SLO Based Quick Revision Notes

Vectors And Equilibrium

Vectors

"Vectors are such quantities that have both magnitude and direction"

Examples :-

Weight , velocity , force , displacement

Vector = Magnitude . Direction

$\vec{A} = |\vec{A}| \cdot \hat{A}$

Representation Of Vectors

Symbolic Representation

1. By bold face letter either capital OR small
F , **f**

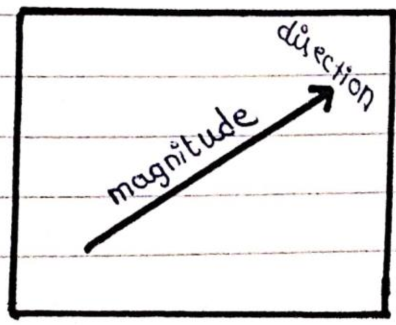
2. By simple face letter with an arrow over it
$\vec{F}$, $\vec{a}$

Graphical Representation

It is represented by an arrow

* **length** of the arrow = **magnitude**

* **arrow head** point the **direction**



MAGNITUDE OF VECTOR

→ Numeric value of vector is called its **magnitude**

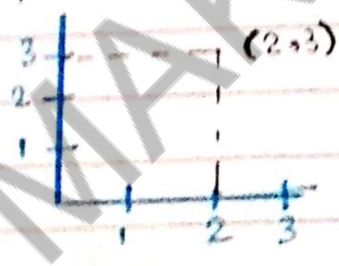
→ It is represented as
$|\vec{F}|$, $|\vec{B}|$

GEOMETRIC VECTORS

"Geometric vectors are those that are considered without reference to coordinate axes."

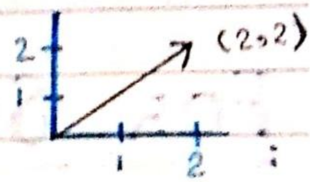
COORDINATE AXES

"coordinate axes are any sets of values that indicate the position of point in a given reference system."



ALGEBRIC VECTOR

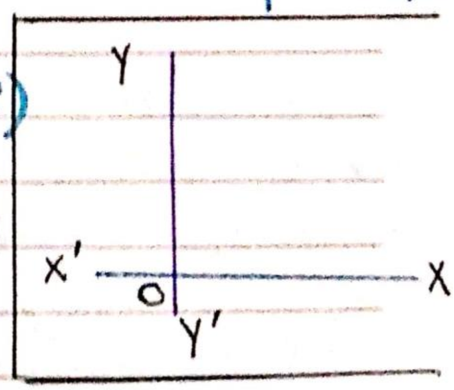
"Algebric vectors are related to coordinate system."



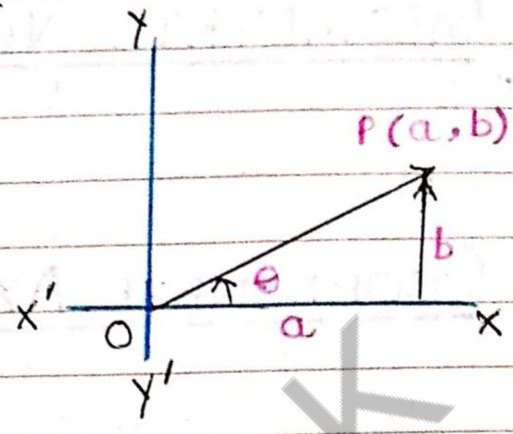
CARTESIAN COORDINATE SYSTEM

The cartesian coordinate system (number plane) consists of

- horizontal line called x-axis (XOX')
- vertical line called y-axis (YOY')
- The x-axis and y-axis intersect at a right angle (90°) at point 'O' called origin



For Plane

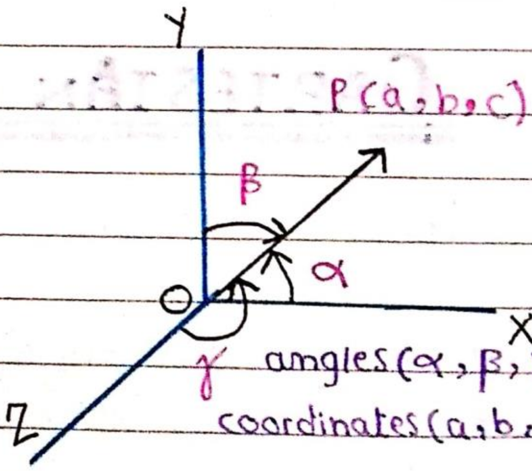


$\vec{r} = a\hat{x} + b\hat{y}$

$|r| = \sqrt{a_x^2 + b_y^2}$

- Two mutually ⊥ lines form plane.
- θ is the distance from x-axis of the vector.
- The rectangular components of vector in plane are 2
- In order to locate point in plane we need 2 coordinate.
- In order to locate point in plane, we need 1 angle.
- The angle is measured from +ive x-axis in anticlockwise direction.

For Space



angles (α, β, γ)

coordinates (a, b, c)

$a\hat{x} + b\hat{y} + c\hat{z}$

- Three mutually ⊥ lines form space.
- In order to locate a point in space we need 3 coordinates.
- In order to locate a point in space, we need 3 angles
- The rectangular components of vector in space are 3

Type	Definition	Formula/ Representation	Example	Explanation
PARALLEL VECTOR	Vectors that have same direction but unequal magnitude	$\vec{A}$ ⟶ $\vec{B}$ ⟶ $\lvert\vec{A}\rvert \neq \lvert\vec{B}\rvert$	like parallel forces.	When 2 vectors are in same direction and have same angle but vary in magnitude then it is known as ∥ vectors.
ANTI-PARALLEL VECTORS	vector which are in opposite direction and do not have same direction	⟶ $\vec{A}$ $\vec{B}$ ⟵	When tension is acting in upward direction and weight in down-ward direction	These vectors are exactly opposite to each other and their magnitude is also not same. The ∠ b/w them is 180°
NEGATIVE VECTOR	Vectors that have same magnitude but opposite direction	⟶ $\vec{A}$ ⟵ $\vec{A} = -\vec{A}$	$\lvert A\rvert = \lvert B\rvert$ equal magnitude $A\uparrow = B\downarrow$ opposite direction.	If vector AB point from left to right, then vector BA will point from right to left.

