QUICK NOTES

CHAPTER TWO

VECTORS AND EQUILIBRIUM

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VECTORS

Definition:

"Vector is a mathematical quantity having both magnitude and direction. Some quantities (such as weight, velocity or friction) require both a magnitude (or size) and a direction for a complete description and are called vectors."

→ REPRESENTATION

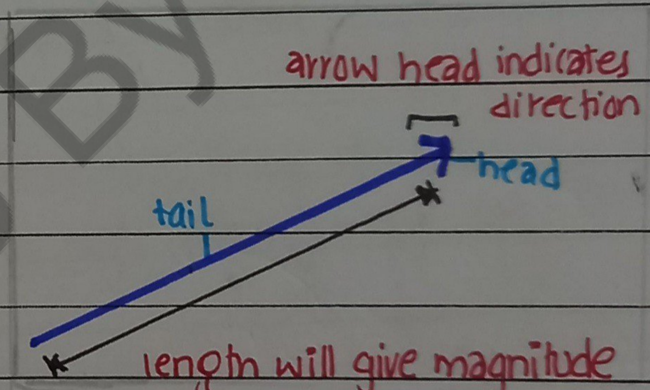
A. Symbolic representation

an arrow head or bar over it. [$\vec{A}$] Bold face either capital or small

⇒ to represent or indicate only magnitude of vector, following modification will be made,

$|\vec{A}|$

B. Graphical representation.



⇒ select a suitable scale. for example 1cm = 1N.

→ TYPES

→ **Geometric vectors**; are those which are not placed in a cartesian plane.

→ **Cart vectors**; are those which are placed in a cartesian plane.

→ **Unit vector**; is the dimensionless vector having magnitude 1 and is used to represent direction of a vector

e.g unit vector for vector $\vec{A}$ is $\hat{A}$ ← cap is representation of unit vector.

a vector $\vec{A}$ can be written as $\vec{A} = \underbrace{|A|}_{\text{magnitude}} \hat{A}$] direction.

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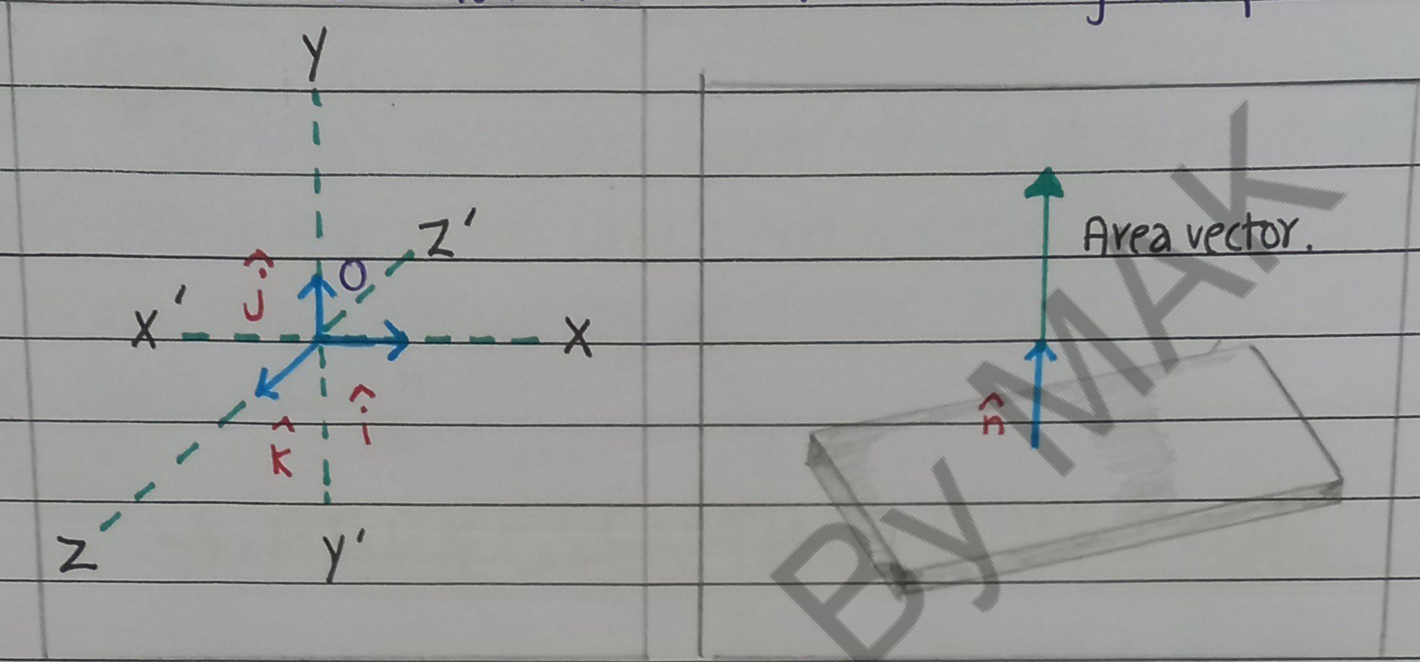
- Unit vector along x-axis is represented as $\hat{i}$.
- Unit vector along y-axis is represented as $\hat{j}$.
- Unit vector along z-axis is represented as $\hat{k}$.
- Unit vector $\hat{n}$ is normal or $\perp$ to a surface at a given point.



▲ Representations of some important unit vectors

> **Note:-** Unit vectors only indicate directions, they do not change magnitudes or dimensions of anything.

