ acceleration ⇒ change in velocity

HORIZONTAL COMPONENT	VERTICAL COMPONENT
$V_{ix} = V_{fx}$	V_{iy} = variable · $[V_{iy} = 0]$
$V_x = 0\ m/s$	$V_y \neq 0$
$a_x = 0\ m/s^2$	$a_y = -g$
$V_{ix} = V_{fx}$	$V_{iy} \neq V_{fy}$



TRAJECTORY : The path followed by projectile is called Trajectory.



HORIZONTAL DISTANCE COVERED BY PROJECTILE

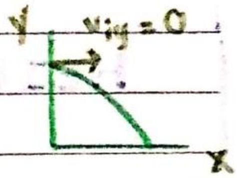
$S = v_i t + \frac{1}{2} a t^2$

$S_x = v_{ix} t + \frac{1}{2} a_x t^2$

$X = v_{ix} t + \frac{1}{2} (0) t^2$

$X = v_{ix} t$

VERTICAL DISTANCE COVERED BY PROJECTILE



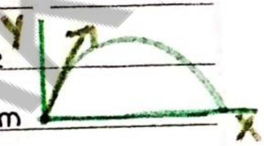
$S = v_{iy} t + \frac{1}{2} a_y t^2$

$Y = 0t + \frac{1}{2} g t^2$

$Y = \frac{1}{2} g t^2$ [when projectile is hit from certain height]

$Y = v_{iy} t - \frac{1}{2} g t^2$

[when projectile is targeted from ground level]



HORIZONTAL VELOCITY

$V_{fx} = v_{ix} + a_x t$

$V_{fx} = v_{ix} + (0) t$

$V_{fx} = v_{ix} = V_0 \cos\theta$

horizontal 'v' is constant so initial and final will be same ($v_{ix} = v_{fx}$)

VERTICAL VELOCITY

$V_{fy} = v_{iy} + a_y t$

$V_{fy} = V_0 \sin\theta + g t$ [under gravity]

$V_{fy} = V_0 \sin\theta - g t$ [against gravity]

MAGNITUDE OF VELOCITY

According to "PYTHAGORAS THEOREM"

$$V = \sqrt{V_{fx}^2 + V_{fy}^2}$$

$$V = \sqrt{(V_0 \cos\theta)^2 + (V_0 \sin\theta - g t)^2}$$

DIRECTION OF VELOCITY

$$\theta = \tan^{-1} \frac{V_{fy}}{V_{fx}}$$

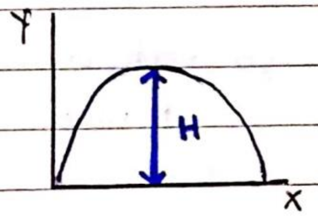
$$\theta = \tan^{-1} \frac{V_0 \sin\theta - g t}{V_0 \cos\theta}$$

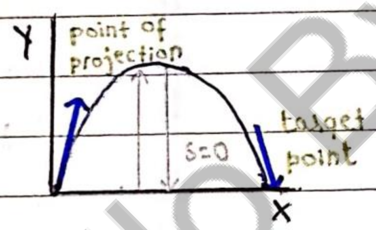
How to Remember the formulas:
- V_0 is in numerator
- g is present in denominator
- sin θ is present in all the 3 formulas

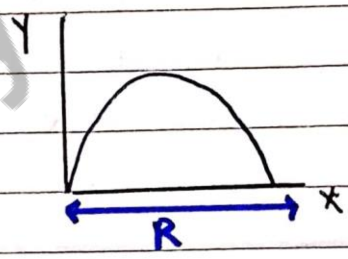
PROPERTIES OF PROJECTILE

HEIGHT	TIME OF FLIGHT	RANGE
DEFINITION		
Maximum vertical distance covered by projectile is called height of projectile	Time required by projectile from point of projection to target point is called time of flight.	Maximum horizontal distance covered by projectile is called range of projectile

DIAGRAM







DERIVATION

Height

According to 3rd eq of motion

$$2a_yS_y = V_{fy}^2 - V_{iy}^2$$

$$2(-g)H = 0 - (V_0\sin\theta)^2$$

$$-2gH = -V_0^2\sin^2\theta$$

$$H = \frac{V_0^2\sin^2\theta}{2g}$$

V_0 = original velocity

g = acceleration due to gravity

θ = angle of original velocity from x-axis.

Time of Flight

According to 2nd eq of motion

$$S_y = v_{iy}t + \frac{1}{2}a_yt^2$$

$$(0) = v_0\sin\theta\, t + \frac{1}{2}(-g)t^2$$

$$-V_0\sin\theta\, t = -\frac{1}{2}gt^2$$

$$T = \frac{2V_0\sin\theta}{g}$$

Summit time is equal to half of the total time of flight $T' = T/2$

$$T' = \frac{2V_0\sin\theta/g}{2}$$

$$T' = \frac{V_0\sin\theta}{g}$$

Range

According to 2nd eq of motion

$$S_x = V_{ix}t + \frac{1}{2}a_xt^2$$

$$R = V_{ix}t + \frac{1}{2}(0)t^2$$

$$R = V_{ix}t + 0$$

$$R = V_0\cos\theta\left(\frac{2V_0\sin\theta}{g}\right)$$

$(2\sin\theta\cos\theta = \sin 2\theta)$

$$R = \frac{V_0^2 2\sin\theta\cos\theta}{g}$$

$$R = \frac{V_0^2\sin 2\theta}{g}$$

MAXIMUM RANGE [R_{max}]

R_{max} When

$$\theta = 45^\circ$$
$$2\theta = 90^\circ$$

$$R = \frac{V_o^2 \sin 2(45^\circ)}{g}$$

2 projection angles for the same Range

$$R_{\theta_1} = R_{\theta_2}$$
$$45 \pm 5$$
$$R_{45+5} = R_{45-5}$$
$$\frac{V_o^2}{g}\sin 2(50) = \frac{V_o^2}{g}\sin 2(40)$$
$$\frac{V_o^2}{g}(0.98) = \frac{V_o^2}{g}(0.98)$$

At what angle the height and Range of projectile are same ?

$$H = R$$
$$\frac{V_o^2 \sin^2\theta}{2g} = \frac{V_o^2}{g}\sin 2\theta$$
$$\frac{\sin\theta . \sin\theta}{2} = 2\sin\theta\cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = 4$$
$$\tan\theta = 4$$
$$\theta = \tan^{-1}(4)$$
$$\theta = 76^\circ$$