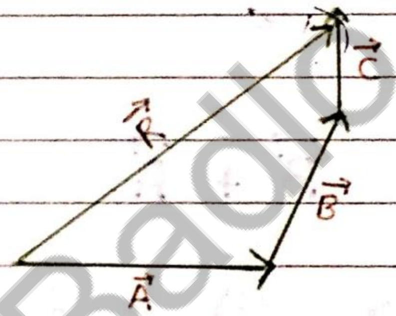
VECTOR ADDITION IS COMMUTATIVE

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

HEAD TO TAIL RULE

"The method of addition in which head of 1st vector is joined with tail of second vector and head of 2nd vector joins tail of 3rd vector and upto so on such that the head of resultant coincides with the head of last vector and its tail coincides with tail of first vector."

EXAMPLE



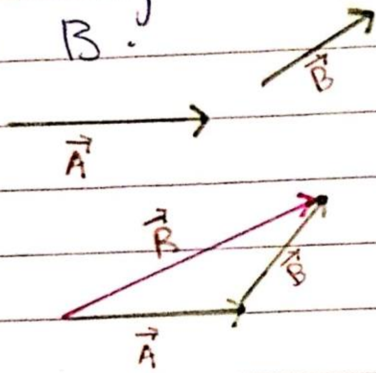
RESULTANT VECTOR

"The vector having combined effect of all vectors taken together is called resultant vector."

e.g $\vec{R}=\vec{A}+\vec{B}$

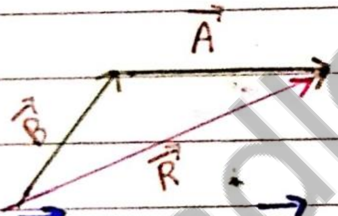
PROVE $\Rightarrow$ $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Consider 2 vectors $\vec{A}$ and $\vec{B}$. By head to tail rule, head of vector $\vec{A}$ joins with tail of vector B.



$$\vec{R} = \vec{A} + \vec{B} \qquad \text{———} \quad (i)$$

ALSO,



$$\vec{R} = \vec{B} + \vec{A} \qquad \text{———} \quad (ii)$$

COMPARING Eq (i) and (ii)

$$\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

HENCE,

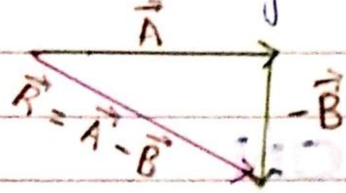
VECTOR ADDITION IS COMMUTATIVE

SUBTRACTION OF VECTOR

→ Subtraction of vector is like addition of vector.

→ It is the subtraction b/w one +ive vector and one -ive vector.

→ It is the addition of -ive vector.



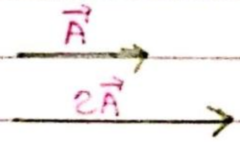
$$\vec{R} = \vec{A} + (-\vec{B})$$
$$\vec{R} = \vec{A} - \vec{B}$$

MULTIPLICATION OF VECTOR

CASE - I

When a vector is multiplied by +ive scalar number (e.g 2), its magnitude increases but direction remain same.

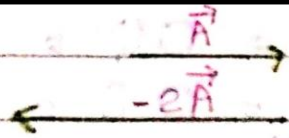
$2 \times \vec{A} \nearrow = 2A \nearrow$



CASE - II

When a vector is multipl...
Resultant vector increases ...
reverses.

$-2 \times \vec{A} \nearrow = -2A \swarrow$



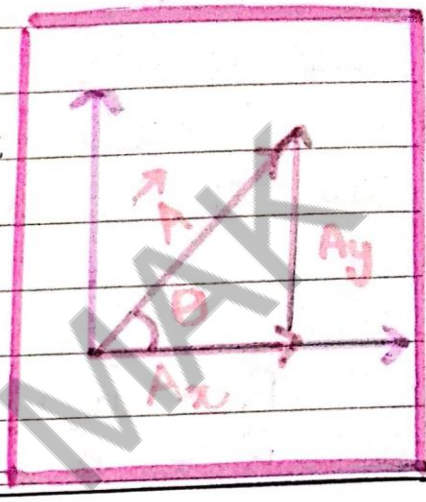
CASE - III

When a scalar physical quantity is multiplied by a vector, a new vector quantity is obtained.

$\vec{F} = m \times \vec{a}$
$\vec{P} = m \times \vec{v}$
$\vec{d} = \vec{v} \times \vec{t}$

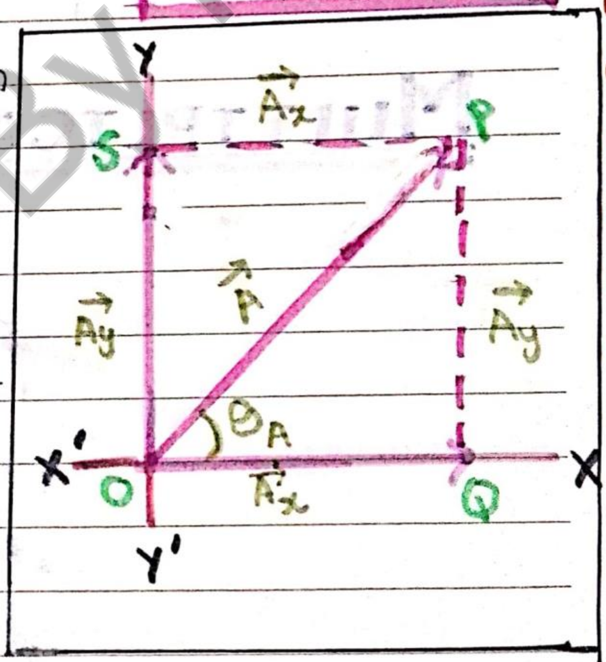
RESOLUTION OF VECTOR

"The process of splitting a vector into two or more vectors is called resolution of a vectors."



→ The only operation that can't be operated on vector is division.

EXPLANATION



* Consider a vector $\vec{A}$ in cartesian coordinate system which is represented by line OP making an angle θ
* Make the horizontal component of the vector by drawing ⊥ from point P on x-axis which meet axis at Q.
* Make the verticle component of vector by drawing ⊥ from point P on y-axis which meet at S

$\vec{A_x} = OQ$ = horizontal component of vector $\vec{A}$
$\vec{A_y} = OS$ = vertical component of vector $\vec{A}$
$\vec{A} = OP$ = vector A.