→ **Null vector:** it's a vector having arbitrary (random) direction and has zero magnitude.

- denoted as $\vec{0}$
- obtained by addition, subtraction and (x) cross multiplication of vectors. For example when a vector $\vec{A}$ is subtracted from itself : $\vec{A} + (-\vec{A}) = \vec{0}$

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CARTESIAN COORDINATE SYSTEM

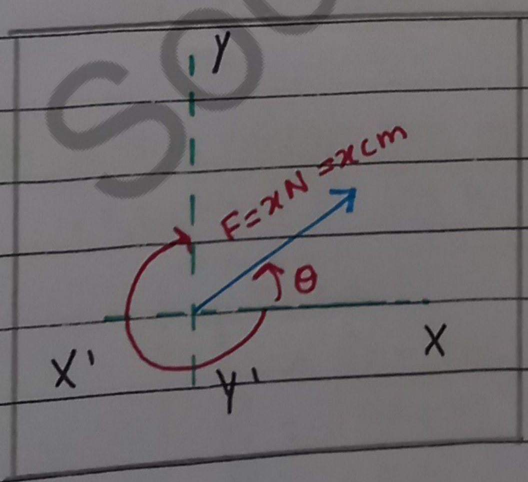
Definition:

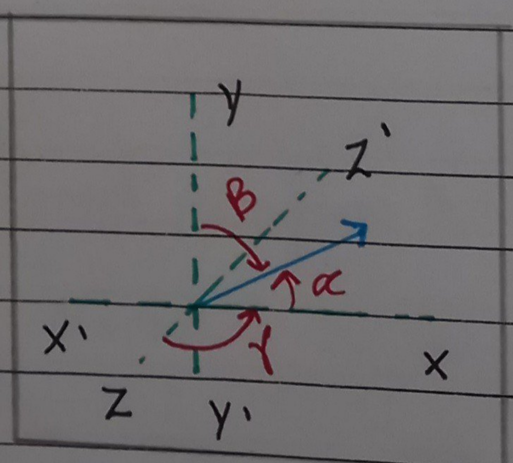
"Cartesian coordinate system consists of a horizontal line called the x-axis (XOX') and a vertical line called the y-axis (YOY') intersecting at a right angle 90° at a point O called origin."

→ REPRESENTING A VECTOR

→ In plane:	→ In space:
• Two dimensional representation	• 3 dimensional representation
• Two mutually perpendicular lines are drawn.	• 3 mutually perpendicular lines are drawn.
• Vector can be drawn along x and y axis.	• Vector can be drawn along x, y and z-axis.
• Angle is measured from +ve x-axis in anticlockwise direction.	• α, β and γ are angle symbols for x, y & z axis.

Diagram





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ADDITION OF VECTORS

⇒ Vectors may be added geometrically by drawing them to a common scale and placing them head to tail. Joining the tail of the first vector with the head of the last will give another vector which is the sum of these vectors called **resultant vector.**

→ ADDING TWO VECTORS

→ If two vectors are parallel

- They'll add like scalars.

$\longrightarrow$ $F_1 = 2N$

$\longrightarrow$ $F_2 = 2N$

$\vec{F_1} + \vec{F_2} = 4N$

$\theta = 0^\circ$

→ If two vectors are anti parallel

- They'll subtract like scalars.

$\longrightarrow$ $F_1 = 10N$

$\longleftarrow$ $F_2 = 8N$

$\vec{F_1} - \vec{F_2} = 2N$

$\theta = 180^\circ$

→ If two vectors are perpendicular

- Resultant $\vec{R}$ will be obtained through pythagoras theorem

$\uparrow$ $F_2 = 10N$

$\longrightarrow$ $F_1 = 5N$

$\vec{R} = \sqrt{F_1^2 + F_2^2} = \sqrt{25+100}$

$\vec{R} = 15N$

$\theta = 90^\circ$

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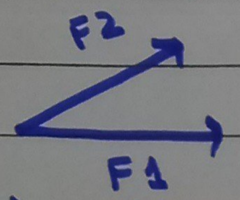
If vectors are at an angle θ

- Resultant is found through the formula given below.

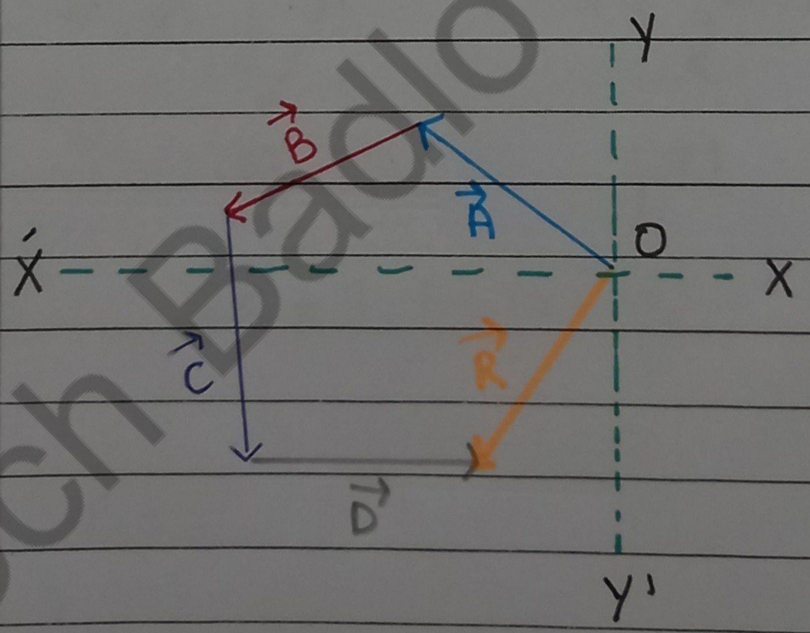


$$\vec{R} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

ADDING MORE THAN 2 VECTORS

- Addition can be extended to any number of vectors.