Addition Of Components

Sum of components is equal to the magnitude of the vector.

$$\vec{A} = \vec{A_x} + \vec{A_y} \quad \text{— (i)}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

Components As Vectors

$$\cos\theta = \frac{\text{base}}{\text{hyp}} \qquad \sin\theta = \frac{\text{Perp}}{\text{hyp}}$$

$$\cos\theta = \frac{A_x}{A} \qquad \sin\theta = \frac{A_y}{A}$$

$$A_x = A\cos\theta \quad \text{— (ii)} \qquad A_y = A\sin\theta \quad \text{— (iii)}$$

Putting eq (ii) and (iii) in eq (i)

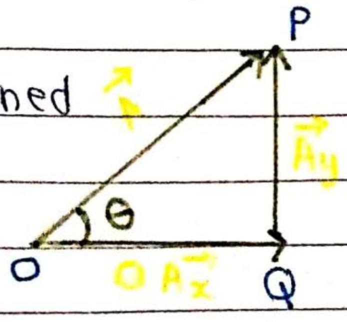
$$\vec{A} = \vec{A_x} + \vec{A_y}$$

$$\vec{A} = A\cos\theta + A\sin\theta$$

$$\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$$

Magnitude Of Vector

Magnitude of vector can be determined by using 'Pythagoras Theorem'



$$(\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2$$

$$\sqrt{(\text{hyp})^2} = \sqrt{(\text{base})^2 + (\text{perp})^2}$$

As,

hyp = A , base = A_x , perp = A_y

So,

$$|A| = \sqrt{A_x^2 + A_y^2}$$

For Space

$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

DIRECTION OF VECTOR

Direction of vector can be determined from its components.

$$\tan\theta = \frac{\text{Perp}}{\text{base}}$$

$$\tan\theta = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

ADDITION OF VECTORS BY RECTANGULAR COMPONENTS

COMPONENT OF VECTOR

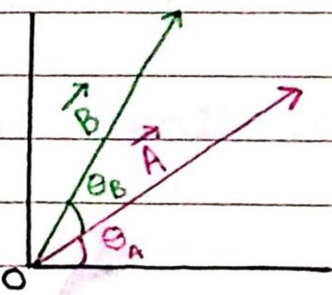
"Part of vector which gives its effective value in particular direction"

RECTANGULAR COMPONENT

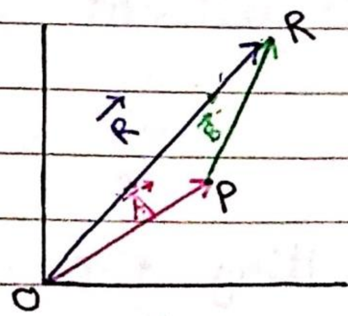
"The components of vector which are perpendicular to each other."

EXPLANATION

1. Consider 2 vectors $\vec{A}$ and $\vec{B}$, making an angle θ_A and θ_B with x-axis. We will add them according to head to tail rule.

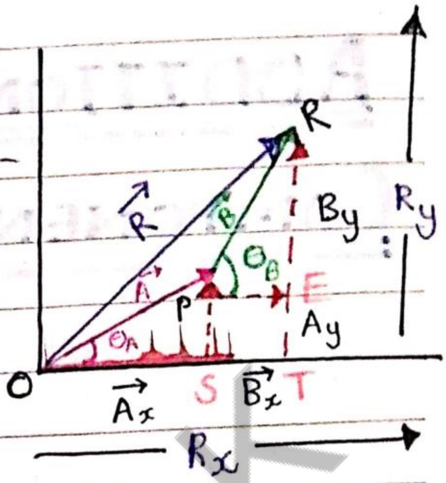


2. Representative lines for vector $\vec{A}$ and $\vec{B}$ are OP and PR. Join O and R which will be equal to resultant vector of $\vec{A}$ and $\vec{B}$



$OP = \vec{A}$ = vector A
$PR = \vec{B}$ = vector B
$OR = \vec{R}$ = resultant vector

3. Now we will resolve vector $\vec{A}$ and $\vec{B}$ by drawing perpendicular from their head.



$OS = A_x$ $\quad ST = B_x$

$ET = A_y$ $\quad ER = B_y$

MATHEMATICAL EXPLANATION

Vector A can be written as

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

Vector B can be written as

$$\vec{B} = B_x\hat{i} + B_y\hat{j}$$

Vector R in terms of rectangular components

$$\vec{R} = R_x\hat{i} + R_y\hat{j} \quad \text{— (i)}$$

Resultant vector along x-axis

$$\vec{R_x} = A_x\hat{i} + B_x\hat{i}$$

$$\vec{R_x} = (A_x + B_x)\hat{i}$$

Resultant vector along y-axis

$$\vec{R_y} = A_y\hat{i} + B_y\hat{j}$$

$$\vec{R_y} = (A_y + B_y)\hat{j}$$

Putting the value of R_x and R_y in eq (i)

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

Magnitude of $\vec{R}$ can be determined by PYTHAGORAS THEOREM

$$Hyp^2 = base^2 + perp^2$$
$$OR^2 = OT^2 + TR^2$$
$$R^2 = R_x^2 + R_y^2$$
$$R = \sqrt{R_x^2 + R_y^2}$$
$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

Direction of $\vec{R}$

$$\tan\theta = \frac{R_y}{R_x}$$
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

SUMMARY

1. Resolve the vectors into their components.
2. Find out R_x and R_y.
3. Write $\vec{R}$ in terms of x and y component.
4. Find the magnitude $[R = \sqrt{R_x^2 + R_y^2}]$ and direction $[\theta = \tan^{-1}(R_y/R_x)]$ of resultant vector.

ADDITION OF n-VECTORS

n-vector can be written as:

$$\vec{R} = (A_x + B_x + C_x + \dots)\hat{i} + (A_y + B_y + C_y + \dots)\hat{j}$$

For Magnitude

$$|R| = \sqrt{(A_x + B_x + C_x + \dots)^2 + (A_y + B_y + C_y + \dots)^2}$$

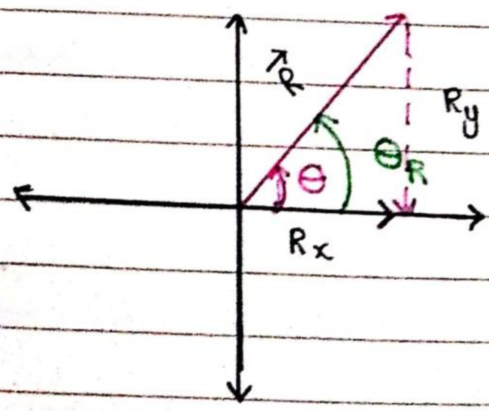
For Direction

$$\theta = \tan^{-1}\left[\frac{A_y + B_y + C_y + \dots}{A_x + B_x + C_x + \dots}\right]$$

DETERMINATION OF θ_R IN QUADRANTS

θ = angle b/w vector and neighbouring x-axis.
θ_R = angle b/w vector and +ive x-axis.

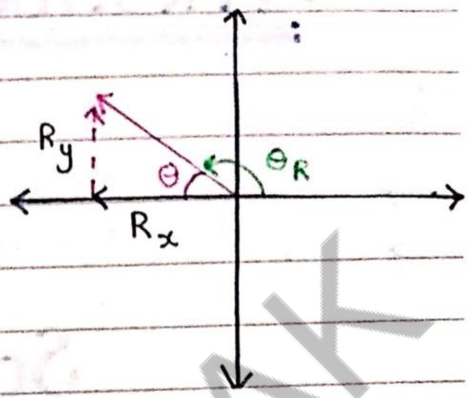
1st Quadrant



→ R_x = +ive
→ R_y = +ive

$$\boxed{\theta_R = \theta}$$

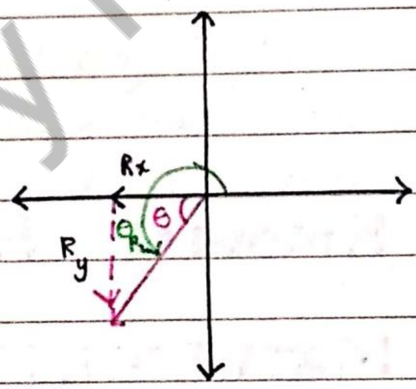
2^{nd} Quadrant



- → R_x = -ive
- → R_y = +ive

$$\boxed{\theta_R = 180 - \theta}$$

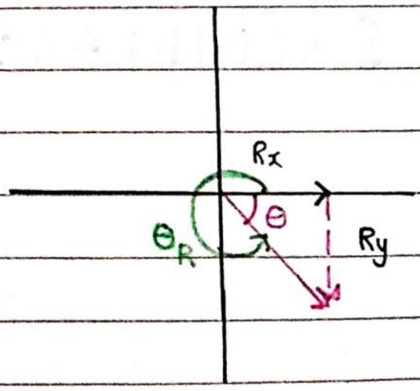
3^{rd} Quadrant



- → R_x = -ive
- → R_y = -ive

$$\boxed{\theta_R = 180 + \theta}$$

4^{th} Quadrant



- → R_x = +ive
- → R_y = -ive

$$\boxed{\theta_R = 360 - \theta}$$

(PRODUCT OF VECTORS)

There are 2 types to multiply a vector :

- → Scalar Product OR Dot Product.
- → Vector Product OR Cross Product.

SCALAR / DOT Product

STATEMENT:

"When a vector is multiplied by a vector and the resultant obtained is scalar physical quantity, so this product is called scalar product."

KNOWN AS:

Dot Product.

MATHEMATICALLY:

Scalar = vector • vector

$$C = \vec{A} \cdot \vec{B}$$

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

AB = magnitude

θ = angle b/w $\vec{A}, \vec{B}$

EXAMPLES:

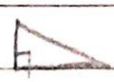
$$W = \vec{F} \cdot \vec{d}$$

$$P = \vec{F} \cdot \vec{V} \quad \text{(power)}$$

$$\phi = \vec{E} \cdot \vec{A}$$

$$K.E = \frac{1}{2} m (\vec{V} \cdot \vec{V})$$

1) cos θ

2) Base 0° 

Dot Product Hold Commutative Property

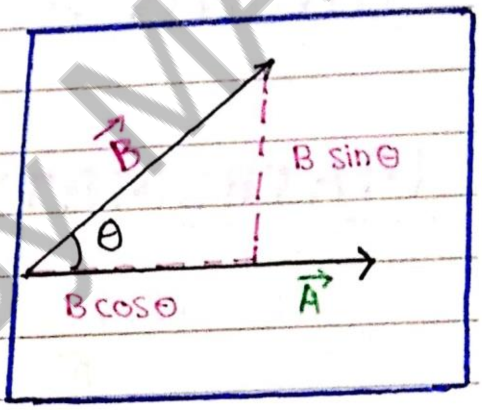
$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

$$AB\cos\theta = BA\cos\theta$$

* Addition in vectors
↓
Head to tail Rule

* Multiplication
tails are joined

L.H.S = $\vec{A}\cdot\vec{B}$ = AB cos θ



$\vec{A}\cdot\vec{B} = AB\cos\theta$

= (magnitude of A)(projection of $\vec{B}$ along $\vec{A}$)

OR

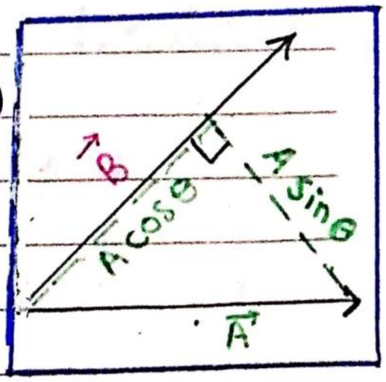
= (magnitude of A)(effective component of $\vec{B}$ along $\vec{A}$)

OR

= (magnitude of A) (component of $\vec{B}$ parallel to $\vec{A}$)

* Component of B ⇒ AB cos θ

R.H.S = $\vec{B}\cdot\vec{A}$ = BA cosθ



$\vec{B}\cdot\vec{A} = BA\cos\theta$

= (magnitude of B)(projection of $\vec{A}$ on $\vec{B}$)

OR = (magnitude of B)(effective component of $\vec{A}$ along $\vec{B}$)

OR = (magnitude of B)(component of $\vec{A} \parallel \vec{B}$)

* Component of A ⇒ BA cosθ

L.H.S = R.H.S

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

VECTOR PRODUCT

STATEMENT:

"When 2 vectors are multiplied and the resultant obtained is a vector quantity, so this type of product is known as vector product"

KNOWN AS:

Cross Product.

MATHEMATICALLY:

vector × vector = vector

$$\vec{A} \times \vec{B} = \vec{C}$$

$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$$

AB = magnitude

θ = angle b/w A and B

$\hat{n}$ = Unit vector ⊥ plane containing A and B

EXAMPLES

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{F} = q(\vec{v} \times \vec{B})$ Magnetic Force acting on charge in M.F

$\vec{F} = I(\vec{L} \times \vec{B})$ Force acting on current.