For example,
consider four vectors $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ in xy-plane.



- To determine the resultant measure the length of R and convert it back according to the given scale.

Vector Addition is commutative :-

vector addition holds commutative property i.e.,

$$\boxed{\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{D} + \vec{C} + \vec{B} + \vec{A}}$$

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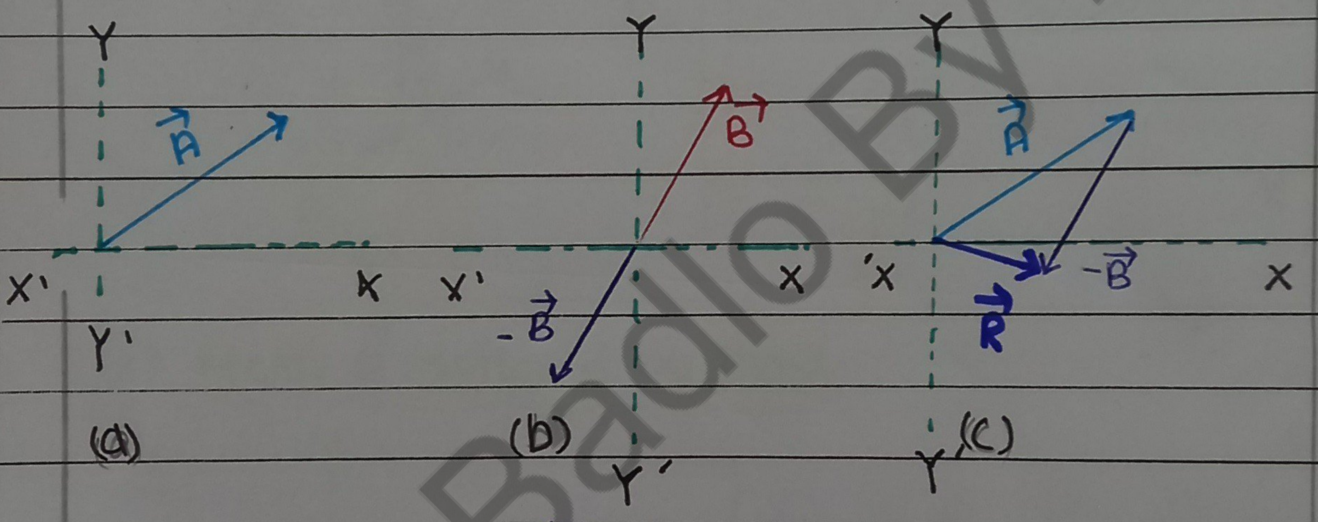
→ SUBTRACTION OF VECTORS

- Subtraction of one vector from another means addition of the negative vector with the first.
- First the negative of the vector which is being subtracted is determined
- Lastly rules of vector addition are followed (head to tail rule) to determine resultant vector.

For example

subtract vector $\vec{B}$ from $\vec{A}$.



$$\boxed{\vec{R} = \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}}$$

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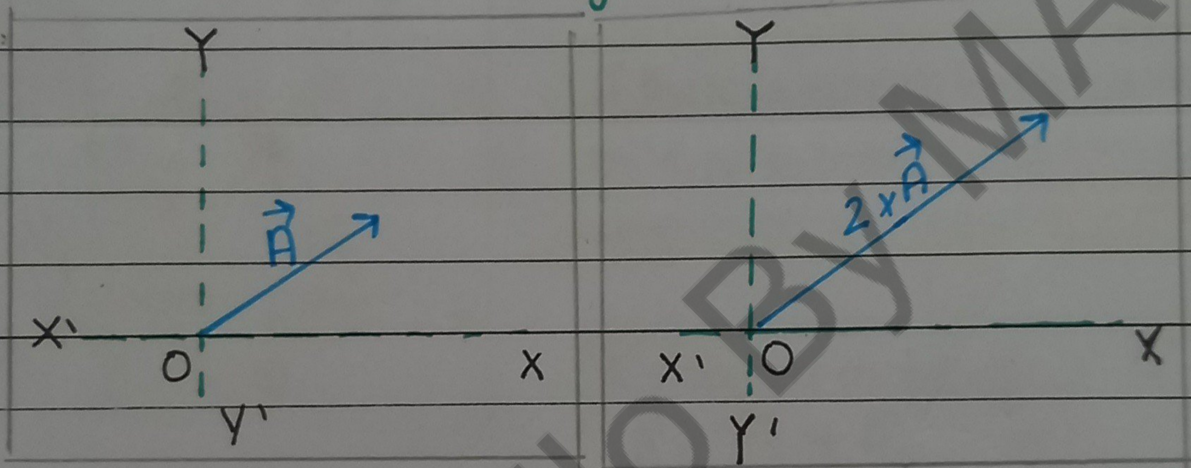
MULTIPLYING VECTOR BY A SCALAR