VECTOR PRODUCT IS ANTI-COMMUTATIVE

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$
$$AB \sin\theta \, \hat{n} \neq BA \sin\theta \, \hat{n}$$
$$AB_y \, \hat{n} \neq BA_y \, \hat{n}$$

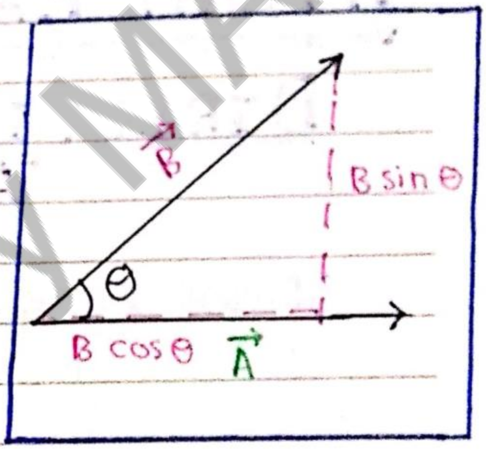
L.H.S → $\vec{A} \times \vec{B}$

$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$

= (magnitude of A)(effective component of B ⊥ A)

OR

= (magnitude of A)(projection of B ~~⊥~~ A)



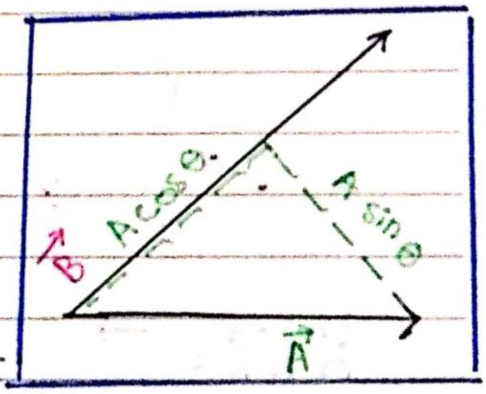
R.H.S → $\vec{B} \times \vec{A}$

$\vec{B} \times \vec{A} = BA \sin\theta \, \hat{n}$

= (magnitude of B)(effective component of A ⊥ B)

OR

= (magnitude of B)(projection of A ⊥ B)



$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

But

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

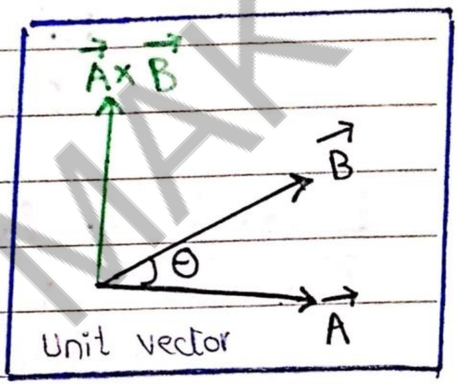
/ /

UNIT VECTOR

→ $\hat{n}$ is the unit vector

→ It gives the direction of $\vec{A} \times \vec{B}$.

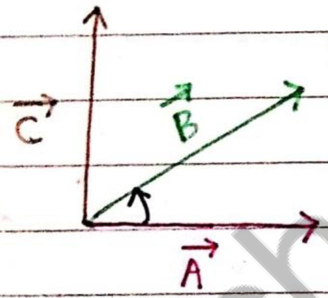
→ $\hat{n}$ is always perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

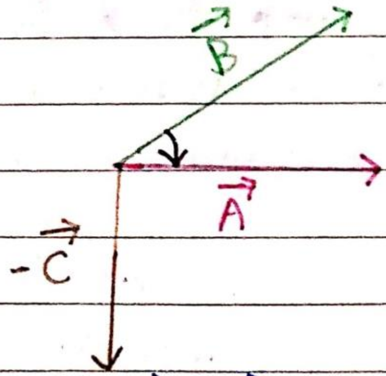


RIGHT HAND RULE

DIRECTION OF UNIT VECTOR $\hat{n}$

"Rotate fingers of right hand from first vector to second vector, the stretched thumb will point in the direction of resultant vector."





$\vec{A} \times \vec{B} = \vec{C}$

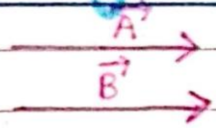
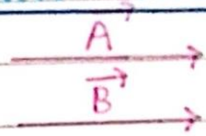
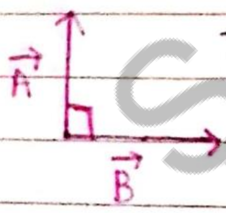
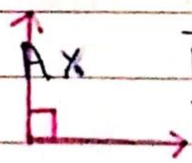
$AB \sin\theta \, \hat{n}$

$\vec{B} \times \vec{A} = -\vec{C}$

$-AB \sin\theta \, \hat{n}$

→ $\vec{C}$ is toward North

→ $\vec{C}$ is toward South

Scalar **PROPERTIES OF DOT PRODUCT**	Vector **PROPERTIES OF CROSS PRODUCT**
When $\theta = 0$ OR $\vec{A} \parallel \vec{B}$ OR max magnitude  $\vec{A}\cdot\vec{B} = AB\cos\theta$ $\vec{A}\cdot\vec{B} = AB\cos 0$ $\vec{A}\cdot\vec{B} = AB(1)$ $\vec{A}\cdot\vec{B} = AB$ (max)	**When $\theta = 0$ OR $\vec{A} \parallel \vec{B}$ OR min magnitude**  $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$ $\vec{A}\times\vec{B} = AB\sin(0)\,\hat{n}$ $\vec{A}\times\vec{B} = AB(0)$ $\vec{A}\times\vec{B} = 0$ min
For Unit Vector $\hat{i}\cdot\hat{i} = \lvert\hat{i}\rvert\lvert\hat{i}\rvert\cos(0)$ $\hat{i}\cdot\hat{i} = (1)(1)(1)$ $\hat{i}\cdot\hat{i} = 1$ Similarly: $\hat{j}\cdot\hat{j} = 1$ $\hat{k}\cdot\hat{k} = 1$	**For Unit Vector** $\hat{i}\times\hat{i} = \lvert\hat{i}\rvert\lvert\hat{i}\rvert\sin(0)\,\hat{n}$ $\hat{i}\times\hat{i} = (1)(1)(0)\,\hat{n}$ $\hat{i}\times\hat{i} = 0$ Similarly: $\hat{j}\times\hat{j} = 0$ $\hat{k}\times\hat{k} = 0$
When $\theta = 90°$ OR $\vec{A} \perp \vec{B}$ OR min magnitude  $\vec{A}\cdot\vec{B} = AB\cos\theta$ $\vec{A}\cdot\vec{B} = AB\cos 90$ $\vec{A}\cdot\vec{B} = AB(0)$ $\vec{A}\cdot\vec{B} = 0$ (min)	**When $\theta = 90°$ OR $\vec{A} \perp \vec{B}$ OR max magnitude**  $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$ $\vec{A}\times\vec{B} = AB\sin(90)\,\hat{n}$ $\vec{A}\times\vec{B} = AB(1)$ $\vec{A}\times\vec{B} = AB$ max