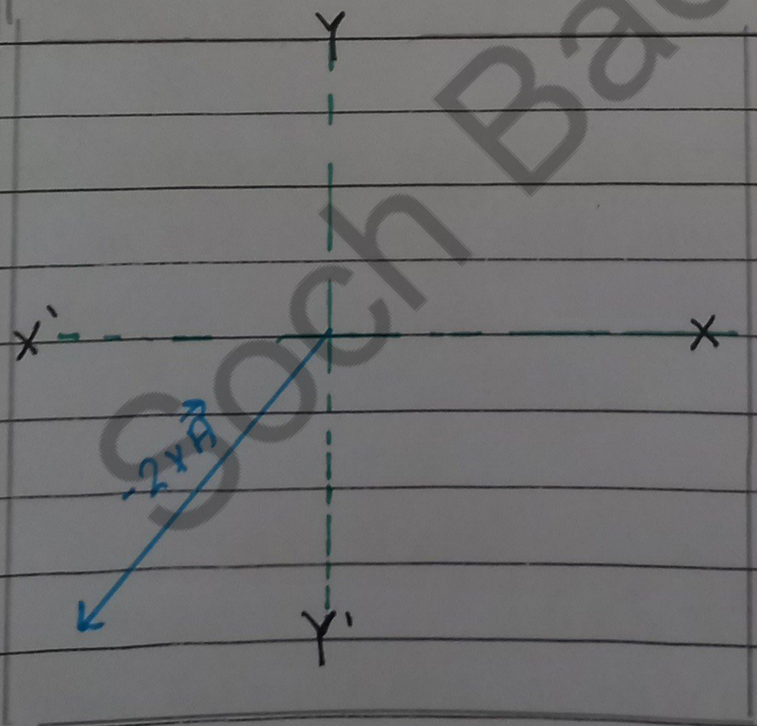
- If 'k' is a scalar multiplied with a vector $\vec{A}$, the length of $\vec{A}$ becomes 'k' times.
- The direction of $\vec{A}$ depends upon the value of 'k' i.e, if k is negtive, direction of $\vec{A}$ reverses.

→ For example

(i) vector $\vec{A}$ multiplied by $k=2$.



(ii) vector $\vec{A}$ multiplied by $k=-2$.



$k=0$ then $k\vec{A} = \vec{0}$

$k>0$ then $+k \cdot \vec{A} = \vec{kA}$

$k<0$ then $-k \cdot \vec{A} = -\vec{kA}$

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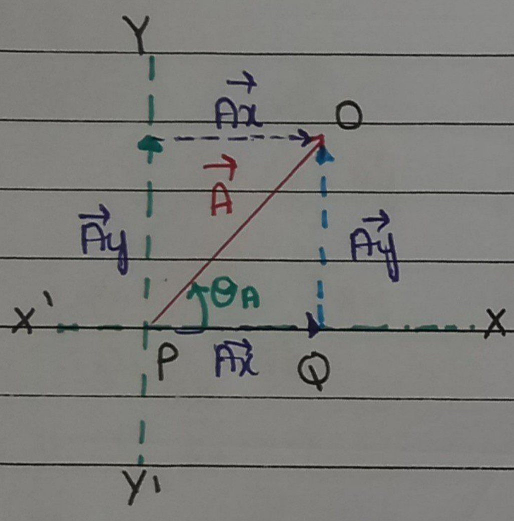
RESOLUTION OF VECTORS

Definition:

"The process of splitting a vector into two or more vectors is called resolution of a vector. The vectors so obtained are called the components of vectors."



vector represented in terms of its components.

If these components in which a vector is splited, perp. to each other then such components are called "rectangular components of vector"

Rectangular components of $\vec{A}$;

$\vec{A} = \vec{A_x} + \vec{A_y}$ or

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

→ Components expressed in terms of vectors

In the diagram given above, OPQ is a right angle triangle. therefore

$\cos\theta = \frac{Base}{hyp} = \frac{A_x}{A}$, $\sin\theta = \frac{Perp}{hyp} = \frac{A_y}{A}$

$A_x = A\cos\theta$, $A_y = A\sin\theta$

hence, $\boxed{\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}}$

From right ∠ΔOPQ, using Pythagorus theorem

$(hyp)^2 = (base)^2 + (perp)^2$

or $\boxed{|A| = \sqrt{A_x^2 + A_y^2}}$ and $\boxed{\theta = \tan^{-1}\frac{A_y}{A_x}}$

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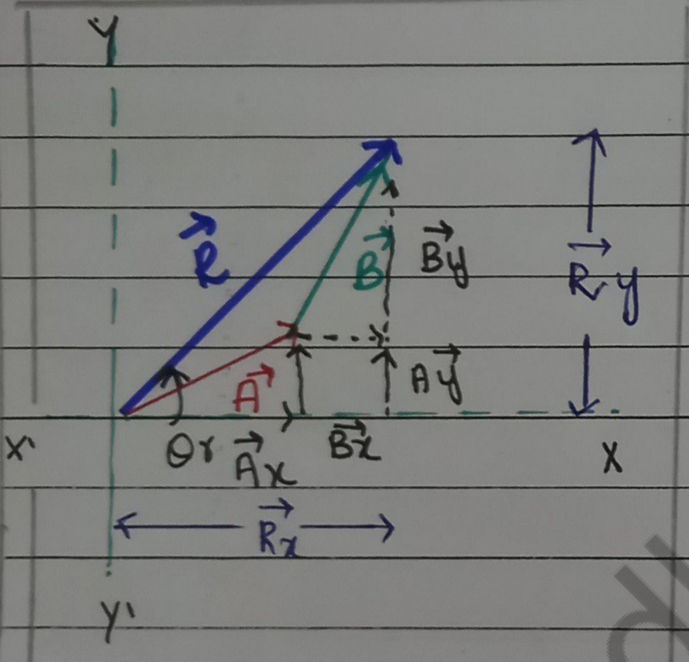
ADDITION OF VECTORS BY RECTANGULAR COMPONENTS

"The analytical method for addition of vectors is called addition of vectors by rectangular components."



$$\vec{R} = \vec{R}x + \vec{R}y \quad \ldots i$$
$$\vec{R}x = \vec{A}x + \vec{B}x \quad \ldots ii$$
$$\vec{R}y = \vec{A}y + \vec{B}y \quad \ldots (iii)$$
$$\vec{R} = Rx\,\hat{i} + Ry\,\hat{j} \quad \ldots (iv)$$

→ CALCULATIONS:

From the figure we get the points (i), (ii), (iii) and (iv). therefore

$$\vec{R} = (Ax + Bx)\hat{i} + (Ay + By)\hat{j} \qquad eq\ 1$$

By the rectangular components magnitude is

$$|R| = \sqrt{Rx^2 + Ry^2} \qquad eq\ 2$$

combining eq 1 & 2 we get

$$\boxed{R = \sqrt{(Ax + Bx)^2 + (Ay + By)^2}}$$

the direction is $\theta r = \tan^{-1}\dfrac{Ry}{Rx}$ or $\boxed{\tan^{-1}\dfrac{Ay + By}{Ax + Bx}}$

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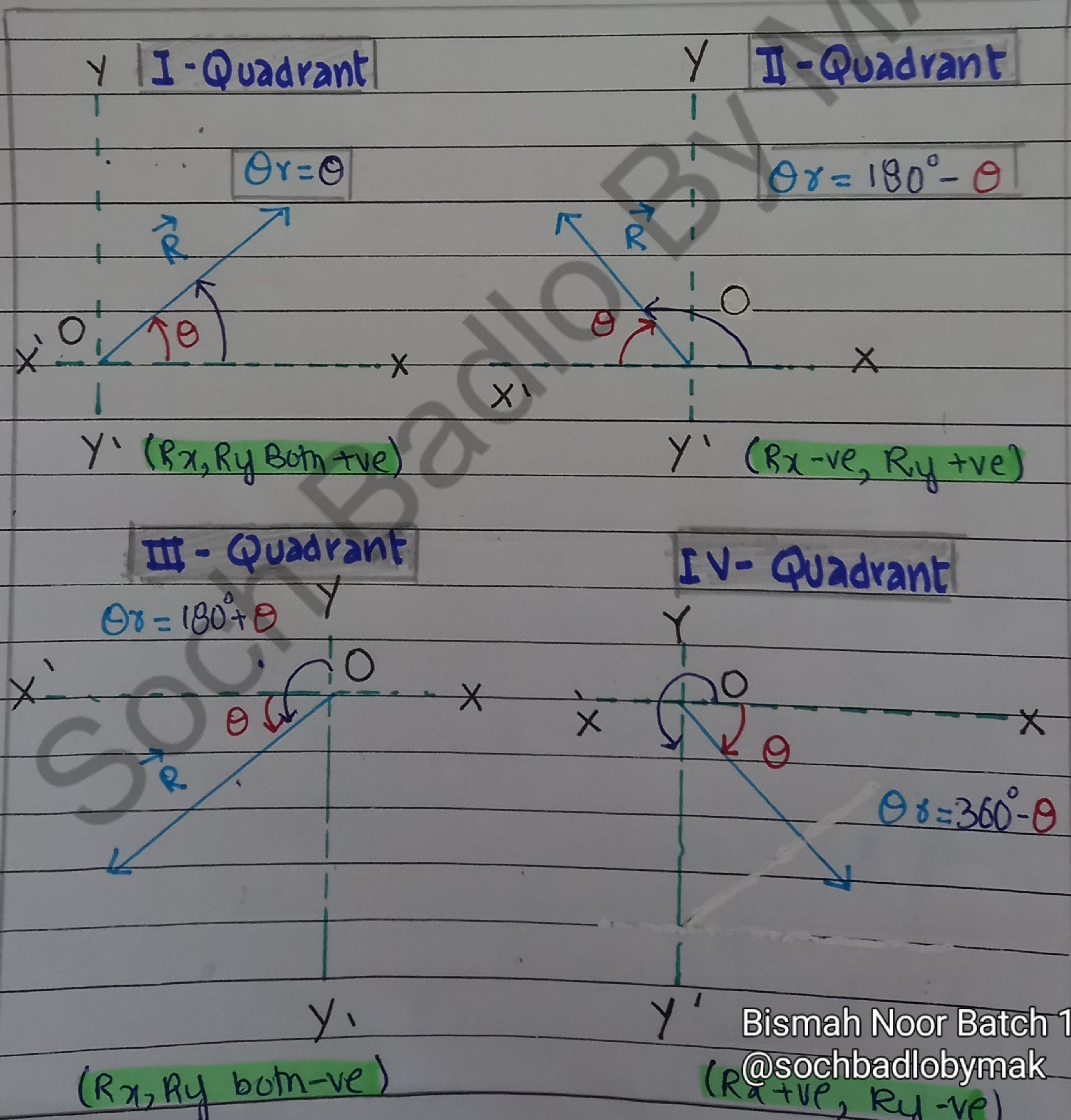
→ DETERMINATION OF ANGLE