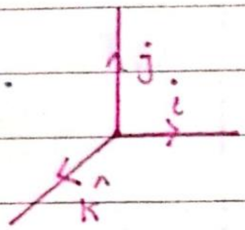
For Unit Vector

$\hat{i} \cdot \hat{j} = |\hat{i}||\hat{j}| \cos 90$

$\hat{i} \cdot \hat{j} = (1)(1)(0)$

$\hat{i} \cdot \hat{j} = 0$

$\hat{j} \cdot \hat{k} = 0$



For Unit vector

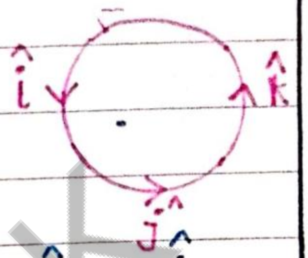
$\hat{i} \times \hat{j} = |\hat{i}||\hat{j}| \sin 90 \, \hat{n}$

$\hat{i} \times \hat{j} = (1)(1)(1)$

$\hat{i} \times \hat{j} = 1$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{i} = -\hat{k}$

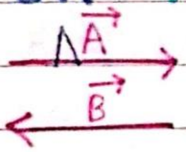


Similarly

$\hat{j} \times \hat{k} = \hat{i}$ $\quad$ $\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\quad$ $\hat{i} \times \hat{k} = -\hat{j}$

When $\theta = 180°$ OR $\vec{A} \uparrow\downarrow \vec{B}$ OR -ive magnitude



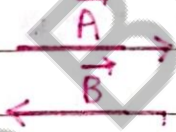
$\vec{A} \cdot \vec{B} = AB \cos \theta$

$A \cdot B = AB \cos(180)$

$\vec{A} \cdot \vec{B} = AB(-1)$

$\vec{A} \cdot \vec{B} = -AB$

When $\theta = 180°$ OR $\vec{A} \uparrow\downarrow \vec{B}$ OR +ive magnitude



$\vec{A} \times \vec{B} = AB \sin \theta \, \hat{n}$

$\vec{A} \times \vec{B} = AB \sin 180 \, \hat{n}$

$\vec{A} \times \vec{B} = AB (0) \, \hat{n}$

$\vec{A} \times \vec{B} = 0$

Self Product

$\vec{A} \cdot \vec{A} = AA \cos \theta$

$\vec{A} \cdot \vec{A} = AA \cos(0)$

$\vec{A} \cdot \vec{A} = A^2 (1)$

$\vec{A} \cdot \vec{A} = A^2$

FOR UNIT VECTOR

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

Self Product

$\vec{A} \times \vec{A} = AA \sin \theta \, \hat{n}$

$\vec{A} \times \vec{A} = AA \sin(0) \, \hat{n}$

$\vec{A} \times \vec{A} = A^2 (0)$

$\vec{A} \times \vec{A} = 0$

FOR UNIT VECTOR

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

PROPERTIES OF DOT PRODUCT

Multiplication of 2 different vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$
$$= A_xB_x(\hat{i}\cdot\hat{i}) + A_yB_y(\hat{j}\cdot\hat{j}) + A_zB_z(\hat{k}\cdot\hat{k})$$
$$= A_xB_x + A_yB_y + A_zB_z$$

★ $A_yB_y(\hat{i}\cdot\hat{j})$
$$= A_yB_y(ij\cos 90^\circ)$$
$$= A_yB_y\cos 90^\circ$$
$$= A_yB_y(0)$$
$$= 0$$

PROPERTIES OF CROSS PRODUCT

Multiplication of 2 different vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

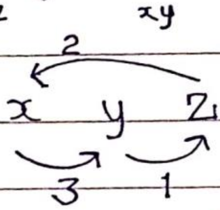
1st Method

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

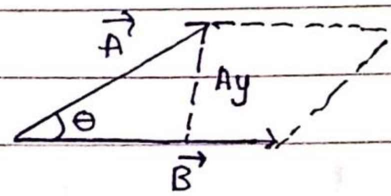
$$= (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - B_xA_y)\hat{k}$$

2nd Method

$$(A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$



AREA OF ||gm



$$\vec{A}\times\vec{B} = \text{Area of ||gm}$$
$$= b\times h$$
$$= B(A_y)$$
$$= BA\sin\theta$$

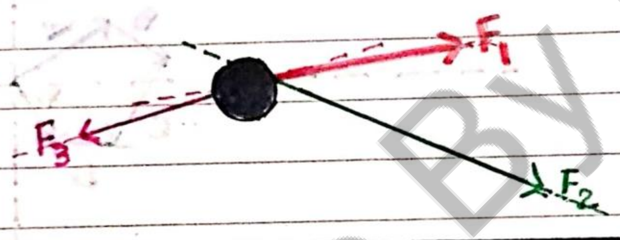
~(CONCURRENT FORCES)~

DEFINITION

"Two or more forces acting on a body having same line of action are called concurrent forces"

EXAMPLE

F_1, F_2 and F_3 are concurrent forces:



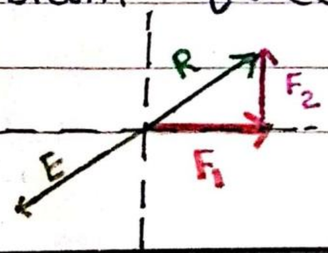
EQUILIBRANT FORCE

Concurrent forces can be balanced by a single force called equilibrant force.

⇒ Equilibrant force is equal but opposite to resultant force.

EXAMPLE

E is equilibrant force.



Torque Or Moment Of Force

Definition

"Turning effect of forces is called torque."