- We determine angle 'θ' irrespective of signs of R_x and R_y.
- After determining 'θ', values are added or substracted from it with respect to the quadrant in which $\vec{R}$ is present in order to determine θ_R.

$$\boxed{\theta = \tan^{-1}\frac{R_y}{R_x}}$$

→ Figure:



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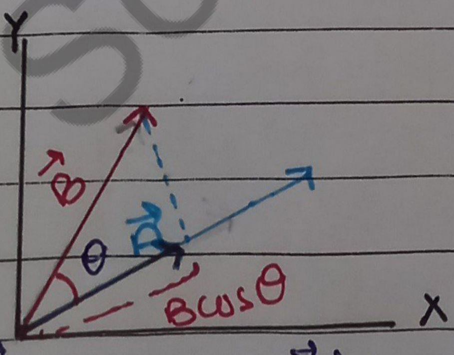
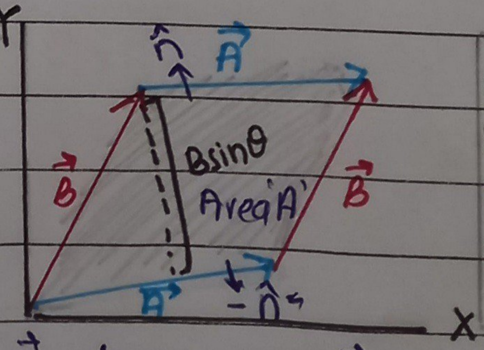
PRODUCT OF VECTORS

SCALAR PRODUCT	VECTOR PRODUCT
Definition	
• When a vector is multiplied by a vector and the resultant obtained is a scalar quantity, such type of vector multiplication is called scalar product.	• When a vector is multiplied by a vector and the resultant obtained is a vector quantity, such type of vector multiplication is called vector product.
$\boxed{\text{vector} \cdot \text{vector} = \text{scalar}}$	$\boxed{\text{vector} \times \text{vector} = \text{vector}}$
• Also called dot product.	• Also called cross product.
Mathematical form	
• $\vec{A}\cdot\vec{B} = AB\cos\theta$	• $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$
Commutative law	
• Obeys commutative law. i.e, $\vec{B}\cdot\vec{A} = BA\cos\theta$, $\vec{B}\cdot\vec{A} = AB\cos\theta$	• Doesn't obey commutative law i.e, $\vec{B}\times\vec{A} = -\vec{B}\vec{A}\sin\theta$, $\vec{A}\vec{B}\sin\theta = \vec{A}\times\vec{B}$ hence $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$ its anti commutative.
Examples	
• Examples are work, power etc.	• Examples are torque, angular momentum etc.
Diagram	



$\vec{A}\cdot\vec{B}$ = (magnitude of $\vec{A}$) (component of B parallel to $\vec{A}$) = $AB\cos\theta$



Area of ||gm = B×H
Base×Height
= vector product

$\vec{A}\times\vec{B}$ = (magnitude of $\vec{A}$) component of $\vec{B} \perp$ to $\vec{A}$) = $AB\sin\theta\,\hat{n}$

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PROPERTIES

DOT PRODUCT

- When $\theta = 0$ or $\vec{A} \parallel \vec{B}$ or maximum magnitude.

$\longrightarrow \vec{A}$

$\longrightarrow \vec{B}$

- $\vec{A} \cdot \vec{B} = AB\cos\theta$
- $\vec{A} \cdot \vec{B} = AB\cos(0°)$
- $\vec{A} \cdot \vec{B} = AB(1)$

$\vec{A} \cdot \vec{B} = AB(max)$

- When $\theta = 90°$ or $\vec{A} \perp \vec{B}$ or minimum magnitude.

$\vec{A} \uparrow$, $\rightarrow \vec{B}$

- $\vec{A}\vec{B} = AB\cos\theta$

$= AB\cos 90$

$= AB(0)$

$\vec{A} \cdot \vec{B} = AB(min)$

When $\theta = 180°$ or $\vec{A} \uparrow\downarrow B$ or negative magnitude

$\longrightarrow \vec{A}$

$\vec{B} \longleftarrow$

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\vec{A} \cdot \vec{B} = AB\cos(180)$

$\vec{A} \cdot \vec{B} = AB(-1)$

$\vec{A} \cdot \vec{B} = -AB$

CROSS PRODUCT

- When $\theta = 0$ or $\vec{A} \parallel \vec{B}$ or minimum magnitude

$\longrightarrow \vec{A}$

$\longrightarrow \vec{B}$

- $\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$
- $\vec{A} \times \vec{B} = AB\,\sin 0\,\hat{n}$
- $\vec{A} \times \vec{B} = AB(0)\hat{n}$

$\vec{A} \times \vec{B} = AB(min).$

- When $\theta = 90°$ or $\vec{A} \perp \vec{B}$ or maximum magnitude

$\vec{A} \uparrow$, $\rightarrow \vec{B}$

- $\vec{A} \times \vec{B} = AB\sin 90\hat{n}$

$= AB\,~~\sin~~\,1\hat{n}$

$= AB(1)\hat{n})$

$\vec{A} \times \vec{B} = AB(max)$

- When $\theta = 180°$ or $\vec{A} \uparrow\downarrow \vec{B}$ or positive magnitude.

$\longrightarrow \vec{A}$

$\vec{B} \longleftarrow$

$\vec{A} \times \vec{B} = AB\sin\theta\,\vec{n}$

$\vec{A} \times \vec{B} = AB\sin(180)\hat{n}$

$\vec{A} \times \vec{B} = AB(0)\hat{n}$

$\vec{A} \times \vec{B} = 0$

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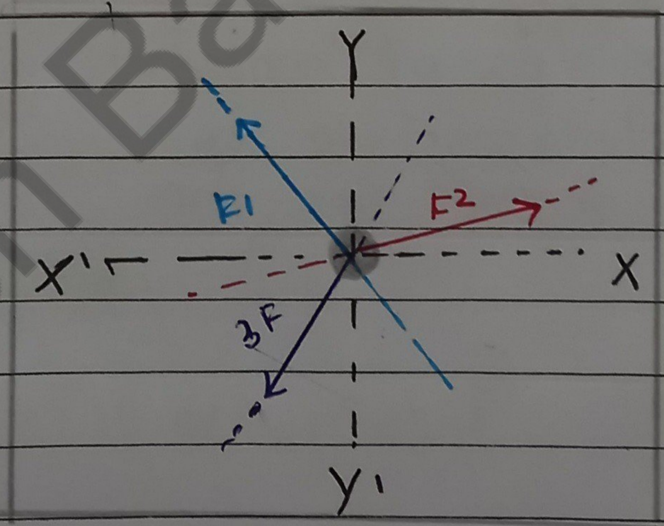
Self Product	Self Product
$\vec{A}\cdot\vec{A} = AA\cos\theta$	$\vec{A}\times\vec{A} = (A \times A)\sin\theta\,\hat{n}$
$\vec{A}\cdot\vec{A} = A^2\cos\theta$	$\vec{A}\times\vec{A} = A^2\sin 0\,\hat{n}$
$\vec{A}\cdot\vec{A} = A^2\cos 0$	$\vec{A}\times\vec{A} = A^2(0)\hat{n}$
$\vec{A}\cdot\vec{A} = A^2 1$	$\vec{A}\times\vec{A} = A^2(0)$
$\vec{A}\cdot\vec{A} = A^2$	$\vec{A}\times\vec{A} = 0$

CONCURRENT FORCES

⇒ **Definition:**

"When two or more forces are acting on a body and the line of action of these forces pass through a common point, the forces are said to be concurrent forces"

⇒ **Diagram**



Two or more concurrent forces can be balanced by a single force called equilibrant F.

⇒ The line of action of these forces (represented by dotted lines) passes through the same point. Hence these forces are said to be concurrent.

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RANDOM MCQ POINTS

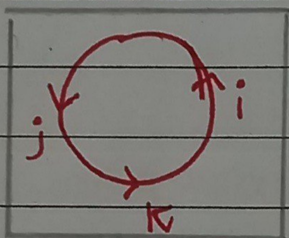
⇒ Dot product of two parallel unit vectors is equal to one. $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

⇒ Dot product of two perpendicular unit vectors is zero. $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 0$

⇒ Cross product of two parallel unit vectors is zero

⇒ Cross product of two perpendicular unit vectors can be determined as

$\hat{i} \times \hat{j} = \hat{k}$
$\hat{k} \times \hat{i} = \hat{j}$
$\hat{k} \times \hat{j} = \hat{i}$



TORQUE OR MOMENT OF FORCE

Definition:- Turning effect of force is called torque or Cross product of moment arm and force is called torque.

⇒ **Mathematical form:-**

$$\boxed{\tau = \vec{r} \times F \text{ or } T = rF\sin\theta\, \hat{n}}$$

⇒ Torque is a vector quantity.

⇒ **Unit**

It has units of Newton metres (N m)

Moment arm:- Perpendicular distance b/w pivot point and force

Pivot point:- A point where a body rotates or tends to rotate.

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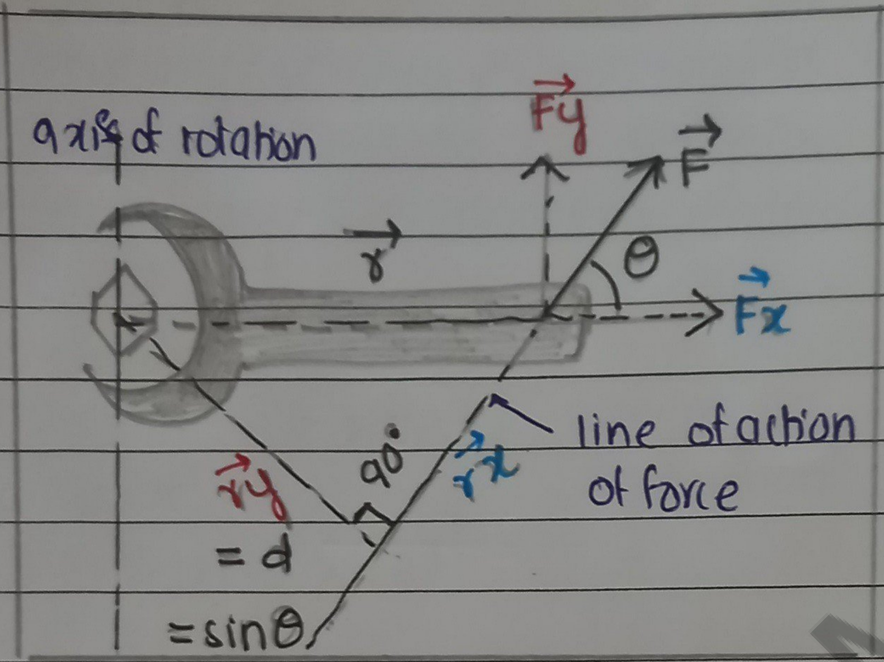

▲ Torque acting on a wrench.