⇒ Also known as moment of force

Mathematical Explanation

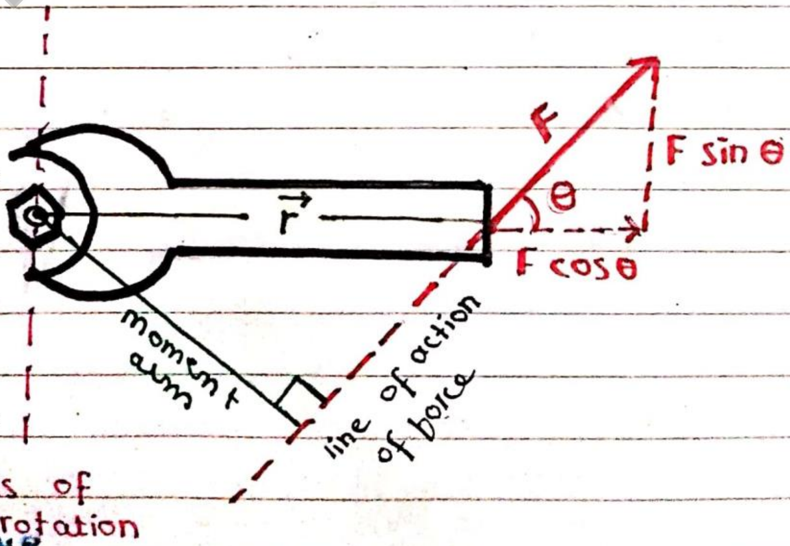
Torque is the **'vector product'** of force and moment arm

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$\vec{\tau} = rF \sin\theta$$
$$\vec{\tau} = rF(\sin\theta)\hat{n} \quad (\hat{n} = \text{unit vector})$$

SI Unit

SI Unit of torque is **Nm**

Explanation



F cos θ is along the direction of $\vec{F}$ so it does not cause rotation

F sin θ is perpendicular to position vector $\vec{r}$ so it is responsible for rotation

It is the effective component of force

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$\tau = \vec{r} \times \vec{F} \sin\theta$$

$\sin\theta$ can also be associated with moment arm

$$\tau = \vec{r} \sin\theta \times \vec{F}$$
$$\tau = F r \sin\theta$$

FACTORS

Torque depends on the following factors :

1. Magnitude of applied force $(\vec{F})$
2. Magnitude of position vector $(\vec{r})$
3. Angle b/w applied force and $\vec{r}$ (θ)

MAX $\vec{\tau}$

When $\theta = 90°$

$\tau = F r \sin\theta$

$\tau = F r \sin 90°$

$\tau = F r \ (1)$

$\tau = F r$

When $\theta = 270°$

$\tau = F r \sin\theta$

$\tau = F r \sin 270°$

$\tau = F r \ (-1)$

$\tau = -F r$

MIN $\vec{\tau}$

When $\theta = 0°$

$\tau = F r \sin\theta$

$\tau = F r \sin 0$

$\tau = F r \ (0)$

$\tau = 0$

When $\theta = 180°$

$\tau = F r \sin\theta$

$\tau = F r \sin 180°$

$\tau = F r \sin (0)$

$\tau = 0$

Direction Of Torque

Direction of torque is determined by 'right hand rule'

"If the fingers of right hand are curled from direction of $\vec{r}$ toward the direction of $\vec{F}$ then 'thumb points direction of torque'"

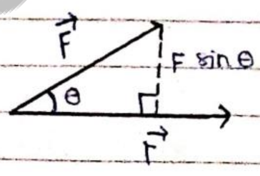
Clockwise Rotation

* If body rotates clockwise its 'τ' is downward
* It is also called clockwise torque
* It is taken +ive

Anticlockwise Rotation

If body rotates anticlockwise its 'τ' is upward
It is also called anticlockwise torque
It is taken +ive

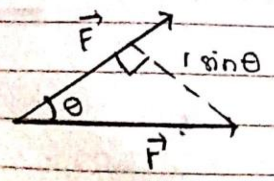
CASE-I



$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} = r F \sin\theta$

CASE-II



$\vec{\tau} = \vec{F} \times \vec{r}$

$\vec{\tau} = F r \sin\theta$

* $\vec{r}$ should be $\perp$ to $\vec{F}$

MOMENT OF COUPLE

DEFINITION

Couple is made of 2 parallel forces acting on the same body, equal in magnitude and opposite in direction and separated by a perpendicular distance.

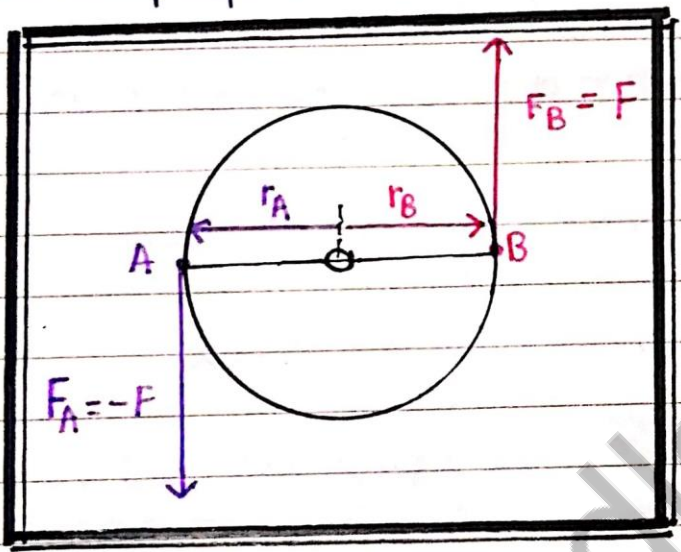
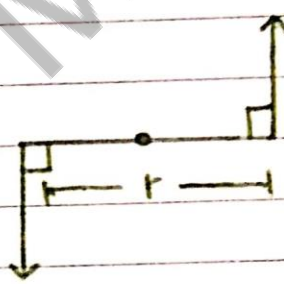


Diagram according to definition



MATHEMATICALLY

$F = -F$

$F_B = F_A$

Total torque due to F_A and F_B

$\vec{\tau} = \vec{r}_A \times \vec{F}_A + \vec{r}_B \times \vec{F}_B$

$\vec{\tau} = \vec{r}_A \times -\vec{F} + \vec{r}_B \times \vec{F}$

$\vec{\tau} = -\vec{r}_A \vec{F} + \vec{r}_B \vec{F}$

$\vec{\tau} = \vec{r}_B \vec{F} + \vec{r}_A \vec{F}$ OR $\vec{\tau} = [\vec{r}_B + (-\vec{r}_A)] \vec{F}$

(-ive sign indicates opposite direction)

$\vec{\tau} = \vec{r} \times \vec{F}$

(EQUILIBRIUM)

DEFINITION

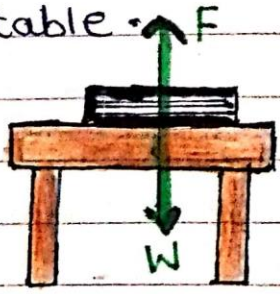
The state in which several forces and torques act together but there is no change in translational motion as well as rotational motion is called equilibrium.