→ Effective component of $\vec{r}$ is $\vec{ry}$ ($\vec{ry}$ = d)
hence d is the effective component of moment arm.

→ Torque is produced is a product of the magnitude of distance to the point of application of force ($\vec{r}$) and the perpendicular component of the force ($F\sin\theta$).

$$\vec{\tau} = (r)(F\sin\theta)\hat{n}$$
$$\text{or } \vec{\tau} = \vec{r} \times \vec{F}$$

The second way to interpret equation is to associated the sine function w/ this distance as $r\sin\theta$ & multiply it with magnitude of force.

→ $\vec{\tau} = (F)(r\sin\theta)\hat{n}$ or $\vec{\tau} = \vec{F} \times \vec{r}$

since $d = r\sin\theta$, $|\vec{\tau}| = Fd$

FACTORS EFFECTING TORQUE:

A. Magnitude of applied force $\vec{F}$.

B. Magnitude of position vector $\vec{r}$

C. Angle between applied force and position vector θ.

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MOMENT OF COUPLE

Definition "Couple is defined as two parallel forces that have the same magnitude but opposite directions and are separated by a distance (perpendicular) d."

→ Mathematically:-

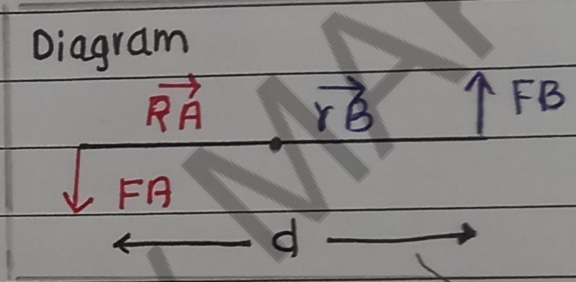
$\vec{\tau} = \vec{rA} \times \vec{FA} + \vec{rB} \times \vec{FB}$

$\vec{FA} = \vec{FB} = \vec{F}$

$\vec{\tau} = \vec{F} \times \vec{rB} + (-\vec{F} \times \vec{rA})$

$\vec{\tau} = \vec{F}(\vec{rB} - \vec{rA})$

$\vec{\tau} = \vec{F}(\vec{r})$ (negative sign only represents direction and there fore it doesn't subtract)



$\vec{\tau} = \vec{r}\vec{F}\sin\theta$

EQUILIBRIUM

Definition:-

"It's the state of a body in which under the action of several forces and torques acting together there is no change in translation motion as well as rotation is called equilibrium."

-Main Points:

The study of objects at equilibrium is called statics

- A body in equilibrium doesn't accelerate.
- A body in equilibrium not net force acting on it.
- A body in equilibrium has no net torque acting on it.
- A body in equilibrium can travel w/ uniform linear and angular velocity.

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TYPES.

⇒ **Static equilibrium :-** When a body is at rest under the action of several forces acting together the body is said to be in static equilibrium.

Example: Book resting on a table under action of mg and reaction of force is in static equilibrium.

⇒ **Dynamic equilibrium:-** When a body is moving at uniform velocity under the action of several forces acting together on it, the body is said to be in dynamic equilibrium.

Example: A paratrooper falling down with constant velocity is said to be in dynamic equilibrium.

Dynamic equilibrium has further two types

(i) Dynamic Translational Equilibrium :-
When a body is moving w/ uniform linear velocity the body is said to be in dynamic translational equilibrium.

Example:
Apple falling down a tree w/ uniform velocity is in this type of equilibrium.

(ii) Dynamic Rotational Equilibrium :-
When a body is moving w/ uniform angular velocity the body is said to be in dynamic rotational equilibrium.

Example:
Compact Disk (CD) rotating in CD player with constant angular velocity is in dynamic rotational equilibrium.

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CONDITIONS OF EQUILIBRIUM

1^{st} Condition of Equilibrium	2^{nd} Condition of equilibrium
Definition	
When the vector sum of all the forces acting on a body is ZERO, then the first condition is satisfied.	For the 2^{nd} equilibrium to be satisfied, anticlockwise torque acting on a body must be equal to clockwise torque.
Mathematically	
$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 .. \vec{F}_n = 0$ or $\boxed{F_{net} = \sum_{i=1}^{i=n} \vec{F_i} = 0}$	$\tau_{net} = \vec{\tau}_1 + \vec{\tau}_2 \cdots \vec{\tau}_n = 0$ or $\boxed{\vec{\tau}_{net} = \sum_{i=1}^{i=n} \vec{\tau}_i = 0}$
Example	
• A book resting on a table, • A car travelling with uniform velocity satisfy 1^{st} condition of equilibrium.	• A see-saw balanced mid air is said to satisfy 2^{nd} condition of equilibrium

COMPLETE EQUILIBRIUM

A complete equilibrium is the one which satisfies both conditions of equilibrium.

Mathematically:

$F_{net} = 0$ as well as $\vec{\tau}_{net} = 0$.

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