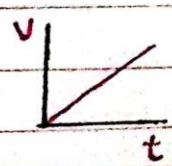
⇒ There is zero acceleration

⇒ The study of objects in equilibrium is called 'statics'

STATES OF EQUILIBRIUM

STATIC EQUILIBRIUM	DYNAMIC EQUILIBRIUM
DEFINITION	
When a body is at rest under action of several forces acting together, the body is said to be in static equilibrium.	When a body is moving at uniform velocity under action of several forces acting together, the body is in dynamic equilibrium
EXAMPLE	
A book resting on the table	Observing a car in motion that moves with constant velocity.



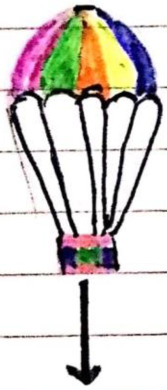


TYPES OF DYNAMIC EQUILIBRIUM

DYNAMIC TRANSLATIONAL EQUILIBRIUM

⇒ When a body is moving with uniform linear velocity, the body is said to be in dynamic translational equilibrium.

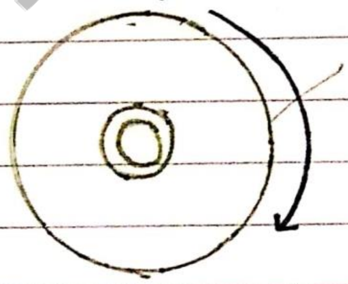
Example : a paratrooper falling down with constant velocity.



DYNAMIC ROTATIONAL EQUILIBRIUM

⇒ When a body is moving with uniform angular velocity, then body is in dynamic rotational equilibrium.

Example : CD rotating in CD player with constant angular velocity.



CONDITIONS OF EQUILIBRIUM

FIRST CONDITION

" The vector sum of all the forces acting on the body must be zero "

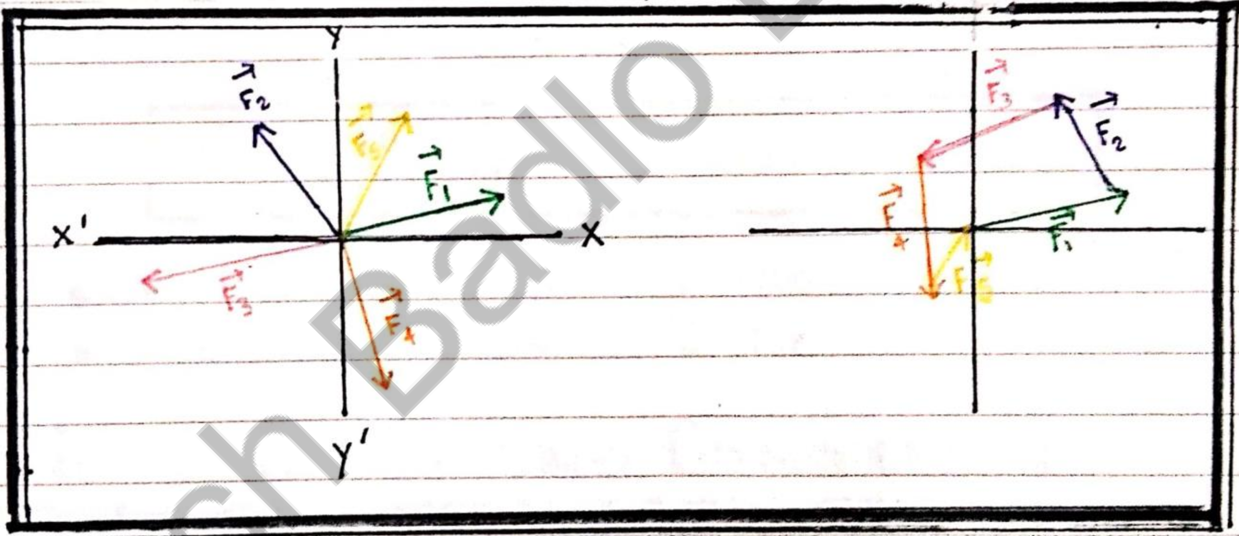
MATHEMATICALLY

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n = 0$$

OR

$$\boxed{\vec{F}_{net} = \sum_{i=1}^{i=n} \vec{F}_i = 0}$$

EXAMPLE



⇒ Resultant force must be a null vector.

⇒ The x and y component of force must also be equal to zero

$$\boxed{\begin{aligned} F_R &= \sum F_x \hat{i} + \sum F_y \hat{j} = 0 \\ F_x \hat{i} &= 0 \quad \text{AND} \quad F_y \hat{j} = 0 \end{aligned}}$$

SECOND CONDITION

"The vector sum of all the torques acting on the body must be equal to zero."

MATHEMATICALLY

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \dots + \vec{\tau}_n = 0$$

OR

$$\boxed{\vec{\tau}_{net} = \sum_{i=1}^{i=n} \vec{\tau}_i = 0}$$

CLOCKWISE / ANTCLOCKWISE TORQUE

Sum of clockwise torque and anticlockwise torque must be equal to zero

$$\boxed{\vec{\tau}_{net} = \sum \vec{\tau}_{clockwise} + \sum \vec{\tau}_{anticlockwise} = 0}$$

- Anticlockwise torque = +ive
- clockwise torque = -ive

NEED OF SECOND CONDITION

In case of couple, the first condition is satisfied but still the object has tendency to rotate. Therefore object is not in equilibrium with respect to rotation. Hence we need 2nd condition for complete equilibrium.

COMPLETE EQUILIBRIUM

⇒ When first condition is satisfied, there is no net force acting on the body. It represent translational equilibrium only

$$\boxed{\vec{F}_{net} = 0 \qquad \vec{a}_{net} = 0}$$

⇒ When second condition is satisfied, there is no net torque acting on the body. It represent rotational equilibrium only

$$\boxed{\vec{\tau}_{net} = 0 \qquad \vec{\alpha}_{net} = 0}$$

For complete equilibrium, both first and second condition must be satisfied.

$$\boxed{\text{Complete equilibrium} \Rightarrow \vec{F}_{net} = 0 + \vec{\tau}_{net} = 0}$$